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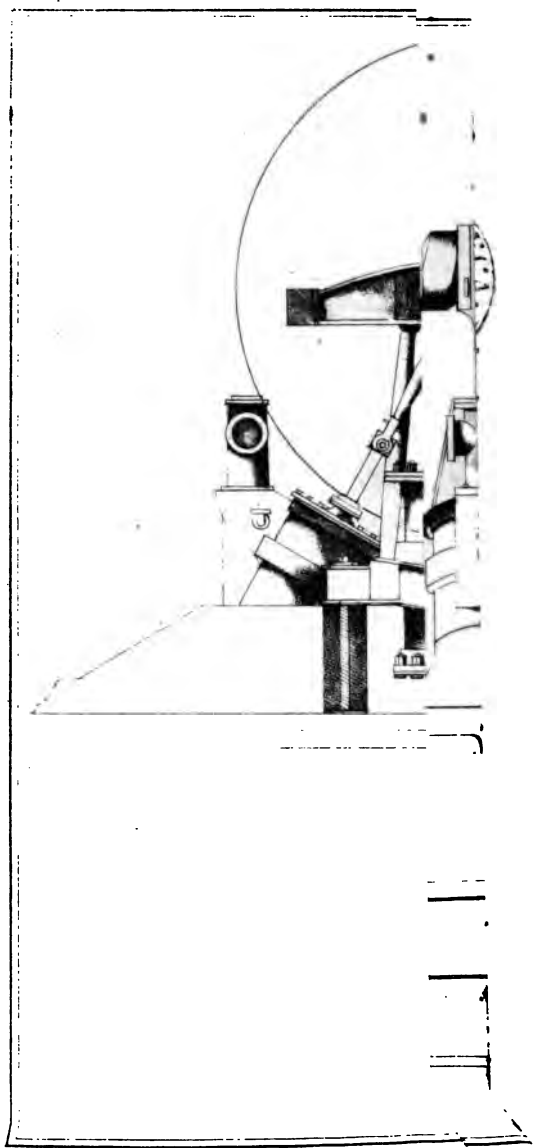
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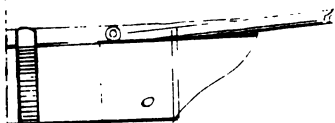
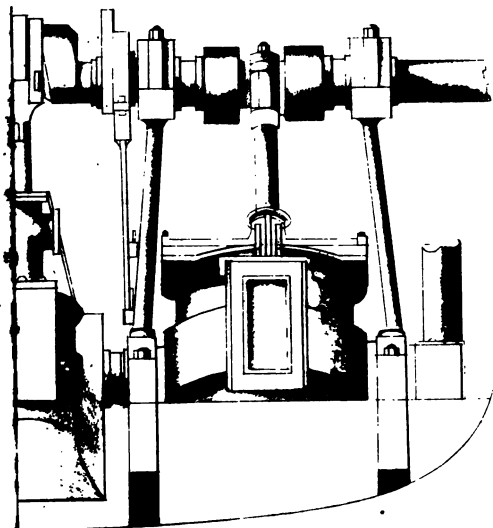
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TEMPLETON'S
MILLWRIGHT AND ENGINEER'S
COMPANION

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A MILLER LITHO

THE
MILLWRIGHT AND ENGINEER'S
POCKET COMPANION

COMPRISING

DECIMAL ARITHMETIC,
TABLES OF SQUARE AND CUBE
ROOTS,
PRACTICAL GEOMETRY,
MENSURATION,
STRENGTH OF MATERIALS,

MECHANIC POWERS,
WATER WHEELS,
PUMPS AND PUMPING ENGINES,
STEAM ENGINES,
TABLES OF SPECIFIC GRAVITY,
&c. &c. &c.



TO WHICH IS ANNEXED, AN APPENDIX OF
A SERIES OF MATHEMATICAL TABLES;
CONTAINING THE CIRCUMFERENCES,
SQUARES, CUBES, AND AREAS OF CIRCLES, SUPERFICIES,
AND SOLIDITY OF SPHERES, &c. &c. &c.

By WILLIAM TEMPLETON,

AUTHOR OF

"THE ENGINEER'S COMMON PLACE BOOK OF PRACTICAL REFERENCE," AND
"LOCOMOTIVE ENGINE POPULARLY EXPLAINED."

WITH LITHOGRAPHIC ILLUSTRATIONS.

THE NINTH EDITION, CORRECTED AND IMPROVED,

By SAMUEL MAYNARD,

EDITOR OF "KEITH'S AND BONNYCASTLE'S MATHEMATICAL WORKS," &c. &c.

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EDITOR'S PREFACE

TO THE

NINTH EDITION.

THE present edition of the following work is the result of a very careful and thorough examination of the former impression. I have not only gone over all the calculations, and corrected whatever numerical errors I could detect, but I have also been anxious to present the several steps of each arithmetical process in the most compact and improved form. Much numerical work is often thrown away by neglecting to take advantage of *cancelling operations*. I have often had occasion to notice this in the revision of the last edition: in the present I have taken care to introduce every simplification into the arithmetical forms that such forms admitted; and I have reason to hope that those who have hitherto countenanced this work will find it even more acceptable in its present improved state.

I would notice, in conclusion, that the Table of useful Numbers often required in calculation, together with their Logarithms, at *pages* 169 to 180 inclusive, which has been computed at considerable labour, will furnish particulars of interest and utility in many practical enquiries.

SAMUEL MAYNARD.

8, Earl's Court,
Leicester Square, London,
30th March, 1852.

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THE MILLWRIGHT AND ENGINEER'S POCKET COMPANION.

EXPLANATION

OF THE SIGNS OR CHARACTERS MADE USE OF IN THE FOLLOWING
WORK.

= signifies Equality; as 4 added to 3 = 7, denotes 4 and 3 added together are equal to 7.

+ signifies Addition; as $5 + 3 = 8$.

— signifies Subtraction; viz. that the latter of the two numbers or quantities between which it is placed is to be subtracted from the former: thus, $5 - 3 = 2$.

× signifies Multiplication; as $5 \times 3 = 15$.

÷ signifies Division; viz. that the former of two quantities or numbers between which it is placed is to be divided by the latter: thus, $15 \div 5 = 3$: the divided quantity or number is called the Dividend, that which divides is called the Divisor. Sometimes the divisor is written under the dividend, with a line drawn between them; as $\frac{15}{5} = 3$.

∴ is to { denotes Proportionals, signifying that the numbers or quantities between which they are placed are proportionals: thus, $2 : 4 :: 8 : 16$, denotes that the number 2 bears the same ratio to 4 as 8 does to 16, and is usually read 2 is to 4 as 8 is to 16.

() Parentheses are used to connect two or more quantities together into one: thus, $(3 + 5) \times 3 = 24$, denotes that the sum of 3 and 5 (which is equal to 8) multiplied by 3 is equal to 24.

3^2 denotes $3 \times 3 = 9$, or the square of 3.

3^3 denotes $3 \times 3 \times 3 = 27$, or the cube of 3.

√ denotes square-root; as, $\sqrt{9} = 3$.

∛ denotes cube-root; as, $\sqrt[3]{27} = 3$.

OF WEIGHTS AND MEASURES.

Avoirdupois Weight is the only weight made use of in mechanical calculations; and all metals, save gold and silver, are weighed by it: hence it is not requisite here to take any other into consideration.

		Fr. Grammes.
	1 Dram..... =	1.772
16 Drams	= 1 Ounce	28.347
16 Ounces	= 1 Pound	453.544
28 Pounds	= 1 Quarter	12.699 kilog.
4 Quarters	= 1 Hundred-wt. ..	50.797 "
20 Hundred-wt. .	= 1 Ton	1015.939 "

NOTE.—5760 Troy grains = 1 pound Troy; and 7000 Troy grains = 1 pound Avoirdupois: hence, 175 pounds Troy = 144 pounds Avoirdupois.

Or, Avoirdupois	lbs. × 1.21528 =	Troy lbs.
Do.....	ounces × .9115 =	Do. ounces.
Troy	lbs. × .823 =	Avoir. lbs.
Do.....	ounces × 1.1 =	Do. ounces.
Do.....	grains × .03657 =	Do. Drams.
Also, Avoirdupois	lbs. × .00893 =	Cwts.
And, Do.....	lbs. × .000447 =	Tons, nearly.

TABLES,

Showing the relative Proportion between Foreign Weights and the Avoirdupois Pound.

1. FRENCH WEIGHTS.—DECIMAL SYSTEM.

1 Milligramme	=	.0154 Troy grains.
1 Centigramme	=	.1543 "
1 Decigramme	=	1.5434 "
1 Gramme	=	15.4340 "
1 Decagramme	=	154.3400 "
1 Hectogramme	=	1543.4000 "
1 Kilogramme	=	2.20486 lbs. Avoirdupois.
1 Myriagramme.....	=	22.0486 "
1 Quintal	=	1 cwt. 3 qrs. 24½ lbs. nearly.
1 Millier or Bar = 100 Quintals	=	9 tons 16 cwt. 3 qrs. 12½ lbs.

2. SYSTEME USUEL.

The Kilogramme	=	1000 Grammes	=	2 lbs. 3 oz.	4½ Drs. Avor.	
The Livre Usuel	=	500	"	=	1 " 10½ "	
The Half	=	250	"	=	8 " 13½ "
The Quarter	=	125	"	=	4 " 6½ "
The Eighth	=	62½	"	=	2 " 3½ "
The Once	=	31 3	"	=	1 " 1½ "
The Half	=	15½	"	=	8½ "
The Quarter	=	7 8	"	=	4½ "
The Gros	=	3 9	"	=	2½ "

3. VARIOUS FOREIGN WEIGHTS IN POUNDS AVOIRDUPOIS.

Places & names of weights.	Lbs.	Places & names of weights.	Lbs.
Alexandria } rotola for- fora	·9367	Genoa peso sottile	·6988
" } rotola zaidino	1·335	" } peso gros-o	·7687
" } rotola zauro.	2·07	Hamburgh.....pound	1·068
" } rotola mina...	1·67	Havannah.....pound	1·075
Amsterdam ... old pound	1·09	Leghorn.....pound	·749
" new ditto	2·205	Madras.....vis	3·125
Antwerp old pound	1·033	Malabar visay	3·001
" new ditto	2·205	Malta rotola	1·750
Bahia, Lisbon, and Oporto } Aragal	1·012	Mocha.....rotola	1·125
Barcelona.....pound	·882	Mogadore } commercial	1·19
Batavia catty	1·36	" } pound	1·781
Bergen	1·1013	Naplesrotola	1·904
Christiania, and Copenhagen ... } pound	1·1013	Odessa and	·9026
Bombay.....seer	·7	Petersburgh } pound	1·08
Bremen.....pound	1·099	Port-au Prince } livre	1·08
Buenos Ayres, Cadiz, Lima, Malaga, Val- paraiso, and } pound	1·015	and } poids de	1·08
Vera Cruz ...	2·053	Port Louis } marc	·9217
Calcutta.....seer	1·333	Rigapound	1·01
Canton & Manilla... catty	1·09	Rio de Janeiro.....aragal	2·205
Cape Townpound	2·83	Rotterdam.....pound	2·82
Constantinopleoke	1·033	Smyrnaoke	·9365
Dantzic & Memel...pound	1·033	Stockholm } commercial	·75
		" } pound	1·235
		" } iron weight	1·0517
		Triestepound	·6641
		Venice peso grosso	
		" } peso sottile	

NOTE.—America, the British West Indies, Gibraltar, and Van Diemen's Land use the pound Avoirdupois, as in England.

EXAMPLE 1.—Suppose I purchase an article in London which weighs 50 lbs. Avoirdupois, what will it weigh in Amsterdam according to their new weight?

$$50 \text{ lbs.} \div 2.205 = 22.676 \text{ lbs. of Amsterdam.}$$

EX. 2.—An article that weighs 60 lbs. in Leghorn, according to their weight, what will it weigh in lbs. Avoirdupois?

$$·749 \times 60 = 44.94 \text{ lbs. Avoirdupois nearly.}$$

LONG MEASURE.

		Fr. Metres.
12 Inches.....	= 1 Foot.....	= 3048
3 Feet.....	= 1 Yard.....	= 9144
6 Feet.....	= 1 Fathom.....	= 18288
5½ Yards.....	= 1 Pole or rod....	= 50292
40 Poles.....	= 1 Furlong.....	= 2011679
8 Furlongs or 1760 yards..	= 1 Mile.....	= 16093436
3 Miles.....	= 1 League.....	= 48280307
Surveying Chain = 22 yards, consists of 100 links, and each link = 7·92 inches.		

FRENCH LONG MEASURE.—DECIMAL SYSTEM.

French.		English.
1 Millimetre.....	=	·03937 inches.
1 Centimetre.....	=	·39370 "
1 Decimetre.....	=	3·93701 "
1 Metre.....	=	39·37009 "
1 Decametre.....	=	3280841 feet.
1 Hectometre.....	=	32808408 "
1 Kilometre.....	=	109361358 yards.
1 Myriametre.....	=	1093613583 "

SYSTEME USUEL.

Usuel.		Metrical.		English.
1 Ligne	=	2·31	Millimetres	= ·091 inches.
1 Pouce	=	2·77	Centimetres	= 1·090 "
1 Pied	=	3·33	Decimetres	= 13·110 "
1 Aune	=	12·	Decimetres	= 3 feet 11·24 inches.
1 Toise	=	2·	Metres	= 6 feet 6·74 inches.

THE LINEAL FOOT OF VARIOUS COUNTRIES, GIVEN IN
ENGLISH INCHES.

	Inches.		Inches.
Amsterdam & Ant- werp.....	= 11·143	Canton.....	= 12·65
Bahia, Lisbon, and Rio de Janeiro..		Dantzic and Memel.	= 11·3
Bergen, Copenha- gen, Cape Town, Christiana, and Hamburgh	= 12·36	Port-au-Prince and	= 12·8
		Port Louis	
		Riga.....	= 10·79
		Stockholm	= 11·684
		Venice.....	= 13·68

NOTE.—The English foot is used generally throughout America, the British West Indies, Russia, and Van Diemen's Land.

LENGTH OF A MILE IN DIFFERENT COUNTRIES, GIVEN IN
ENGLISH YARDS.

	Yards.		Yards.
Dantzic	8475	Poland	8101
Denmark	8244	Portugal	6760
Flanders	6864	Prussia	8237
Germany	6859	Russia	1167
Hanover	11559	Scotland	1984
Holland	8101	Spain	4635
Hungary	9113	Sweden	11700
Ireland.....	2240	Switzerland	9153
Netherlands.....	1093	Tuscany	1808

SUPERFICIAL MEASURE.

		Fr. Sq. Metres.
144 Square inches	= 1 Sq. foot ..	= 0.0929
9 Square feet	= 1 Sq. yard..	= 8.361
30½ Square yards	= 1 Sq. pole..	= 25.2928
40 Square poles.....	= 1 Rood	= 1011.7136
4 Roods, or 4840 Sq. yards ..	= 1 Acre	= 4046.8543

A Scotch Acre contains 6084 square yards,
And an Irish Acre contains 7840 square yards.

FRENCH SUPERFICIAL MEASURES.

1 Centiare	=	1.1960 Square yards.
1 Arc (a square decametre).....	=	119.5991 "
1 Decare.....	=	1195.9907 "
1 Hectare	=	11959.9067 "
		or 2 acres 1 rood 35 perches.

SOLID MEASURE.

		Fr. Cubic Metres.
1728 Cubic inches	= 1 Cubic foot	= 0.283
27 Cubic feet	= 1 Cubic yard.....	= 7.645
42 Cubic feet	= 1 Ton of Shipping..	= 1.1892

A Load of unhewn timber.....	=	40 Cubic feet.
" squared do.....	=	50 "
" 1 inch plank	=	600 Square feet.
" 1½ inch do.....	=	400 "
" 2 inch do.....	=	300 "

NUMBER OF CUBIC FEET IN A TON OF VARIOUS BODIES.

Names of Bodies.	Cubic feet in a ton.	Names of Bodies.	Cubic feet in a ton.
Marble	13·06	Beech	42·
Granite	13·5	Teak	48·
Common Stone ..	14·22	Mahogany	34·
Paving do.....	14·82	Lignum Vitæ	27·
Sand	23·6	Maple & Riga Fir	47·8
Coal	28·6	Larch	65·8
Tallow	38·	Pitch Pine	54·2
English Oak	37·	Oil	39·
American do.....	41·	Proof Spirits	38·6
Ash	42·	Distilled Water..	35·8
Elm.....	53·	Sea do. ..	34·8

A Gallon of Linseed Oil weighs 9·32 *lbs.* Avoirdupois.
 „ Distilled Water 10· „
 „ Sea Water 10·32 „
 „ Proof Spirits 9·27 „

IMPERIAL WINE MEASURE.

1 Gill = 8·665 cubic inches.
 4 Gills = 1 Pint = 34·659 „
 2 Pints = 1 Quart = 69·318 „
 4 Quarts.... = 1 Gallon .. = 277·273843570 „
 10 Gallons .. = 1 Anker.... = 1·604 cubic feet.
 18 Gallons .. = 1 Runlet .. = 2·888 „
 42 Gallons .. = 1 Tierce.... = 6·739 „
 63 Gallons .. = 1 Hogshead = 10·109 „
 84 Gallons .. = 1 Puncheon = 13·478 „
 126 Gallons .. = 1 Pipe = 20·218 „
 252 Gallons .. = 1 Tun..... = 40·435 „

FRENCH MEASURES OF CAPACITY.

1 Millitre = ·06102 cubic inches .
 1 Centilitre..... = ·61024 „
 1 Decilitre = 6·10238 „
 1 Litre (a cubic decimetre) = 61·02379 „
 1 Decalitre..... = 610·23792 „
 1 Hectolitre = 3·5315 cubic feet.
 1 Kilolitre = 35·3147 „
 1 Myrialitre = 353·14694 „
 The Litron usuel = 62·45 cubic inches.

A TABLE

*Showing the relative Value between the British Imperial Gallon
and Foreign Measures of Capacity.*

Places and Names of Measures.	Imp. Gall.	Places and Names of Measures.	Imp. Gall.
Amsterdam...mingle	266	Havannah....arroba	3415
".....kan	220	Leghorn..wine fiasco	499
Antwerp....stoopen	608	".....oil fiasco	443
".....litre	220	Lisbon.....almude	3641
Barcelona .. cortane	2270	Malta.....caffiso	4582
Bordeaux.....velte	1672	Mocha.....cudra	1666
Cadiz .. great arroba	3540	Naples ..wine barilla	9164
" .. small arroba	3124	".....oil staja	2226
Constantinople..alma	1146	Oporto.....almude	5311
Dantzic.. beer anker	12925	Petersburgh...wedro	2707
" .. wine anker	9915	Rotterdamstoop	564
Genoa...wine barilla	16349	Stockholmkanne	575
".....oil barilla	14162	Trieste.....boccali	312
Gibraltargallon	909	Venice...wine sechii	2377
Hamburgh...stubjen	797	".....oil miro	3356

NOTE.—The new Imperial standards of capacity, length, and weight, are not only to be adopted throughout Great Britain and Ireland, but also in all its colonies and dependencies.

IMPERIAL ALE AND BEER MEASURE.

	1 Gill..... =	8.665 cubic inches.
4 Gills..... =	1 Pint..... =	34.659 "
2 Pints..... =	1 Quart..... =	69.318 "
4 Quarts..... =	1 Gallon..... =	277.274 "
9 Gallons..... =	1 Firkin..... =	1.444 cubic feet.
18 Gallons..... =	1 Kilderkin.... =	2.888 "
36 Gallons..... =	1 Barrel..... =	5.776 "
54 Gallons..... =	1 Hogshead.... =	8.665 "
72 Gallons..... =	1 Puncheon.... =	11.553 "
108 Gallons..... =	1 Butt..... =	17.330 "

NOTE.—The old Ale Gallon contained 282 cubic inches, and the old Wine Gallon contained 231; hence,

Imperial Gallons.... ×	·98324	= old Ale Gallons.
Imperial Gallons.... ×	1.20032	= old Wine Gallons.
Old Ale Gallons.... ×	1.01704	= Imperial Gallons.
Old Wine Gallons... ×	·83311	= "
Cubic feet..... ×	6.23210	= "
Cubic inches..... ×	·00361	= "

IMPERIAL DRY MEASURE.

	1 Gill	=	8·665 cubic inches.
4 Gills	= 1 Pint	=	34·659 "
2 Pints	= 1 Quart	=	69·318 "
4 Quarts	= 1 Gallon	=	277·274 "
2 Gallons	= 1 Peck	=	554·548 "
4 Pecks	= 1 Bushel	=	1·2837 cubic feet.
8 Bushels	= 1 Quarter	=	10·2700 "
40 Bushels	= 1 Way	=	51·3500 "
20 Bushels	= 1 Last	=	102·6999 "

NOTE.—The Winchester bushel contained 2150·42 cubic inches, and the Imperial bushel contains 2218·192 cubic inches, hence,

Imperial bushels \times 1·0315157 = Winchester bushels,
and Winchester bushels. \times ·9694472 = Imperial bushels.

A bushel of wheat is reckoned = 60 *lbs.* Avoirdupois.

"	barley	"	= 47	"
"	oats	"	= 38	"
"	pease	"	= 64	"
"	beans	"	= 63	"

HEAPED IMPERIAL MEASURE OF CAPACITY FOR COALS,
CULM, LIME, FISH, POTATOES, FRUIT, AND OTHER
GOODS.

	The Gallon..	=	351·936 cubic inches.
2 Gallons	= 1 Peck	=	703·871 "
4 Pecks	= 1 Bushel	=	2815·487 "
3 Bushels	= 1 Sack	=	4·888 cubic feet, <i>nearly</i> .
12 Sacks	= 1 Chaldron ..	=	58·656 "

DIMENSIONS OF DRAWING PAPER IN FEET AND INCHES.

Wove Antiquarian..	4 feet 4 inches	by	2 feet 7 inches.
Double Elephant....	3 " 4 "	by	2 " 2 "
Atlas	2 " 9 "	by	2 " 2 "
Columbier	2 " 9½ "	by	1 " 11 "
Elephant	2 " 3¾ "	by	1 " 10½ "
Imperial	2 " 5 "	by	1 " 9½ "
Super royal	2 " 3 "	by	1 " 7 "
Royal	2 " 0 "	by	1 " 7 "
Medium	1 " 10 "	by	1 " 6 "
Demy	1 " 7½ "	by	1 " 3½ "

DIMENSIONS OF IMPERIAL CONICAL LIQUID MEASURES.

Diameters.

Two Gallon ..	Top $2\frac{1}{2}$ in. ..	Bottom $11\frac{1}{2}$ in. ..	Depth 12'66498 in.
Gallon	" 2	" 9	" 10'28258
Half Gallon ..	" $1\frac{3}{4}$	" 7	" 8'23406
Quart	" $1\frac{1}{4}$	" $5\frac{3}{4}$	" 6'33249
Pint	" 1	" $4\frac{1}{2}$	" 5'14129
Half Pint.....	" $\frac{7}{8}$	" $3\frac{1}{2}$	" 4'11703
Gill	" $\frac{5}{8}$	" $2\frac{7}{8}$	" 3'16625

DIMENSIONS OF IMPERIAL CYLINDRICAL DRY MEASURES.

Diameters and Depths.

Eighth of a Peck.....	a cylinder of	4'45232 inches.
Forpit or Half Gallon..	"	5'60957
Gallon or Half Peck ..	"	7'06762
Peck	"	8'90464
Half Bushel	"	11'21914
Bushel	"	14'13524
Quarter.....	"	28'27048

NOTE.—Multiply the decimal part by 8, the product will equal eighths of an inch, and decimal parts of an eighth of an inch.

DECIMAL FRACTIONS.

A Decimal Fraction derives its name from the Latin, *decem*, "ten," which denotes the nature of its numbers, representing the parts of an integral quantity, divided into a tenfold proportion.

NUMERATION

Teacheth to read or write any number proposed, either by words or characters.

In Decimal Fractions, the integer, or whole thing, as a gallon, a pound, a yard, &c., is supposed to be divided into ten equal parts, called tenths; those tenths into ten equal parts, called hundredths; and those hun-

dredths into ten equal parts, called thousandths; and so on, without end. So that a denominator of a decimal being always known to consist of a unit, with as many ciphers as the numerator has places, is, therefore, never expressed, being understood to be 10, 100, 1000, 10000, &c., according as the numerator consists of 1, 2, 3, 4, or more figures; thus, instead of $\frac{2}{10}$, $\frac{24}{100}$, $\frac{211}{1000}$, the numerators only are written with a dot or comma before them, thus, .2, .24, .211.

If a unit or whole quantity of any description, as a gallon, a pound, a foot, &c., be divided into ten equal parts, the decimal represents as many of those parts as the decimal figure expresses,—thus, .7 means seven of those parts, or seven-tenths; but if the decimal consisted of two figures, unity would be understood to be divided into a hundred equal parts, of which the decimal represents as many as the figure expresses,—thus .65 means sixty-five of those parts, or sixty-five hundredths; and if the decimal consisted of three figures, unity would be supposed to be divided into a thousand equal parts, of which the decimal represents as many as the number expresses,—thus .625 is six hundred and twenty-five of those parts; or, suppose the decimal .0625, unity would be understood to be divided into 10000 equal parts; but the value of decimal figures is made more plain by means of the following

TABLE.

Tenths5
Hundredths56
Thousandths567
Ten thousandths5678
Hundred thousandths56789

Thus, .5 is read five-tenths; .56 is read five-tenths and six hundredths, or fifty-six hundredths; .567 is read five-tenths, six hundredths, and seven-thousandths, or five hundred and sixty-seven thousandths; and so on, as in the table.

Ciphers to the right hand of decimals cause no difference in their value; for $\cdot 5$, $\cdot 50$, $\cdot 500$ are decimals of the same value, being each equal to $\frac{1}{2}$; that is, $\cdot 5 = \frac{5}{10}$, $\cdot 50 = \frac{50}{100}$, $\cdot 500 = \frac{500}{1000}$. But if ciphers are placed on the left hand of decimals, they diminish their value in a tenfold proportion; thus, $\cdot 3$, $\cdot 03$, $\cdot 003$, are three-tenths, three-hundredths, and three-thousandths, and answer to the vulgar fractions $\frac{3}{10}$, $\frac{3}{100}$, $\frac{3}{1000}$, respectively.

A whole number and decimal are thus expressed, $85\cdot 75$, $85\cdot 04$, &c.

REDUCTION OF DECIMALS.

By reduction we change vulgar fractions, and the lesser parts of coin, weight, measure, &c., into decimals, and find the value of any decimal given.

Because decimals increase their value towards the left hand, and decrease their value towards the right hand, in the same tenfold proportion with integers, or whole numbers, they may be annexed to whole numbers, and worked in all respects as whole numbers; hence, if simple arithmetic be well understood, there is little more to be learned than the placing of the separating point—the rule for which ought to be well attended to

1.—*To reduce a vulgar fraction to a decimal of an equal value.*

RULE.—Add a cipher, or ciphers, to the numerator, and divide by the denominator, the quotient will be the decimal required.

EXAMPLE.—Reduce $\frac{14}{32}$ to a decimal.

32)14·0000(·4375

128

120

96

240

224

160

160

Thus you may take any number of ciphers at pleasure, but their number will be best ascertained when the work is finished; then you must have as many decimal figures as you have taken annexed ciphers in dividing; and if there are not so many in the quotient, you must prefix ciphers to the left hand of it,—thus, $\frac{1\cdot0000}{32} = \cdot03125$.

Sometimes the quotient figures will repeat continually, as $\frac{2}{3}$, thus, $\frac{2\cdot0}{3} = \cdot6'$, then it is called a repetend, and the first figure which repeats may be dashed, to distinguish it from a terminate decimal.

Sometimes two, three, or more figures will repeat, as $\frac{12}{33}$, thus, $\frac{12\cdot00}{33} = \frac{4\cdot00}{11} = \cdot36'$; such are called compound repetends or circulates, and the first and last of the repeating figures may be dashed.

2.—*To reduce the lesser parts of coin, weights, measures, &c., to decimals.*

RULE.—Divide the least name by such number as will reduce it to the next greater; to the decimal so obtained prefix the given number of the same name, then divide by such number as will reduce it to the next greater, always annexing ciphers to the dividend, as occasion may require: thus proceed till it be reduced to the decimal of the required integer. Or, reduce the given parts to a simple quantity, by reducing them to the lowest name mentioned; annex ciphers thereto, and divide by such numbers as will reduce them to the name required. Or, reduce the given parts to a vulgar fraction, and that fraction to a decimal.

EXAMPLE 1.—Reduce $17s. 10\frac{1}{2}d.$ to the decimal of a pound sterling.

Here $\frac{1\cdot0d.}{2} = \cdot5d.$; and $\frac{10+\cdot5}{12}s. = \frac{10\cdot500}{12}s. = \cdot875s.$;

then $\frac{17 + \cdot875}{20}l. = \frac{17\cdot8750}{20}l. = \cdot89375l.$ is the decimal required.

EXAMPLE 2.—Reduce 2 feet 9 inches to the decimal of a yard.

Here $2ft. 9in. = 33$ inches; and $36in. = 1$ yard; hence, by vulgar fractions, $\frac{33}{36} = \frac{33\cdot0000}{36} = \frac{11\cdot0000}{12} = \cdot916\bar{6}yd.$ as required.

To find the value of any given decimal.

RULE.—Multiply the decimal given by the number of parts of the next inferior denomination, cutting off the decimals from the product; then multiply the remainder by the next inferior denomination; thus proceeding till you have brought the least known parts of the integer.

EXAMPLE 1.—Required the value of $\cdot89375$ of a pound sterling.

$$\begin{array}{r}
 \cdot89375l. \\
 \hline
 20 \\
 \hline
 17\cdot87500 \text{ shillings.} \\
 \hline
 12 \\
 \hline
 10\cdot500 \text{ pence.} \\
 \hline
 4 \\
 \hline
 2\cdot0 \text{ farthings.} \\
 \hline
 \text{Ans. } 17s. 10\frac{1}{2}d.
 \end{array}$$

EXAMPLE 2.—Reduce $\cdot625$ of a hundred weight to its proper terms.

Here $\cdot625 \times 4 \text{ qrs.} = 2\cdot500 \text{ qrs.}$, and $\cdot5 \times 28 \text{ lbs.} = 14\cdot0 \text{ lbs.}$; whence $2 \text{ qrs. } 14 \text{ lbs.} = \text{Ans.}$

**A TABLE OF RECIPROCALs,
FOR OBTAINING DECIMAL EQUIVALENTS.**

No.	Recip.	No.	Recip.	No.	Recip.	No.	Recip.	N	Recip.
1	1.000000	51	.019608	101	.009901	151	.006623	201	.004975
2	.500000	52	.019231	102	.009804	152	.006579	202	.004950
3	.333333	53	.018868	103	.009709	153	.006536	203	.004926
4	.250000	54	.018519	104	.009615	154	.006494	204	.004902
5	.200000	55	.018182	105	.009524	155	.006452	205	.004878
6	.166667	56	.017857	106	.009434	156	.006410	206	.004854
7	.142857	57	.017544	107	.009346	157	.006369	207	.004831
8	.125000	58	.017241	108	.009259	158	.006329	208	.004808
9	.111111	59	.016949	109	.009174	159	.006289	209	.004785
10	.100000	60	.016667	110	.009091	160	.006250	210	.004762
11	.090910	61	.016393	111	.009009	161	.006211	211	.004739
12	.083333	62	.016129	112	.008929	162	.006173	212	.004717
13	.076923	63	.015873	113	.008850	163	.006135	213	.004695
14	.071429	64	.015625	114	.008772	164	.006098	214	.004673
15	.066667	65	.015385	115	.008696	165	.006061	215	.004651
16	.062500	66	.015152	116	.008621	166	.006024	216	.004630
17	.058824	67	.014925	117	.008547	167	.005988	217	.004608
18	.055556	68	.014706	118	.008475	168	.005952	218	.004587
19	.052632	69	.014493	119	.008403	169	.005917	219	.004566
20	.050000	70	.014286	120	.008333	170	.005882	220	.004545
21	.047619	71	.014085	121	.008264	171	.005848	221	.004525
22	.045455	72	.013889	122	.008197	172	.005814	222	.004505
23	.043478	73	.013699	123	.008130	173	.005780	223	.004484
24	.041667	74	.013514	124	.008065	174	.005747	224	.004464
25	.040000	75	.013333	125	.008000	175	.005714	225	.004444
26	.038462	76	.013158	126	.007937	176	.005682	226	.004425
27	.037037	77	.012987	127	.007874	177	.005650	227	.004405
28	.035714	78	.012821	128	.007813	178	.005618	228	.004386
29	.034483	79	.012658	129	.007752	179	.005587	229	.004367
30	.033333	80	.012500	130	.007692	180	.005556	230	.004348
31	.032258	81	.012346	131	.007634	181	.005525	231	.004329
32	.031250	82	.012195	132	.007576	182	.005495	232	.004310
33	.030303	83	.012048	133	.007519	183	.005464	233	.004292
34	.029412	84	.011905	134	.007463	184	.005435	234	.004274
35	.028571	85	.011765	135	.007407	185	.005405	235	.004255
36	.027778	86	.011628	136	.007353	186	.005376	236	.004237
37	.027027	87	.011494	137	.007299	187	.005348	237	.004219
38	.026316	88	.011364	138	.007246	188	.005319	238	.004202
39	.025641	89	.011236	139	.007194	189	.005291	239	.004184
40	.025000	90	.011111	140	.007143	190	.005263	240	.004167
41	.024390	91	.010989	141	.007092	191	.005236	241	.004149
42	.023810	92	.010870	142	.007042	192	.005208	242	.004132
43	.023256	93	.010753	143	.006993	193	.005181	243	.004115
44	.022727	94	.010638	144	.006944	194	.005155	244	.004098
45	.022222	95	.010526	145	.006897	195	.005128	245	.004082
46	.021739	96	.010417	146	.006849	196	.005102	246	.004065
47	.021277	97	.010309	147	.006803	197	.005076	247	.004049
48	.020833	98	.010204	148	.006757	198	.005051	248	.004032
49	.020408	99	.010101	149	.006711	199	.005025	249	.004016
50	.020000	100	.010000	150	.006667	200	.005000	250	.004000

The numbers in the table are the denominators of the fraction hence, multiply the reciprocal of the denominator by the numerator of the fraction, and the product is the decimal equivalent.

Thus, suppose the decimal equivalent of $7\frac{1}{16}$ be required:—

Reciprocal of 16 = .0625 \times 7 = .4375 its decimal equivalent.

ADDITION OF DECIMALS.

RULE.—Arrange the numbers under each other, according to their several values; find the sum, as in addition of whole numbers, and cut off for decimals as many figures to the right hand as there are decimals in any one of the given numbers.

EXAMPLE.—What is the sum of 23·45, 7·849, 543·2, 8·6234, 253·004?

23·45
7·849
543·2
8·6234
253·004

836·1264

If any of the decimals be repetends, bring all to the form of repetends, and make them begin and end together; then, in adding, increase the sum of the first column by as many units as would arise to carry to it if they were continued farther; and you will have a circulate in the sum beginning and ending like the others. Thus,

·7500'
·6666'
·8888'
·8750'
·4444'

3·6249' = 3·625.

2·50'0000'
3·66'66666'
7·69'69696'
14·37'23723'

28·23'60087'

The repetend of '6', the circulate of 6'9' and '3'72', are continued till their periods begin and end together. It may easily be observed that there would be two to carry to the first column if it were carried any farther.

NOTE.—It is not always necessary to attend to the rule for repetends and circulates; three or four decimal figures, according to the rule, being sufficiently near the truth for common calculations.

SUBTRACTION OF DECIMALS.

RULE.—Place the numbers directly under each other according to their several values, subtract as in whole numbers, and cut off for decimals, as in addition.

EXAMPLE.—Subtract 35.87043 from 132.005 .

132.005 If both be single repetends, make them
 35.87043 end together; and if there be occasion to

 96.13457 borrow at the first figure, borrow 9 only
instead of 10;

Thus, $.83'$ If both be circulates, or one a repetend
 $.66'$ and the other a circulate, continue both till

 $.16'$ their periods begin and end together; then if
there should be occasion to borrow at the
following figure, where they continued that
figure farther, carry one to the first figure; and if the
numbers be in different denominations, reduce them till
they be alike.

Subtract $\frac{834}{999}$ from $1\frac{2}{3}$; thus, $1\frac{2}{3} = 1.666'$
and $\frac{834}{999} = .834'$

 $.831'$

MULTIPLICATION OF DECIMALS.

RULE.—Place the factors under each other, and multiply them together, as in whole numbers; then point off as many figures from the product (*counting from right to left*) as there are decimal places in both factors; observing, if there be not enough, to annex as many ciphers to the left hand of the product as will supply the deficiency.

EXAMPLE.—Multiply $.4375$ by $.125$.

$.4375$ Here the product of $.4375$ by $.125$ is
 $.125$ $.0546875$; and as there are three places of

 21875 decimals in the multiplier, and four in the
 8750 multiplicand, a cipher must be added on the
 4375 left hand of the product to reduce it to its

 $.0546875$ proper terms.

To multiply a repetend by a single figure, add 1 to the first product for every 9 therein, so will you have a repetend in the product; and if there be several figures in the multiplier, do so with each product, and make them begin and end together; then add them as so many repetends.

If the multiplicand be a circulate, consider the increase that would arise to the first product if the multiplicand were continued farther: thus do with each product, make them begin and end together, and add them by the rule for adding circulates.

To contract the operation so as to retain only as many decimals in the product as may be thought necessary.

RULE.—Place the unit figure of the multiplier under that figure of the multiplicand whose place is the last to be retained in the product, and dispose of the rest so that they may all stand in contrary order to that in which they are usually placed.

Then, in multiplying, reject all the figures to the right hand of the multiplying digit, and set down the product so that the right hand figures may fall in a straight line under each other; observing to increase the first figure of every line with what would arise by carrying 1 from 5 to 14,—2 from 15 to 24,—3 from 25 to 34, &c., from the product of the two preceding figures when you begin to multiply; and the sum will be the product required.

EXAMPLE.—Multiply 27·14986 by 92·41035, retaining four decimals in the product.

<i>Common way.</i>	<i>Contracted way.</i>
27·14986	27·14986
92·41035	53014·29
13574930	24434874
8144958	542997
2714986	108599
10859944	2715
5429972	81
24434874	14
2508·9280650510	2508·9280

DIVISION OF DECIMALS.

RULE.—Prepare your decimals as directed for multiplication, divide as in whole numbers, cut off as many figures for decimals in the quotient as the number in the dividend exceeds the number in the divisor, namely, make the number of decimal figures in the divisor and quotient together equal to the number in the dividend.

EXAMPLE.—Divide 173.5425 by 3.75 .

$$\begin{array}{r}
 3.75 \overline{) 173.5425} \quad (46.278 \\
 \underline{1500} \\
 2354 \\
 \underline{2250} \\
 1042 \\
 \underline{750} \\
 2925 \\
 \underline{2625} \\
 3000 \\
 \underline{3000} \\
 \hline
 \hline
 \end{array}$$

Although you may take additional ciphers at pleasure, care must be had in reckoning the number taken in dividing for decimals in the dividend; and if you put the decimal point in the quotient at any part of the operation, continuing the operation afterwards will not cause the point to be removed.

If there should not be so many figures in the quotient as there should be decimals, prefix ciphers on the left hand to make up the number.

EXAMPLE.—Divide 1.4850 by 247.5 .

Thus, $\frac{1.4850}{247.5} = .006$. And if there be not as many decimal figures in the dividend as in the divisor, you may annex a sufficient number of ciphers; and if there be not a remainder, you must add ciphers to the right hand of the quotient till you have taken as many in the dividend as will make the decimal figures therein equal to those in the divisor: thus,—

$$\frac{1.4856}{247.5} = 6000.$$

A TABLE

Of the Fractional Parts of an Inch when divided into thirty-two parts; likewise a Foot of twelve inches reduced to Decimals.

Parts.	Decimals.	Parts.	Decimals.	Parts of a foot.	Decimals.
$\frac{1}{2} + \frac{1}{32} =$	·96875	$\frac{1}{2} + \frac{1}{32} =$	·46875	11 =	·9166'
$\frac{1}{2} + \frac{1}{16} =$	·9375	$\frac{1}{2} + \frac{1}{16} =$	·4375	10 =	·8333'
$\frac{1}{2} + \frac{1}{8} =$	·90625	$\frac{1}{2} + \frac{1}{8} =$	·40625	9 =	·75
$\frac{1}{2} =$	·875	$\frac{1}{2} =$	·375	8 =	·6666'
$\frac{1}{2} + \frac{1}{32} =$	·84375	$\frac{1}{2} + \frac{1}{32} =$	·34375	7 =	·5833'
$\frac{1}{2} + \frac{1}{16} =$	·8125	$\frac{1}{2} + \frac{1}{16} =$	·3125	6 =	·5
$\frac{1}{2} + \frac{1}{8} =$	·78125	$\frac{1}{2} + \frac{1}{8} =$	·28125	5 =	·4166'
$\frac{1}{2} =$	·75	$\frac{1}{2} =$	·25	4 =	·3333'
$\frac{1}{2} + \frac{1}{32} =$	·71875	$\frac{1}{2} + \frac{1}{32} =$	·21875	3 =	·25
$\frac{1}{2} + \frac{1}{16} =$	·6875	$\frac{1}{2} + \frac{1}{16} =$	·1875	2 =	·1666'
$\frac{1}{2} + \frac{1}{8} =$	·65625	$\frac{1}{2} + \frac{1}{8} =$	·15625	1 =	·0833'
$\frac{1}{2} =$	·625	$\frac{1}{2} =$	·125	$\frac{1}{2} =$	·072916'
$\frac{1}{2} + \frac{1}{32} =$	·59375	$\frac{1}{2} =$	·09375	$\frac{1}{4} =$	·0625
$\frac{1}{2} + \frac{1}{16} =$	·5625	$\frac{1}{2} =$	·0625	$\frac{1}{8} =$	·052083'
$\frac{1}{2} + \frac{1}{8} =$	·53125	$\frac{1}{2} =$	·03125	$\frac{1}{16} =$	·04166'
$\frac{1}{2} =$	·5			$\frac{1}{32} =$	·03125
				$\frac{1}{64} =$	·02083'
				$\frac{1}{128} =$	·010416'

The utility of this table will appear evident by means of the following example:—

Suppose a board or plate to be $30\frac{1}{2}$ inches long, $8\frac{1}{2}$ inches broad, and $\frac{3}{8} + \frac{1}{16}$ of an inch in thickness; required its contents in cubic inches.

Here $30 \cdot 25 \times 8 \cdot 625 = 260 \cdot 90625$, and $260 \cdot 90625 \times \cdot 4375 = 114 \cdot 14648$, &c., the number of cubic inches. = *Ans.*

OF THE SQUARE ROOT.

When a number is multiplied by itself, as 6×6 , or 9×9 , &c., it produces the square or second power of that number; and the number itself is called the root of that square.

A root consisting of a single figure is found by inspection of the following table:—

Root	1	2	3	4	5	6	7	8	9
Squares	1	4	9	16	25	36	49	64	81
Cubes	1	8	27	64	125	216	343	512	729

To extract or find the square root of any number which consists of more figures than one.

RULE.—Make a point or dot over every second figure, commencing at the right hand, by which the given square will be pointed into periods of two figures each, except the first or left-hand period, which will sometimes have but one.

The unit figure must always be the latter figure in the period; for the decimal point must be between the periods, and not in the middle of a period.

Find the greatest root in the first period, which write in the quotient or root, and the square thereof under the same period; subtract therefrom, and to the remainder annex the next period for a dividend.

Double the quotient for a divisor; see how often the divisor is contained in the dividend, with this consideration, that the answer must be the unit's figure of the divisor.

Write the answer in the quotient, also in the unit place of the divisor; then multiply the divisor, so completed by the last quotient figure; write the product under the dividend, and subtract therefrom; to the remainder annex the next period for a new dividend.

Thus proceed with every period ; and if there be still a remainder, annex pairs of ciphers for additional periods, till you have a competent number of decimals in the root.

Vulgar fractions, &c., may be reduced to decimals.

The periods which are whole numbers give whole numbers, and decimal periods give decimals in the root.

EXAMPLE 1.—What is the square root of 76176 ?

$$\begin{array}{r}
 \dot{7}6\dot{1}7\dot{6}(276 \\
 \underline{4} \\
 47)361 \\
 \underline{329} \\
 546)3276 \\
 \underline{3276}
 \end{array}
 \quad \text{The proof} = 276 \times 276 = 76176.$$

EXAMPLE 2.—Required the square root of .75.

$$\begin{array}{r}
 \dot{.}7\dot{5}(\dot{.}866 \\
 \underline{64} \\
 166)1100 \\
 \underline{966} \\
 1726)10400 \\
 \underline{10356} \\
 \hline
 .44 \text{ remainder.}
 \end{array}
 \quad \text{The proof} = .866^2 + .000044 = .75.$$

EXAMPLE 3.—Required the square root of .000854.

$$\begin{array}{r}
 \dot{.}0008\dot{5}4(\dot{.}029 \text{ and } \dot{.}029 \\
 \underline{4} \\
 49)454 \\
 \underline{441} \\
 \hline
 \text{Remainder } 13
 \end{array}
 \quad
 \begin{array}{r}
 \dot{.}029 \\
 \underline{\dot{.}029} \\
 261 \\
 \underline{58} \\
 \hline
 \dot{.}000841 \\
 + 13 \\
 \hline
 \dot{.}000854 = \text{Proof.}
 \end{array}$$

OF THE CUBE ROOT.

When a square is multiplied again by its root, as $6 \times 6 \times 6$, it produces the cube or third power of that root.

Single cubes are found by inspection of the preceding table.

To extract the root of any number that consists of more than one figure.

RULE.—Point the given cube into periods of three figures, and so that the unit figure be the last in its period; then from the first period subtract the greatest cube it contains; put the root as a quotient, and to the remainder bring down the next period for a dividend.

Find a divisor by multiplying the square of the root by 300; see how often it is contained in the dividend; and the answer gives the next figure in the root.

Multiply the divisor by the last figure in the root. Multiply all the figures in the root by 30, except the last; and that product by the square of the last. Cube the last figure in the root; add these three last found numbers together, and subtract this sum from the dividend; to the remainder bring down the next period for a new dividend, and proceed as before.

EXAMPLE.—Required the cube root of 444194947.

$$\begin{array}{r}
 444194947(763 \\
 343 \\
 \hline
 7 \times 7 \times 300 = 14700)101194 \\
 95976 \\
 \hline
 76 \times 76 \times 300 = 1732800)5218947 \\
 5218947 \\
 \hline
 \end{array}$$

<p>1. Divisor 14700</p> $ \begin{array}{r} 6 \\ \hline 88200 \\ 7 \times 30 \times 36 = 7560 \\ 6 \times 6 \times 6 = 216 \\ \hline 95976 \end{array} $	<p>2. Divisor 1732800</p> $ \begin{array}{r} 3 \\ \hline 5198400 \\ 76 \times 30 \times 9 = 20520 \\ 3 \times 3 \times 3 = 27 \\ \hline 5218947 \end{array} $
---	---

Besides the preceding, there is another, and, perhaps, a better way of extracting the cube root, and which we shall attempt to render as intelligible as possible, by an explanation of the following example:—

EXAMPLE.—Required the cube root of 926859375.

1st Cipher.	2nd Cipher.	
0	0	926859375(975
9	81	729
9	81	197859
9	162	183673
18	24300	14186375
9	1939	14186375
270	26239	
7	1988	
277	2822700	
7	14575	
284	2837275	
7		
2910		
5		
2915		

After having divided the number into periods of three figures each as before, place before it two ciphers, at moderate distances from each other, and from the number itself, as represented in the example; then find, by a reference to the table, *page* 20, the nearest cube contained in 926, the first period, and place its root, which is 9, in the quotient, as the first figure of

the required root. Place it also under the first cipher, and by going through the process of addition, we obtain 9. Multiply this by 9, and place the product under the second cipher. By again going through the form of addition we get 81, which being multiplied by 9, becomes 729. This is placed under the first period of figures, 926, and subtracted from it, and to the remainder the second period 859, is annexed. Then recommencing at the column of the first cipher, we again place a 9, and add up. To the sum we add another 9, and obtain 27. We now multiply the 18 by 9, and place the product 162 under the 81 of the second column, and by addition we obtain 243. We now turn

to the first column, and annex one cipher to the 27, making it 270; and in a similar manner annex two ciphers to the 243 of the second column. All this is done preparatory to finding the second figure of the quotient, or of the required root.

To find that figure we divide the number 197859 of the third column by 24300 of the second column. The quotient would appear to be 8: this, however, is found on trial to be too large, and we therefore take 7, which answers. We add this 7 to the first column, and multiply the sum 277 by 7, placing the product 1939 in the second column. Then, by adding up, we obtain 26239, the product of which by 7 we place in the third column and subtract, and to the remainder we annex the last period of figures 375. Recommencing at the first column, we add 7 to the 277, and to the sum 284 we add 7 again. We now multiply the sum 284 by 7, and place the product in the second column and add up. Then to the 291 in the first column we annex one cipher, and to the 28227 in the second column two ciphers.

To find the third figure of the root we divide the number 14186375 in the third column by the 2822700 in the second column, and the quotient is found to be 5. This we add to the first column, and multiply the sum 2915 by 5, and place the product under the 2822700 of the second column. Upon adding we obtain 2837275, whose product by 5 is placed under the 14186375 of the third column, and, being exactly equal to it, there is no remainder upon subtracting, consequently the work is finished, and 975 is the required cube root.

N.B.—The above method is not only useful for extracting the cube root, but also for that of any other root, attention being paid to the following directions:—

Instead of dividing the number whose root is to be extracted into periods of three figures each, we divide it into periods of as many figures each as correspond to the order of the root; as, for example, four figures for the fourth root, five for the fifth, &c.; but the number of ciphers which we employ must be one less than that which corresponds to the order of the root, as three ciphers for the fourth root, and four for the fifth root. It must be borne in mind, too, that every new figure which we place in the quotient must be added to the first column as many times as correspond to the order of the root, and that the number of products added to the second column be one less than the above number, and that added to the third two less, and so on.

Lastly, before finding a new figure for the quotient, we annex one cipher to the number in the first column, two to that in the second, three to that in the third, and so on, until we arrive at the last column, where, instead of annexing ciphers, we bring down the numbers that make up the next period.

If, after extracting the root, we have a remainder, we can continue the quotient to decimals, by annexing to the remainder as many ciphers as there are figures in a period, and then proceeding as before; and if the number whose cube root is to be extracted consist of a whole number and a decimal, we divide it into periods by commencing at the unit's place, and proceeding towards the left hand to divide the whole number, and towards the right to divide the decimal; and if the decimal do not contain a sufficient number of figures to make up the last period, we supply the deficiency with ciphers.

The following example will, perhaps, make the subject a little more plain :—

EXAMPLE.—Required the fourth root of 285762·321.

1st Cipher.	2nd Cipher.	3rd Cipher.	
0	0	0	285762·3210(23·12
2	4	8	16
—	—	—	—
2	4	8	125762
2	8	24	119841
—	—	—	—
4	12	32000	..59213210
2	12	7947	48986321
—	—	—	—
6	2400	39947	102268890000
2	249	8721	98739268336
—	—	—	—
80	2649	48668000	..3529621664 rem.
3	258	318321	
—	—	—	—
83	2907	48986321	
3	267	319243	
—	—	—	—
86	317400	49305564000	
3	921	64070168	
—	—	—	—
89	318321	49369634168	
3	922		
920			
1	319243		
921	923		
1			
922	32016600		
1	18484		
923			
1	32035084		
9240			
2			
9242			

Involution and Evolution of numbers are very conveniently performed upon the *Engineer's Slide Rule*, for when the slide is set straight at both ends, C is a line of squares, and D a line of roots; consequently, against any number upon D is its square upon C, and against any number upon C is its root upon D.

EXAMPLE 1.—What is the square of 16?

Opposite 16 upon D is 256, the square number upon C.

EXAMPLE 2.—Required the square root of 625.

Opposite 625 upon C is 25 upon D, the root required.

The cube root is performed by inverting the slide, and setting the number to be cubed upon B to the same number upon D, and against 1 or 10 upon D is the cube number upon B. Also, set the cube number upon B to 1 or 10 upon D, and where two numbers of equal value meet upon the lines B and D is the root required.

EXAMPLE 1.—Required the cube of 9.

Set 9 upon B to 9 upon D, and against 10 upon D is 729 upon B.

EXAMPLE 2.—Required the cube root of 343.

Set 343 upon B to 10 upon D, and against 7 upon B is 7 upon D, the root required.

These lines also serve to multiply the square of any number, any number of times: thus,—

To find the product of 6 times 6, multiplied by 3.

Set 3 upon B to 6 upon D, and against 10 upon D is 108 upon B.

To find the root of a number consisting of integers and decimals.

RULE.—Multiply the difference between the root of the integral part of the given number, and the root of the next higher integer number, by the decimal part of the given number, and add the product to the root of the integral part of the number given; the sum will be the root of the number required, correct in all cases of the square root to 3 places, and in the cube root to 7, if the roots of the integral parts are correct to 4 places of decimals.

PRACTICAL GEOMETRY.

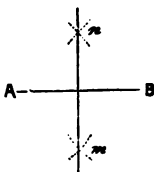
Geometry is the science which treats of that species of quantity called magnitude, as represented by lines, surfaces, and solids.

Practical Geometry is that art by which we are enabled to turn the rules of the science to a practical account.

PROBLEM I.

To divide a given line into two equal parts.

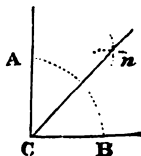
From A and B as centres, with any distance greater than half the length of the line, describe arcs cutting each other in m and n ; then a line drawn through the points m and n will divide the line into two equal parts, as required.



PROBLEM II.

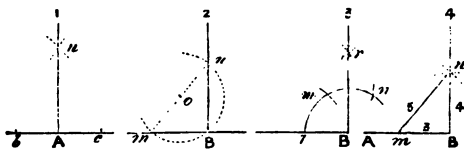
To divide a given angle into two equal parts.

From the point C as a centre, with any distance at pleasure, describe the arc A B; and from A and B as centres, with the same or any other convenient distance, describe arcs cutting each other in n ; then a line drawn from the point C, through n , will divide the angle as required.



PROBLEM III.

From any given point in a right line, to erect a perpendicular.



1.—On each side of the point A, take equal distances,

as $A b$, $A c$; from b and c , as centres, with any radius greater than $b A$ or $c A$, describe arcs cutting each other in n ; then will a line drawn from the point A through n be the perpendicular required.

2.—Take any point o , and with o as centre, and $o B$ as radius, describe the arc $m B n$, cutting the line in m and B ; draw a line from m through the centre o , and continue it until it cut the opposite side of the arc in n ; then the line which joins n and B is the perpendicular required.

3.—With the point B as centre, and with any radius, describe the arc $l m n$, cutting the line in l ; with l as centre, and the same radius, cut the arc in m ; and with m as centre, and the same radius, cut the arc again in n . Now, with m and n as centres, and with any radius, describe arcs cutting each other in r , then the line joining r and B is the perpendicular required.

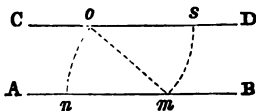
4.—From the point B , on the line $A B$, take three equal parts (as feet, inches, &c.) to m ; and from m and B as centres, describe arcs cutting each other in n , making the distance from B to n four parts, and from m to n five parts, then will the line $B n$ be the perpendicular required.

PROBLEM IV.

Through a given point o , to draw a straight line $C D$, parallel to a given straight line $A B$.

Take any point m in the line $A B$; with m as a centre and radius $m o$, describe the arc $o n$; with n as a centre and the same distance describe the arc $s m$.

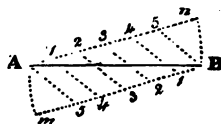
Take the arc $o n$ in your compasses, and apply it from m to s ; through o , s , draw $C D$, and it will be the parallel as required.



PROBLEM V.

To divide a right line into any number of equal parts.

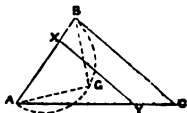
Let A B be the line that is to be divided; then at the point A draw a line making any angle with the line A B and at B draw another line parallel to it. Upon each of these lines, beginning at the points A and B, cut off as many equal parts as you require the line A B to be divided into, as A 1, 1 2, 2 3, &c., B 1, 1 2, 2 3, &c.; then draw lines joining the points A and 5, 1 and 4, 2 and 3, &c., and the lines so drawn will cut A B into the required number of equal parts.



PROBLEM VI.

To divide a triangle into two equal parts, and still retain its original form.

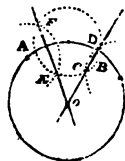
Let A B C be the given triangle to be divided, bisect one of its sides as A B, and describe the semicircle A G B; bisect the semicircle in G, and at a distance from A, equal to A G or B G, draw the line x y, parallel to B C, which is the line of equal division as required.



PROBLEM VII.

Through any three points out of a right line to describe the circumference of a circle.

From the middle point as a centre, with any convenient distance, describe the circle, or arcs of a circle, as A and B, and from the other points, with the same distance, describe arcs cutting the circle in C D and E F; draw lines through C D and E F, and where

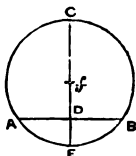


they intersect each other at o is the centre of the circle required.

PROBLEM VIII.

To find the centre of a given circle.

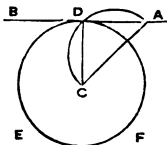
Draw any chord AB , bisect it in D , and through D draw EC perpendicular to AB ; then bisect EC , and the point of section f will be the centre of the circle.



PROBLEM IX.

From a given point to draw a tangent to a circle.

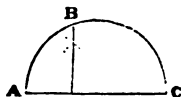
Let A be the point from which it is required to draw a tangent to the circle $DE F$. Join A and the centre C , and upon the line AC describe the semicircle ADC , then through D , the point at which the circles intersect, draw the line AB , which will be the tangent required.



PROBLEM X.

To find a mean proportional between two given right lines, or the side of a square equal to a given rectangle.

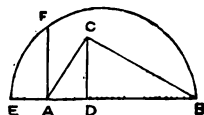
Upon a right line as a diameter equal to both given lines, describe the semicircle ABC , and where the two lines meet, or between their respective lengths, erect a perpendicular to the semicircle at B , and the perpendicular will be the mean proportion or side of the required square, equal to the given rectangle.



PROBLEM XI.

To find the side of a square which shall be equal in area to a given triangle.

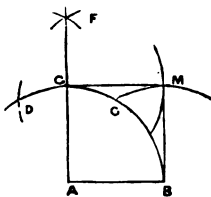
Let $A B C$ be the given triangle. From C let fall $C D$ perpendicular to $A B$, and produce the line $B A$ to E , making $A E$ equal to half the perpendicular $C D$; then upon $E B$ describe the semicircle $E F B$, and from the point A erect a perpendicular cutting the circle in F ; then $A F$ will be the side required.



PROBLEM XII.

Upon a given right line to construct a square.

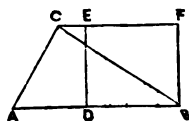
Let $A B$ be the line upon which it is required to construct a square. With A as centre, and $A B$ as radius, describe the arc $B C D$; with B as centre, and the same radius, cut the arc in C ; and with C as centre, and the same radius, cut the arc again in D ; then with C and D as centres, and with equal radii, describe arcs cutting each other in F , and from F draw $F A$, cutting the circle in G . Then with G and B as centres, and the distance $A B$ as radius, describe arcs cutting each other in M ; join $G M$ and $B M$, and the figure $G M B A$ is the required square.



PROBLEM XIII.

To make a rectangle equal to a given triangle.

Let $A B C$ be a triangle, to which it is required to make a rectangle equal. Bisect $A B$ in D , and at D erect a perpendicular; from B draw a line parallel to $D E$, and from C

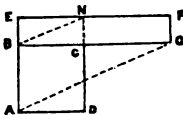


draw a line parallel to $A B$; then the figure $D E F B$ is the rectangle required.

PROBLEM XIV.

To produce a rectangle equal to a given square.

Suppose $A B C D$ be the given square, also $B E$ one end of the required rectangle, draw $E F$ parallel to $B C$, join $B N$, continue the side of the square $B C$, and draw the line $A G$ parallel with $B N$, until it intersects at G , then $B G$ is the side of the rectangle required.

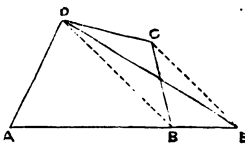


PROBLEM XV.

To make a triangle equal to a given quadrilateral figure.

Let $A B C D$ be the given quadrilateral figure.

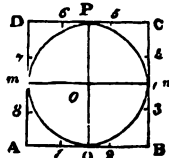
Join D and B , and from C draw $C E$ parallel to $B D$; produce $A B$ to E , and join D and E ; then the triangle $A D E$ is the triangle required.



PROBLEM XVI.

To circumscribe a square about a given circle.

Draw two diameters at right angles as $m n$ and $O P$; from $m n$ $O P$, as centres, with the radius of the circle, describe arcs cutting each other in $A B C$ and D ; join $A B$, $B C$, $C D$, $D A$, and $A B C D$ will be the square required.



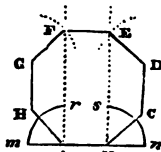
And from A as a centre, with the distance $A o$, cut

the lines A B, A D, in 2 and 7; from B as a centre cut the lines B A, B C, in 1 and 4; from C as a centre cut the lines C B, C D, in 3 and 6; and from D as a centre cut the lines D C, D A, in 5 and 8; join 1, 8; 2, 3; 4, 5; and 6, 7; and 1, 2, 3, 4, 5, 6, 7, 8, will be a regular octagon.

PROBLEM XVII.

Upon a right line to describe an octagon.

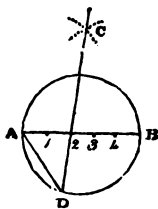
On the extremities of one side A B, erect the perpendiculars A F and B E; continue the line A B to A *m* and B *n*, forming the angles *m* A *r* and *n* B *s*; bisect the angles with the lines A H and B C; make each of those lines equal to A B; make H G and C D the same length, and parallel to A F and B E: from G and D as centres with the radius A B, describe arcs cutting A F and B E; join G F, F E, and E D, then A B C D E F G H will be the octagon required.



PROBLEM XVIII.

In a given circle to inscribe any regular polygon.

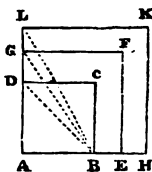
Divide the diameter A B into as many equal parts as the polygon is required to have sides; from A and B as centres, with the distance A B, describe arcs cutting each other in C; draw a line through the second division, meeting the circumference at D; join A D, and it will be the side of the polygon required.



PROBLEM XIX.

To find the side of a square that shall be any number of times the area of a given square.

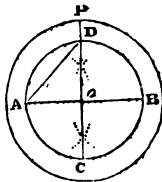
Let $A B C D$ be the given square, then will the diagonal $B D$ be the side of a square $A E F G$, double in area to the given square $A B C D$; and if the diagonal be drawn from B to G , it will be the side of a square $A H K L$, three times the area of the square $A B C D$, or the diagonal $B L$ will equal the side of a square four times the area of the square $A B C D$, &c.



PROBLEM XX.

To find the diameter of a circle that shall be any number of times the area of a given circle.

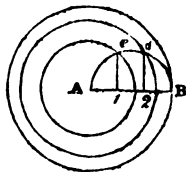
Let $A B C D$ be the given circle; draw the two diameters $A B$ and $C D$ at right angles to each other, and the chord $A D$ will be the radius of the circle $o P$, twice the area of the given circle nearly; and half of this chord $A D$ will be the radius of a circle that will contain half the area, &c.



PROBLEM XXI.

To divide a given circle into any number of co-centric parts equal to each other.

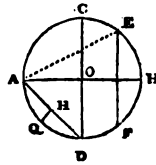
Upon the radius $A B$ describe the semicircle $A e d B$; divide $A B$ into the proposed number of equal parts, as 1, 2, &c.; erect the perpendiculars $1 e$, $2 d$, &c., meeting the semicircle in e and d ; then from the centre A , and radii $A e$, $A d$, &c., describe circles; so shall the circle be divided into the proposed number of equal parts as required.



PROBLEM XXII.

To find the side of a square nearly equal in area to a given circle.

Draw the two diameters AB and CD at right angles to each other, bisect the radius OC by a line from one end of the diameter at A , meeting the circumference in E , then will the line AE be the side of a square nearly equal in area to the given circle.



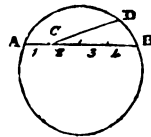
And if the line EF be drawn parallel to CD , it will be $\frac{1}{4}$ of the circumference nearly.

Or three times the diameter AB or CD , and once the versed sine QH , of the angle AOD , will be the circumference nearly.

PROBLEM XXIII.

To find a right line that shall be nearly equal to any given arc of a circle.

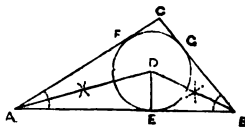
Divide the chord AB into four equal parts, set one part on the circumference from B to D , draw a line from C , the first division on the chord; and twice the length of the line CD will be the length of the arc nearly.



PROBLEM XXIV.

To describe the largest possible circle in a triangle.

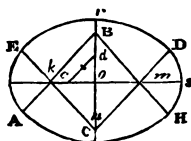
Let ABC be a triangle. Bisect the two angles A and B , (by Problem II.) and from D , where the lines AD and BD meet, draw DE perpendicular to AB ; then, with D as centre, and DE as radius, describe the circle EFG , which is the one required.



PROBLEM XXV.

To describe an ellipsis, the transverse and conjugate diameters being given.

From o , as a centre, with the difference of the transverse and conjugate semi-diameters, set off $o c$ and $o d$; draw the diagonal $c d$, and continue the line $o c$ to k , by the addition of half the diagonal $c d$, then will the distance $o k$ be the radius of the centres that will describe the ellipsis; draw the lines $A B$, $C D$, $C E$, and $B H$, cutting the semi-diameters of the ellipsis in the centres $k B m n$; then with the radius $m s$, and with k and m as centres, describe the arcs $D H$ and $A E$; also, with the radius $n r$, and with n and B as centres, describe the arcs $E D$ and $A H$, and the figure $A E D H$ will be the ellipsis required.



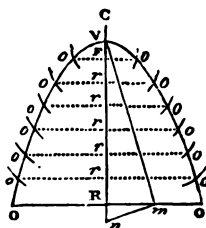
PROBLEM XXVI.

To describe a parabola, any ordinate to the axis and its abscissa being given.

Let $V R$ and $R o$ be the given abscissa and ordinate; bisect $R o$ in m , join $V m$, and draw $m n$ perpendicular to it, meeting the axis in n ; make $V C$ and $V F$ each equal to $R n$, then will F be the focus of the curve.

Take any number of points, $r, r, \&c.$, in the axis, and draw the double ordinates of an indefinite length.

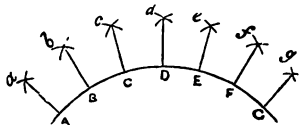
From F , as a centre, with the radii $C F$, $C r$, $\&c.$, describe arcs cutting the corresponding ordinates in the points $o o o o, \&c.$, and the curve $o V o$ drawn through all the points of intersection, will be the parabola required.



PROBLEM XXVII.

To draw at the circumference of a circle lines tending towards the centre when the centre is inaccessible.

Mark off upon any portion of the circumference any number of equal parts, and with any radius greater than the length of one division, but less than that of two, and with the centres A, B, C, D, &c., describe arcs cutting each other in *b*, *c*, *d*, &c.; then the required lines may be drawn by joining *a* A, *b* B, *c* C, and so on.

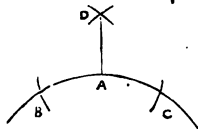


To draw the end lines a A, g G.

With the distance A *b* as radius, and with the centres B and F, describe the arcs *a* and *g*; then, with the distance B *b* as radius, and the centres A and G, describe arcs cutting the former ones, and at the points of intersection *a* and *g* draw A *a* and g G, which will be the required straight lines.

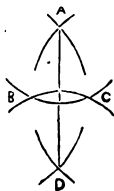
From any given point in the circumference (as A in the annexed figure) to draw a line tending towards the centre.

With A as centre, and with any radius, cut the circle in B and C; then, with B and C as centres and a radius greater than the former, describe arcs cutting each other in D; join D and A, and the line D A will be the one required.



To draw from a point, without the circumference, a line tending towards the centre.

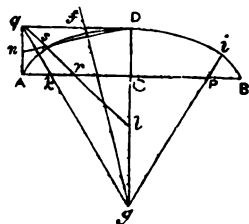
Let A be the given point. With A as centre, and with any convenient radius, describe an arc cutting the circumference in B and C; with B and C as centres, and the distance B A as radius, describe arcs cutting each other in A and D; then join A and D, and the line so drawn will be the one required.



PROBLEM XXVIII.

To describe an elliptical arc, the width and rise of span being given.

Bisect the chord or width of span $A B$, and at the point of section C erect a perpendicular $C D$ equal to the height of span; erect a perpendicular also at A , making $A q$ equal to $C D$; join q and D , and bisect $A q$ in n , and $A C$ in r ; join n and D , and from q through r draw the line $q l$, meeting the line $D C$ produced. Now, bisect the line $s D$ in f , and at the point of section erect at right angles the line $f g$, also meeting the line $D C$ produced, join g and q , and the line so drawn will cut the line $A B$ in k ; make $C P$ equal to $C k$, and through P draw the line $g i$; then, with g as centre and $g D$ as radius, describe the arc $s D i$, and with k and P as centres and $k A$ and $P B$ as radii, describe the arcs $A s$ and $B i$; the construction of the arc will then be completed.



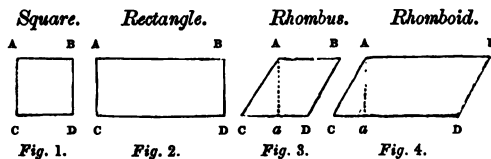
MENSURATION.

Mensuration is the method of calculating the comparative magnitudes of figures; and it is divided into two parts,—Mensuration of Superficies or Surfaces, and Mensuration of Solids.

The magnitude of a surface is called its area, and is the space enclosed between its boundary lines.

The magnitude of a body is called its solid contents, and is expressed in cubic feet, inches, &c.

MENSURATION OF SUPERFICIES.



A **SQUARE** is a quadrilateral figure which has all its sides equal, and all its angles right angles.

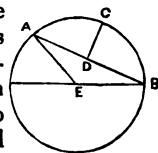
A **RECTANGLE** is a four-sided figure which has its angles right angles, and its opposite sides parallel.

A **RHOMBUS** is a parallelogram whose sides are equal, but whose angles are not right angles.

A **RHOMBOID** is a parallelogram whose adjacent sides are unequal, and whose angles are not right angles.

A **TRAPEZOID** is a four-sided figure which has but two of its sides parallel.

A **CIRCLE** is a figure bounded by one line called the circumference; and is such, that all lines drawn to the circumference from a certain point within the figure called the centre are equal to each other. Any of these lines is called



a radius; and a line drawn through the centre, terminating both ways in the circumference, is called a diameter. The portion of circle cut off by a diameter is called a semicircle.

An **ARC** of a circle is any portion of the circumference.

A **SEGMENT** of a circle is a figure contained by an arc and its chord.

A **VERSED SINE** is a line drawn from the middle of a chord perpendicular to the circumference.

A **SECTOR** of a circle is a figure contained by two radii and an arc, as *ACB E*.

Some useful Properties of Numbers employed in the Solutions of the following Problems.

1. Half the sum of any two numbers increased by half their difference, will give the greater number; and half their sum diminished by half their difference, will give the less number.

2. The quotient arising from the division of the sum of two or more numbers, is equal to the sum of the quotients arising from the division of the parts separately, by the same divisor.

3. Any three of the four following quantities of a division sum, *viz.* divisor, dividend, quotient, and remainder, being given, the fourth may be found by the following formula. Let *d* = the divisor; *D* = the dividend; *q* = the quotient; and *r* = the remainder. Then we shall have

$$d = \frac{D-r}{q}; D = (d \times q) + r; q = \frac{D-r}{d}; \text{ and } r = D - (q \times d).$$

4. An even number cannot divide or measure an odd number, nor a greater a less.

5. A given number is divisible by 2 if the last digit is even; it is divisible by 4 if the last two digits are divisible by 4; it is divisible by 8 if the last three digits are divisible by 8; and in general it is divisible by 2^n , if the last *n* digits are divisible by 2^n .

6. A number is divisible by 3 if the sum of the digits is divisible by 3; such number also may be divided by 6 if, besides this, the last digit is even; it is also divisible by 9 if the sum of the digits can be divided by 9. The method of proof, by casting out the nines, in Addition, Multiplication, and Division, depends upon this theorem.

7. Every number that has the last digit 5 or 0, such number is divisible by 5 in both cases, and by ten in the latter case.

8. A number is divisible by 11 when the sum of the digits in the odd places (counting from the right or unit's place) be equal to the sum of the digits in the even places; or if the difference of these sums be divisible by 11, the number itself is divisible by 11.

9. If any two numbers be separately divided by 9 or 3, and the two remainders multiplied together, and that product divided by 9 or 3, the last remainder will be the same as if you divided the product of the first two numbers by 9 or 3.

10. If any number ending with 1, 3, 7, or 9, be the numerator or denominator of a fraction, and will not divide by 3, 7, or 9, that fraction is *generally* in its lowest terms, for

Every number must terminate in one	}	0 1 2 3 4 5 6 7 8 9
or other of the ten digits		
But no even number can be a prime	}	. . . 2 . 4 . 6 . 8 .
number; hence take away		
We have remaining		0 1 . 3 . 5 . 7 . 9
No number terminating in 0 or 5 can	}	0 5
be prime		
Hence it follows that every prime	}
number must terminate in one or		1 . 3 . . . 7 . 9
other of these four digits		

But of such numbers none the sum of whose digits is a multiple of 3 can be prime. Nor any terminating in 1, 3, 7, 9, that is in any power or multiple of another number.

11. Every prime number is of one of the forms $8n + 1$, $8n + 3$, $8n + 5$, $8n + 7$; $12n + 1$, $12n + 5$, $12n + 7$, $12n + 11$; $4n \pm 1$; $6n \pm 1$; $16n \pm 1$, $16n \pm 3$, $16n \pm 5$, $16n \pm 7$; $60n \pm 1$, $60n \pm 7$, $60n \pm 11$, $60n \pm 13$, $60n \pm 17$, $60n \pm 19$, $60n \pm 23$, $60n \pm 29$.

12. When any prime number (above the number 3) is either increased or diminished by unity, the result is always divisible by 6; thus, take for example, $(173 + 1) \div 6 = 174 \div 6 = 29$, and $(5749 - 1) \div 6 = 5748 \div 6 = 958$.

13. When any set of numbers are placed in the form of fractions, with the sign of addition or subtraction between them, and should the whole of these numbers that are in the numerator and denominator contain a common divisor, they may be abbreviated by dividing each of them by the common divisor.

14. When the numerator is equal to the denominator, the fraction is equal to the integer; thus, $\frac{3}{3} = 1$.

15. When the numerator is greater than the denominator, the fraction is greater than the integer; as, $\frac{7}{3} = 1\frac{1}{3}$.

16. If the numerator and denominator of a fraction be either multiplied or divided by the same number, the product or quotient will be a new fraction equal in value to the former; thus, $\frac{3}{3} \div \frac{3}{3} = \frac{1}{1}$, or $\frac{3}{3} \times \frac{3}{3} = \frac{9}{9}$, all of which have the same value, for $\frac{1}{1} = \frac{3}{3} = \frac{9}{9}$.

PROBLEM I.

To find the area of any parallelogram.

RULE.—Multiply the length by the perpendicular height, and the product will be the area.

EXAMPLE.—Required the area of a rhomboid $A B D C$ (*Fig. 4, page 41*) whose length $A B = 20.5$, and perpendicular height $a A = 11.75$.

Here $A B \times a A = 20.5 \times 11.75 = 240.875$, the area of the rhomboid $A B D C$.

NOTE.—In a square (*Fig. 1*), or rectangle (*Fig. 2, page 41*), the perpendicular height is the breadth; therefore, to find the areas of a square and rectangle, multiply the length by the breadth.

PROBLEM II.

To find the area of a trapezoid.

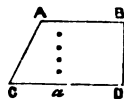
RULE.—Add together the two parallel sides, multiply their sum by the breadth or height, and half the product is the area.

EXAMPLE.—Required the area of a trapezoid whose sides $A B$ and $C D$ are 14.5 and 10.25 , and breadth $a A = 7.25$.

$$\text{Here } \frac{(A B + C D) \times a A}{2} =$$

$$\frac{(14.5 + 10.25) \times 7.25}{2} = 89.71875,$$

the area of the trapezoid $A B D C$.



PROBLEM III.

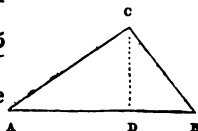
To find the area of a triangle.

RULE.—Multiply one of its sides as a base by a perpendicular let fall from the opposite angle, and take half the product for the area.

Or, from half the sum of the three sides subtract each side separately, and multiply the three remainders so obtained and the half sum together, and the square root of the product will be the area.

EXAMPLE 1.—Required the area of a triangle $A B C$, whose base $A B = 16.5$, and perpendicular $D C = 10.25$.

Here $\frac{A B \times D C}{2} = \frac{16.5 \times 10.25}{2}$
 $= 84.5625$, the area of the triangle $A B C$.



EXAMPLE 2.—What is the area of the triangle $A B C$, whose sides are $B C = 8$, $A C = 12$, and $A B = 16$ respectively?

Here $\frac{1}{2} (A B + A C + B C) = \frac{1}{2} (16 + 12 + 8)$
 $= 18 =$ half the sum of the three sides.

$$\left. \begin{array}{l} 18 - 16 = 2 \\ 18 - 12 = 6 \\ 18 - 8 = 10 \end{array} \right\} \text{the three remainders.}$$

Then $\sqrt{(18 \times 2 \times 6 \times 10)} = \sqrt{2160} = \sqrt{(3^2 \times 4^2 \times 15)} = 12 \sqrt{15} = 12 \times 3.873 = 46.476 =$
the area of the triangle $A B C = \text{Ans.}$

PROBLEM IV.

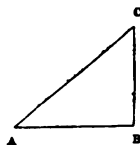
If any two sides of a right-angled triangle be given, the third side may be found by the following rules.

1.—To the square of the base add the square of the perpendicular; and the square root of the sum will be the hypotenuse or longest side.

2.—Multiply the sum of the hypotenuse, and one side by their difference; and the square root of the product will be the other side.

EXAMPLE 1.—Given the base $A B = 16$, and perpendicular $B C = 12$; required the length of the hypotenuse $A C$.

Here $\sqrt{(A B^2 + B C^2)} = \sqrt{(16^2 + 12^2)} = \sqrt{(256 + 144)} = \sqrt{400} =$
 $\sqrt{20^2} = 20$ the length of the hypotenuse $A C$.



EXAMPLE 2.—Given the base $AB = 16$, and hypotenuse $AC = 20$; required the length of the perpendicular BC .

Here $\sqrt{\{(AC + AB) \times (AC - AB)\}} = \sqrt{\{(20 + 16) \times (20 - 16)\}} = \sqrt{(36 \times 4)} = \sqrt{(6^2 \times 2^2)} = 6 \times 2 = 12 = \text{perpendicular } BC = \text{Ans.}$

NOTE.—The diagonal line, or hypotenuse in a square, is equal to the square root of twice the square of the side. And the side of a square is equal to the square root of half the square of its diagonal.

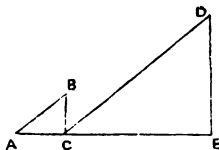
Thus, suppose each side of a square equal 12 feet, find the diagonal.

Here (by Fig. 1, page 41) $\sqrt{(AB^2 \times 2)} = \sqrt{(12^2 \times 2)} = \sqrt{288} = 16.97056$ feet, the diagonal from A to D .

Also, $\sqrt{(16.97056^2 \div 2)} = \sqrt{(288 \div 2)} = \sqrt{144} = 12$ feet, the length of each side.

Similar triangles, or those which are equi-angular to each other, have the sides about their equal angles proportional; thus, in the annexed figure the triangles ABC and CDE are similar, and, therefore, have the sides about the equal angles proportional,

$$\begin{aligned} AC : BC &:: CE : DE; \\ AB : BC &:: CD : DE, \&c. \end{aligned}$$



The utility, then, of the above triangles for practical purposes, as, for instance, ascertaining the heights of buildings, &c., will be seen from the following:—

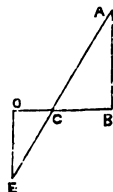
Suppose DE to be an eminence, of which it is required to find the height, and EC the length of the shadow cast by the sun; then, in order to find DE , we may erect perpendicularly at C a pole of any known length, as BC , and after measuring the length of its shadow AC , state—as the length of the pole's shadow is to the height of the pole itself, so is the length of the shadow of DE to the height of DE ; or,

$$\text{As } AC : CB :: CE : ED;$$

and supposing $AC = 6$ feet, $BC = 4$ feet, and $CE = 30$ feet, then ED would be 20 feet.

Again, supposing we wished to find the distance between two objects A and B ; draw DB of any length at right angles to AB , and in DB take any point C , through which draw AE ; also, at D , at right angles to DB , draw DE , making the triangle DEC , and state,

$$\text{As } DC : DE :: BC : BA.$$



PROBLEM V.

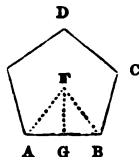
To find the area of any regular polygon from three to fifty sides.

RULE.—Multiply the sum of its sides by a perpendicular drawn from its centre to one of its sides, and take half the product for the area.

Or, multiply the square of the given side of a regular polygon by the numbers in the *sixth* column (*headed area of the polygon*), opposite its name and given number of sides, and the product will be the area.

EXAMPLE 1.—Required the area of the regular pentagon $ABCDE$, each side being 7.5, and perpendicular $FG = 5.1614325$. (*See Table, column 6.*)

$$\begin{aligned} \text{Here } \frac{\text{sum of sides} \times FG}{2} &= \\ \frac{AB \times 5 \times FG}{2} &= \frac{7.5 \times 5 \times 5.1614325}{2} \\ &= 96.77685 \text{ the area of the pentagon } \\ &\quad ABCDE. \end{aligned}$$



EXAMPLE 2.—What is the area of a regular hexagon, each side being 8.75 in length? (*See Table, column 6.*)

$$\text{Here } 8.75^2 \times 2.598 = 199.009375 \text{ the area.} = \text{Ans.}$$

Table of Multipliers for regular Polygons, from three to fifty Sides.

No. of Sides.	Name of the Polygon.	Radius of the circumscribing Circle, that of the inscribed being 1.	Side of the Polygon.	Radius of the circumscribing Circle, the side being 1.	Area of the Polygon.
3	Trigon	2.0000000	1.7320508	.5773503	.4330127
4	Tetragon	1.4142136	1.4142136	.7071068	1.0000000
5	Pentagon	1.2360680	1.1755706	.8506508	1.7204774
6	Hexagon	1.1547005	1.0000000	1.0000000	2.5980762
7	Heptagon	1.1099163	.8677676	1.1523825	3.6399126
8	Octagon	1.0823922	.7653668	1.3065630	4.8284272
9	Nonagon	1.0641778	.6840402	1.4619022	6.1818242
10	Decagon	1.0514622	.6180340	1.6180340	7.6942088
11	Undecagon	1.0422171	.5634652	1.7747331	9.3656415
12	Duodecagon	1.0352762	.5176380	1.9318517	11.1961524
13	Tridecagon	1.0299279	.4786312	2.0892913	13.1857718
14	Tetradecagon	1.0257169	.4450418	2.2469806	15.3345084
15	Pentadecagon	1.0223406	.4158234	2.4048672	17.6423629
16	Hexadecagon	1.0195912	.3901806	2.5629155	20.1093580
17	Heptadecagon	1.0173219	.3674990	2.7210970	22.7355038
18	Octadecagon	1.0154266	.3472964	2.8798853	25.5207681
19	Nonadecagon	1.0138273	.3291892	3.0377692	28.4652110
20	Eicosagon	1.0124651	.3128690	3.1962266	31.5687575
21	Uneicosagon	1.0112954	.2980846	3.3547557	34.8315014
22	Duoicosagon	1.0102833	.2846296	3.5133383	38.2533531
23	Triseicosagon	1.0094016	.2723332	3.6719752	41.8344039
24	Tetreicosagon	1.0086290	.2610524	3.8306488	45.5745246
25	Penteicosagon	1.0079480	.2506664	3.9893649	49.4738444
26	Hexeicosagon	1.0073447	.2410734	4.1481206	53.5323900
27	Hepteicosagon	1.0068077	.2321858	4.3068951	57.7499409
28	Octeicosagon	1.0063276	.2239290	4.4657083	62.1268039
29	Noneicosagon	1.0058966	.2162380	4.6245414	66.6627662
30	Triaccontagon	1.0055083	.2090570	4.7833861	71.3577338
31	Untriacontagon	1.0051571	.2023366	4.9422682	76.2121205
32	Duotriacontagon	1.0048386	.1960344	5.1011600	81.2255440
33	Tritiacontagon	1.0045487	.1901122	5.2600645	86.3980681
34	Tetracontagon	1.0042841	.1845368	5.4189880	91.7298172
35	Pentaccontagon	1.0040420	.1792786	5.5779245	97.2207110
36	Hextriacontagon	1.0038198	.1743114	5.7368565	102.8704680
37	Heptriacontagon	1.0036155	.1696120	5.8958213	108.6797596
38	Octriacontagon	1.0034272	.1651586	6.0547953	114.6481841
39	Nonatricacontagon	1.0032533	.1609332	6.2137666	120.7755354
40	Tesseraccontagon	1.0030922	.1569182	6.3727475	127.0620500
41	Untesseraccontagon	1.0029429	.1530984	6.5317653	133.5082939
42	Dutesseraccontagon	1.0028040	.1494602	6.6907572	140.1130227
43	Trutesseraccontagon	1.0026749	.1459906	6.8497696	146.8771694
44	Tetratesseraccontagon	1.0025544	.1426784	7.0087998	153.8007306
45	Pentatesseraccontagon	1.0024419	.1395130	7.1677935	160.8824925
46	Hexatesseraccontagon	1.0023367	.1364848	7.3268444	168.1245663
47	Heptatesseraccontagon	1.0022382	.1335852	7.4858794	175.5253176
48	Octatesseraccontagon	1.0021457	.1308062	7.6448940	183.0846240
49	Nonatesseraccontagon	1.0020588	.1281404	7.8039765	190.8045888
50	Penteccontagon	1.0019772	.1255810	7.9629855	198.6818125

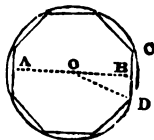
PROBLEM VI.

The side of a regular polygon being given, to find the radius of the circumscribing circle.

RULE.—Multiply the given side by that number in the fifth column of the table which is opposite the number of sides; the product will be the radius of the circumscribing circle.

EXAMPLE 3.—Required the radius O D of the circle circumscribing an octagon, the length of its side C D whereof is 7·6629502 inches. (See Table, column 5.)

Here $DC \times 1\cdot3065630 = 7\cdot6629502$
 $\times 1\cdot3065630 \text{ in.} = 10\cdot0121272 \text{ inches}$
 $= OD. = \text{Ans.}$



PROBLEM VII.

The radius of a circle being given, to find the length of the side of a regular polygon inscribed in it.

RULE.—Multiply the given radius by that number in the fourth column of the table which is opposite the given number of sides; the product will be the length of the side.

EXAMPLE 4.—Given the radius O D = 10·0121272 in., to find C D, the side of the inscribed octagon. (See Table, column 4.)

Here $OD \times \cdot7653668 = 10\cdot0121272 \times \cdot7653668$
 $= 7\cdot6629498 \text{ inches} = CD. = \text{Ans.}$

PROBLEM VIII.

The radius of the circle inscribed in a regular polygon being given, to find the radius of the circle circumscribing that polygon.

RULE.—Multiply the given inscribed radius by that number in the third column of the table which is oppo-

site to the given number of sides ; the product will be the length of the required radius.

EXAMPLE 5.—The radius O B of the inscribed circle of an octagon being 9·25 inches, required the radius O D of the circumscribing circle. (*See Table, column 3.*)

Here $O B \times 1\cdot0823922 = 9\cdot25 \times 1\cdot0823922$ in.
 $= 10\cdot0121278$ in. = O D. = *Ans.*

PROBLEM IX.

Having the diameter of a circle given, to find the circumference ; or the circumference given, to find the diameter.

RULE 1.—By the ratio of Archimedes—as 7 is to 22 so is the given diameter to the circumference.

And, as 22 is to 7 so is the circumference to the diameter ; or, multiply the given circumference by 3·18309*, the product will be the diameter *more nearly*.

2.—Write down the first three odd numbers, each twice, thus, 113355 ; separate these six figures into two parts, as 113 and 355, then we have the ratio of Metius—As 113 is to 355 so is the diameter to the circumference.

And, as 355 is to 113 so is the circumference to the diameter.

NOTE.—The ratio of the diameter of a circle to its circumference has never yet been *exactly* determined : if the diameter be 1, then will the circumference be

greater than 3·14159 2653
 but less than 3·14159 2654.

The ratio of Archimedes is 1 to 3·14285 7143 nearly ; that of Metius, is 1 to 3·14159 2920 nearly.

To four places of decimals we may take 3·1416 as being sufficiently exact for most practical purposes.

EXAMPLE 1.—Required the circumference of a circle when the diameter is 23·5.

* This number is the reciprocal of 3·1416.

Here $\frac{23.5 \times 22}{7} = \frac{517}{7} = 73\frac{6}{7}$, the circumference. =

Ans.

EXAMPLE 2.—The circumference of a circle is $73\frac{6}{7}$, required the diameter.

Here $\frac{73\frac{6}{7} \times 7}{22} = \frac{517}{22} = 23.5$, the diameter. = *Ans.*

EXAMPLE 3.—Required the circumference of a circle whose diameter is 30.

Here $\frac{355 \times 30}{113} = \frac{10650}{113} = 94.248$, the circumference. = *Ans.*

EXAMPLE 4.—What is the diameter of a circle when the circumference is 94.248?

Here $94.248 \div 3.1416 = 30$, the diameter. = *Ans.*

Or thus, $.318309 \times 94.248 = 29.9999866$. = *Ans.* as before.

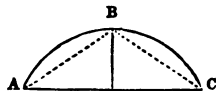
PROBLEM X.

To find the length of any arc of a circle.

RULE.—Subtract the chord of the whole arc from eight times the chord of half the arc; and $\frac{1}{3}$ of the remainder is the length of the arc *nearly*.

EXAMPLE.—Required the length of the arc A B C; the chord of half the arc A B = 19.8, and the chord A C of the whole arc = 34.4.

Here
$$\frac{(A B \times 8) - A C}{3} = \frac{(19.8 \times 8) - 34.4}{3} = \frac{158.4 - 34.4}{3} = \frac{124}{3} = 41.3\frac{1}{3}$$
, the length of the arc A B C. = *Ans.*



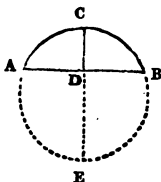
PROBLEM XI.

To find the diameter of a circle by having the chord and versed sine given.

RULE.—Divide the square of half the chord by the versed sine, to the quotient of which add the versed sine, and the sum will be the diameter.

Or, if the sum of the squares of the semichord and versed sine be divided by the versed sine, the quotient will be the diameter of the circle to which that segment corresponds.

EXAMPLE.—Given the chord $AB = 24$, and versed sine $CD = 8$; required the diameter of the circle CE .



Here $\left\{ \left(\frac{AB}{2} \right)^2 \div CD \right\} + CD = (12^2 \div 8) + 8 = (144 \div 8) + 8 = 18 + 8 = 26$, the diameter. = *Ans.*

Or thus,

$\left\{ \left(\frac{AB}{2} \right)^2 + CD^2 \right\} \div 8 = (12^2 + 8^2) \div 8 = (144 + 64) \div 8 = 208 \div 8 = 26$, the diameter, *as before.*

PROBLEM XII.

To find the area of a parabola, or its segment.

RULE.—Multiply the base by the perpendicular height, and two-thirds of the product is the area.

EXAMPLE.—What is the area of a parabola, whose base is 20 feet, and perpendicular height 12?

Here $\frac{2}{3} \times 20 \times 12 = 2 \times 20 \times 4 = 160$ feet, the area.

Table of Versed Sines, in inches, quarters, and fractions of a quarter, by which to ascertain the diameters of circles corresponding to any segment or part of a circle having a chord of three feet.

Length of chord three feet.	Versed sine in inches.		Corresponding diameter in ft. and in.		Versed sine in inches.		Corresponding diameter in ft. and in.		Versed sine in inches.		Corresponding diameter in ft. and in.	
	in.	grs.	ft.	in.	in.	grs.	ft.	in.	in.	grs.	ft.	in.
	6..0		5..0		2..3		10..0		1..2		18..0	
	5..2		5..3		2..2 $\frac{2}{3}$		10..3		1..1 $\frac{7}{8}$		18..6	
	5..1		5..6		2..2 $\frac{1}{2}$		10..6		1..1		19..0	
	5..0		5..9		2..2 $\frac{1}{4}$		10..9		1..1 $\frac{1}{2}$		19..6	
	4..3		6..0		2..2		11..0		1..1 $\frac{1}{4}$		20..0	
	4..2		6..3		2..1 $\frac{1}{2}$		11..3		1..1 $\frac{1}{8}$		21..0	
	4..1		6..6		2..1 $\frac{1}{4}$		11..6		1..0 $\frac{7}{8}$		22..0	
	4..0		6..9		2..1 $\frac{1}{8}$		11..9		1..0 $\frac{3}{4}$		23..0	
	4..0		7..0		2..1		12..0		1..0 $\frac{1}{2}$		24..0	
	3..3		7..3		2..0 $\frac{3}{4}$		12..6		1..0 $\frac{1}{4}$		25..0	
	3..3		7..6		2..0 $\frac{1}{2}$		13..0		1..0		26..0	
	3..2 $\frac{1}{2}$		7..9		2..0 $\frac{1}{4}$		13..6		1..0		27..0	
	3..2		8..0		1..3 $\frac{3}{4}$		14..0		1..0		28..0	
	3..1 $\frac{1}{2}$		8..3		1..3 $\frac{1}{2}$		14..6		0..3 $\frac{3}{4}$		29..0	
	3..1		8..6		1..3 $\frac{1}{4}$		15..0		0..3 $\frac{1}{2}$		30..0	
	3..0		8..9		1..3		15..6		0..3 $\frac{1}{4}$		35..0	
	3..0		9..0		1..2 $\frac{3}{4}$		16..0		0..2 $\frac{3}{4}$		40..0	
	3..0		9..3		1..2 $\frac{1}{2}$		16..6		0..2 $\frac{1}{2}$		45..0	
	2..3 $\frac{1}{2}$		9..6		1..2 $\frac{1}{4}$		17..0		0..2 $\frac{1}{4}$		50..0	
	2..3		9..9		1..2		17..6					

Table of the relative Proportions of the Circle, its equal and inscribed squares.

1. The Diameter of a circle ..	× 8862	} = the side of an equal square.
2. „ Circumference	× 2821	
3. „ Diameter	× 7071	} = the side of an inscribed square.
4. „ Circumference	× 2251	
5. „ Area	× 6366	} = the area of an inscribed square.
6. „ Side of inscribed square ×	1.4142	
7. „ Side of inscribed square ×	4.443	} = the circumference of a circumscribing circle.
8. „ Side of a square	× 1.28	
9. „ Side of a square	× 3.545	} = the circumference of an equal circle.

Examples illustrative of the preceding table.

EXAMPLE 1.—The diameter of a circle is 12·5; required the side of a square equal in area to the given circle.

Here $12\cdot5 \times \cdot8862 = 11\cdot07750$, side of equal square.
= *Ans.*

Ex. 2.—The circumference of a circle being 53·4; required the side of a square equal in area.

Here $53\cdot4 \times \cdot2821 = 15\cdot06414$, side of equal square. = *Ans.*

Ex. 3.—The diameter of a circle being 18; required the side of the greatest square that can be inscribed therein.

Here $18 \times \cdot7071 = 12\cdot7278$, side of inscribed square. = *Ans.*

Ex. 4.—The circumference of a circle is 86; required the side of inscribed square.

Here $86 \times \cdot2251 = 19\cdot3586$, side of inscribed square. = *Ans.*

Ex. 5.—The area of a circle being 371·5; required the area of the greatest square that can be inscribed within the circle.

Here $371\cdot5 \times \cdot6366 = 236\cdot49690$, area of the square.
= *Ans.*

Ex. 6.—The side of a square being 19·375; required the diameter of its circumscribing circle.

Here $19\cdot375 \times 1\cdot4142 = 27\cdot4001250$, diameter. = *Ans.*

Ex. 7.—Required the circumference of a circle to circumscribe a square, each side being 19·375.

Here $19\cdot375 \times 4\cdot443 = 86\cdot083125$, circumference of the circle. = *Ans.*

Ex. 8.—The side of a square being 13·5; required the diameter of a circle equal in area to the given square.

Here $13\cdot5 \times 1\cdot128 = 15\cdot228$, diameter of the circle.
= *Ans.*

Ex. 9.—The side of a square being 13·5; required the circumference of a circle equal in area to the given square.

Here $13\cdot5 \times 3\cdot545 = 47\cdot8575$, circumference of the circle. = *Ans.*

Some of the properties of a circle.

1.—It is the most capacious of all plain figures, or contains the greatest area within the same perimeter or outline.

2.—The areas of circles are to each other as the squares of their diameters, or of their radii.

3.—Any circle whose diameter is double that of another, contains four times the area of the other.

4.—The area of a circle is equal to the area of a triangle whose base is equal to the circumference, and perpendicular equal to the radius.

5.—The area of a circle is equal to the rectangle of its radius, and a right line equal to half its circumference.

6.—The area of a circle is found by squaring the diameter, and multiplying by ·7854; or by multiplying the circumference by the radius, and dividing the product by two.

EXAMPLE 1.—Required the area of a circle, the diameter being 30·5.

Here $30\cdot5^2 \times \cdot7854 = 730\cdot618350$, the area required.

EXAMPLE 2.—What is the area of a circle when the diameter is 1?

In this case the circumference is 3·1416, half of which is 1·5708, and half of 1 = ·5; then $1\cdot5708 \times \cdot5 = \cdot7854$, the area.

PROBLEM XIII.

Having the area of a circle given, to find the diameter.

RULE.—Multiply the area by 452 and divide the product by 355, the square root of the product will be the diameter.

Or, multiply the square root of the area by 1·12838, and the product will be the diameter.

Or, divide the area by ·7854, and extract the square root.

EXAMPLE.—Required the diameter of that circle whose area is 122·71875.

$$\begin{aligned} \text{Here } \sqrt{\frac{122\cdot71875 \times 452}{355}} &= \sqrt{\frac{3927 \times 113}{2840}} \\ &= \sqrt{\frac{392\cdot7 \times 113 \times 284}{284}} = \sqrt{\frac{12602528\cdot4}{284}} = \frac{3550}{284} \\ &= 12\cdot5 \text{ diameter.} = \text{Ans.} \end{aligned}$$

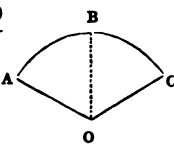
Or thus, $\sqrt{122\cdot71875} = 11\cdot078$; and $11\cdot078 \times 1\cdot12838 = 12\cdot500194 = 12\cdot5 \text{ diameter.} = \text{Ans. as before.}$

PROBLEM XIV.

To find the area of a sector of a circle.

RULE.—Multiply the length of the arc by the radius of the circle, and half the product will be the area.

EXAMPLE.—Required the area of a sector of a circle whose arc A B C = 26·666, and radius B O = 16·9.

$$\begin{aligned} \text{Here } \frac{A B C \times B O}{2} &= \frac{26\cdot666 \times 16\cdot9}{2} \\ &= \frac{450\cdot6554}{2} = 225\cdot3277 \text{ the area of } \end{aligned}$$


the sector A B C O A. = Ans.

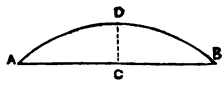
PROBLEM XV.

To find the area of a segment of a circle.

RULE.—Multiply the versed sine by ·626, to the square of the product add the square of half the chord (or multiply the square of the versed sine by ·391876, and to the product add the square of half the chord); multiply twice the square root of the sum by $\frac{2}{3}$ of the versed sine; and the product will be the area.

EXAMPLE.—Required the area of a segment of a circle whose chord $AB = 48$, and versed sine $CD = 18$.

Here $2 \times \sqrt{.391876$



$\times C D^2 + \left(\frac{AB}{2}\right)^2 \times \frac{2 CD}{3} = 2 \times \sqrt{.391876 \times 18^2 + 24^2} \times \frac{2 \times 18}{3}$

$$= 24 \times \sqrt{(.391876 \times 324 + 576)} = 24 \times \sqrt{(126.967824 + 576)} = 24 \times \sqrt{702.967824} = 288 \times \sqrt{4.881721} = 2.20946 \times 288 = 636.32448 \text{ the area.} = \text{Ans.}$$

The following is a near approximate to the preceding rule :

To the cube of the versed sine, divided by twice the length of the chord, add $\frac{2}{3}$ of the product of the chord, multiplied by the versed sine ; and the sum will be the area of the segment nearly. Take the last example :—

Versed sine $CD = 18$, and chord $AB = 48$,

Here $\frac{CD^3}{2 AB} + \frac{2 \times AB \times CD}{3} = \frac{18^3}{2 \times 48} + \frac{2 \times 48 \times 18}{3} = \frac{5832}{96} + 2 \times 48 \times 6 = 60.75 +$

$576 = 636.75$ the area as before nearly.

Or, the area of a segment may be found by finding the area of a sector having the same radius as the segment ; then deducting the area of the triangle, leaves the area of the segment.

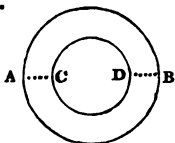
PROBLEM XVI.

To find the area of a circular ring or space included between two concentric circles.

RULE.—Add the inside and outside diameters together, multiply the sum by their difference, and by $.7854$; and the product will be the area.

EXAMPLE.—The diameters of two concentric circles, A B and C D, are 10 and 6; required the area of the ring or space contained between them.

Here $(A B + C D) \times (A B - C D)$
 $\times .7854 = (10 + 6) \times (10 - 6)$
 $\times .7854 = 16 \times 4 \times .7854 =$
 50.2656 the area. = *Ans.*



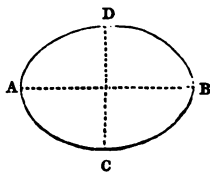
PROBLEM XVII.

To find the area of an ellipse.

RULE.—Multiply the transverse or longer diameter, by the conjugate or shorter diameter, and by .7854, and the product will be the area.

EXAMPLE.—Required the area of an ellipse whose longer diameter A B = 12, and shorter diameter C D = 9.

Here $A B \times C D \times .7854 =$
 $12 \times 9 \times .7854 = 84.8232$ the
 area. = *Ans.*



NOTE.—If half the sum of the two diameters be multiplied by 3.1416, the product will be the circumference of the ellipse, *near enough for most practical purposes.*

Thus $\frac{1}{2} (A B + C D \times 3.1416) = \frac{1}{2} (12 + 9) \times 3.1416 = 21$
 $\times 1.5708 = 32.9868$, the circumference required.

MENSURATION OF SOLIDS.

By solids are meant all bodies, whether solid, fluid, or bounded space, that can be comprehended within length, breadth, and thickness.

PROBLEM I.

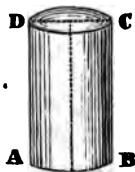
To find the convex surface and solid content of a cylindrical cylinder, or any figure of a cubical form.

RULE 1.—Multiply the circumference of the base by the height of the cylinder, and the product is the convex surface.

RULE 2.—Multiply the area of the base by the height of the cylinder, and the product is the solid content.

EXAMPLE 1.—Required the convex surface of the cylinder A B C D, whose base A B = 32 inches, and perpendicular height B C = 6 feet.

Here $3.1416 \times A B \times B C$
 $= 3.1416 \times 32 \times 72 \text{ inches} =$
 $7238.2464 \text{ square or superficial}$
 $\text{inches, and } 7238.2464 \div 144 =$
 $50.2656 \text{ superficial feet.} = \text{Ans.}$



EXAMPLE 2.—Required the solid content, in cubic inches and cubic feet, of the same cylinder as in Example 1.

Here $(A B^2 \times .7854) \times B C = (32^2 \times .7854) \times 72 = 1024 \times .7854 \times 72 = 57905.9712 \text{ cubic inches,}$
 $\text{and } 57905.9712 \div 1728 = 33.5104 \text{ cubic feet.}$

EXAMPLE 3.—Suppose the cylinder A B C D be intended to contain a fluid, and that the sides and bottom

are each one inch in thickness, how many imperial gallons would it contain?

Here $\{(A B - 2in.)^2 \times .7854\} \times (B C - 1in.) \div 277.274 = \{(32 - 2)^2 \times .7854\} \times (72 - 1) \div 277.274 = (30^2 \times .7854 \times 71) \div 277.274 = (900 \times .7854 \times 71) \div 277.274 = 50187.06 \text{ cubic in.} \div 277.274 = 181 \text{ imperial gallons.} = \text{Ans.}$

Or, since the reciprocal of $1 \div 277.274 = .003607$, therefore $50187.06 \text{ cubic in.} \times .003607 = 181 \text{ imperial gallons} = \text{Ans. as before.}$

PROBLEM II.

To determine the dimensions of any cylindrical vessel, whereby to contain the greatest cubical contents, bounded by the least superficial surface.

RULE.—Multiply the given cubical contents by 2.5465, and the cube root of the product will equal the diameter, and half the diameter equal the depth.

EXAMPLE 1.—Suppose a cylindrical vessel is to be made so as to contain 600 cubic feet, and of such dimensions as to require the least possible materials by which it is constructed, what must be its depth and diameter?

Here $\sqrt[3]{(600 \times 2.5465)} = \sqrt[3]{1527.9} = 11.5177 \text{ feet diameter, and } 11.5177 \div 2 = 5.75875 \text{ feet in depth.}$

NOTE.—If the vessel is to be made with two ends, then the cube root of four times the solidity divided by 3.1416 will equal both the length and diameter, so as to require the least possible quantity of materials in its construction.

EXAMPLE 2.—Suppose a cylindrical vessel is to be made with two ends, so as to contain 600 cubic feet, as in Example 1; required its depth and diameter, so that it may be constructed with the least possible quantity of materials.

Here $\sqrt[3]{\left(\frac{600 \times 4}{3 \cdot 1416}\right)} = \sqrt[3]{764} = 9 \cdot 1418$, the depth and diameter in feet, as required.

Or thus,
 $\sqrt[3]{(600 \times 4 \times \cdot 318309)} = \sqrt[3]{763 \cdot 941600} = 9 \cdot 1416$,
 the *Ans.* as before, nearly.

PROBLEM III.

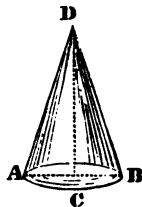
To find the surface and solid content of a cone or pyramid.

RULE 1.—Multiply the circumference of the base by the slant height, and half the product will be the slant surface, to which add the area of the base, and the product will be the whole surface.

RULE 2.—Multiply the area of the base by the perpendicular height, and $\frac{1}{3}$ of the product will be the solid content.

EXAMPLE 1.—Required the convex surface of a cone whose base $AB = 20$ inches, and slant height $BD = 29 \cdot 5$.

Here $\frac{1}{2} (3 \cdot 1416 \times AB \times BD) =$
 $\frac{1}{2} (3 \cdot 1416 \times 20 \times 29 \cdot 5) = 3 \cdot 1416$
 $\times 10 \times 29 \cdot 5 = 926 \cdot 772$ square
 inches, which divided by 144 = $6 \cdot 436$
 superficial feet. = *Ans.*



EXAMPLE 2.—Required the solidity of the cone as above, taking the perpendicular CD at 28 inches.

Here $\frac{1}{3} (\cdot 7854 \times AB^2 \times CD) = \frac{1}{3} (\cdot 7854 \times 20^2 \times 28) = 2932 \cdot 16$ cubic inches, which divided by 1728 = $1 \cdot 697$ cubic feet. = *Ans.*

PROBLEM IV.

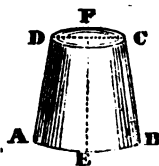
To find the surface of the frustum of a cone or pyramid.

RULE.—Multiply the sum of the perimeters of the

two ends by the slant height, and half the product will be the slant surface; to which add the areas of the two ends, and the product will be the whole surface.

EXAMPLE.—Required the convex surface of the frustum of a cone $A B C D$, whose base $A B = 20$ inches, the slant height $B C = 19$, and top end $C D = 11$ inches.

Here $\frac{1}{2}[(3.1416 \times A B) + (3.1416 \times D C) \times B C] = \frac{1}{2}[3.1416 \times (A B + D C) \times B C] = \frac{1}{2}[3.1416 \times (20 + 11) \times 19] = 1.5708 \times 31 \times 19 = 925.2012$ square inches, which divided by 144 = 6.425 square feet. *Ans.*



PROBLEM V.

To find the solid content of the frustum of a cone.

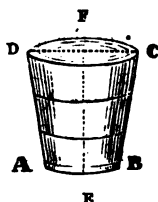
RULE.—To the product of the diameters of the two ends, add the sum of their squares; multiply this sum by the perpendicular height and by .2618, the product is the solid content.

EXAMPLE 1.—Required the solid content of the frustum of a cone in Problem IV., whose perpendicular $E F$ taken = 18 inches, $D C = 11$ inches, and $A B = 20$ inches.

Here $(A B \times D C + A B^2 + D C^2) \times E F \times .2618 = (20 \times 11 + 20^2 + 11^2) \times 18 \times .2618 = (220 + 400 + 121) \times 18 \times .2618 = 741 \times 18 \times .2618 = 3491.8884$ cubic inches, which divided by 1728 = 2.0208 cubic feet. *Ans.*

EXAMPLE 2.—Required the content, in imperial gallons, of the inverted frustum of a cone $A B C D$, whose inner dimensions are $E F = 3\frac{1}{2}$ feet deep, $D C = 18$ inches diameter at bottom, and $A B = 22$ inches diameter at top.

Here $\{(A B \times D C + A B^2 + D C^2) \times E F \times \cdot 2618\} \div 277 \cdot 274 = \{(18 \times 22 + 18^2 + 22^2) \times 42 \times \cdot 2618\} \div 277 \cdot 274 = \{396 + 324 + 484\} \times 42 \times \cdot 2618 \div 277 \cdot 274 = (1204 \times 42 \times \cdot 2618) \div 277 \cdot 274 = 13238 \cdot 7024 \text{ cub. in.} \div 277 \cdot 274 = 47 \cdot 746 \text{ imperial gallons.}$
= Ans.



Or thus :

$13238 \cdot 7024 \times 0 \cdot 60360654 = 47 \cdot 746 \text{ imperial gallons.}$
= Ans. as before.

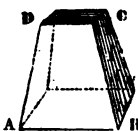
PROBLEM VI.

To find the solid content of the frustum of a pyramid.

RULE.—To the sum of the areas of the two ends, add the square root of their product; multiply this sum by the perpendicular height, and $\frac{1}{3}$ of the product is the solid content.

EXAMPLE.—Required the solid content of the frustum of a pyramid $A B C D$, whose perpendicular height = 24 inches, the area of the base $A B = 144$ square inches, and area of the top end $D C = 64$ square inches.

Here $\{[\text{area at } A B + \text{area at } D C + \sqrt{(\text{area at } A B \times \text{area at } D C)}] \times \text{perp. height}\} \div 3 = \{[144 + 64 + \sqrt{(144 \times 64)}] \times 24\} \div 3 = \{208 + \sqrt{(12^2 \times 8^2)}\} \times 8 = (208 + 12 \times 8) \times 8 = (208 + 96) \times 8 = 304 \times 8 = 2432 \text{ cubic inches, which} \div 1728 = 1 \cdot 4074 \text{ cubic feet.}$
= Ans.



PROBLEM VII.

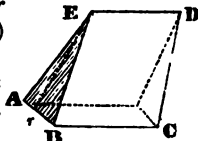
To find the solidity of a wedge.

RULE.—To the length of the edge add twice the length of the base; multiply that sum by the perpen-

dicular height, and by the breadth of the base, and one sixth of the product will be the solidity.

EXAMPLE.—Required the content in cubic inches of the wedge $A B C D E$, whose base $A B C$ is 12 inches long and 4 inches broad, the length of the edge $D E = 10$ inches, and perpendicular height $r E = 20$ inches.

Here $\{(E D + 2 \times B C) \times E r$
 $\times A B\} \div 6 = \{(10 + 2 \times 12)$
 $\times 20 \times 4\} \div 6 = \{(10 + 24) \times$
 $80\} \div 6 = (34 \times 80) \div 6 =$
 $2720 \div 6 = 453 \cdot 3'$ cubic inches.
 $= Ans.$



PROBLEM VIII.

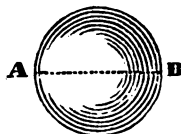
To find the convex surface and solid content of a sphere or globe.

RULE 1.—Multiply the square of the diameter by 3·1416, the product will be the convex superficies.

RULE 2.—Multiply the cube of the diameter by ·5236, and the product is the solid content.

EXAMPLE 1.—Required the convex surface of a sphere, whose diameter $A B = 25\frac{1}{2}$ inches.

Here $A B^2 \times 3 \cdot 1416 = 25 \cdot 5^2 \times$
 $3 \cdot 1416 = 2042 \cdot 8254$ square inches,
 which $\div 144 = 14 \cdot 1863$ square or
 superficial feet. $= Ans.$



EXAMPLE 2.—Required the solid content of a sphere, whose diameter $A B = 25\frac{1}{2}$ inches.

Here $A B^3 \times \cdot 5236 = 25 \cdot 5^3 \times \cdot 5236 = 8682 \cdot 00795$
 cubic inches, which divided by 1728 $= 5 \cdot 0243$ cubic
 feet. $= Ans.$

PROBLEM IX.

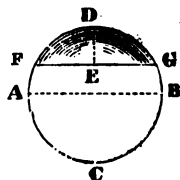
To find the convex surface and solid content of the segment of a sphere.

RULE 1.—Multiply the height of the segment by the whole circumference of the sphere, and the product is the curved surface.

RULE 2.—Add the square of the height to three times the square of the radius of the base; multiply that sum by the height, and by .5236, and the product is the solid content.

EXAMPLE 1.—The diameter A B of the sphere A B C D = 20 inches; what is the convex surface of that segment of it, whose height E D = 8 inches?

Here $3.1416 \times A B \times E D =$
 $3.1416 \times 20 \times 8 = 502.656$ square
 inches which $\div 144 = 3.49$ super-
 ficial feet. = *Ans.*

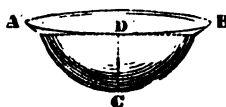


EXAMPLE 2.—The diameter of the base F G of the segment F D G = 18 inches, and perpendicular E D = 8, what is the solid content?

Here $(E D^2 + 3 F E^2) \times D E \times .5236 = (8^2 + 3 \times 9^2) \times 8 \times .5236 = (64 + 243) \times 8 \times .5236 = 307 \times 8 \times .5236 = 1285.9616$ cubic inches, which divided by 1728 = .7442 cubic feet. = *Ans.*

EXAMPLE 3.—Suppose A B C D to be a sugar pan, and that the diameter of the mouth A B is 4 feet, the depth D C being 25 inches, how many imperial gallons will it contain?

Here $\{D C^2 + (\text{half } A B)^2$
 $\times 3\} \times D C \times .5236 = (25^2$
 $+ 2^2 \times 3) \times 25 \times .5236 =$
 $(625 + 576 \times 3) \times 25 \times .5236$



g 3

$= (625 + 1728) \times 25 \times \cdot 5236 = 2353 \times 25 \times \cdot 5236 = 30800\cdot77$ cubic inches, which divided by $277\cdot274 = 111\cdot084$ imperial gallons. = *Ans.*

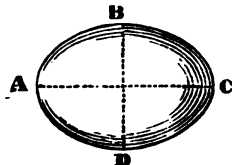
PROBLEM X.

To find the solidity of a spheroid.*

RULE.—Multiply the square of the revolving axis by the fixed axis, and by $\cdot 5236$, and the product will be the solidity.

EXAMPLE 1.—Required the solid content of the prolate spheroid A B C D, whose fixed axis A C = 50, and revolving axis B D = 30.

Here $B D^2 \times A C \times \cdot 5236$
 $= 30^2 \times 50 \times \cdot 5236 = 23562$,
 the solidity. = *Ans.*



EXAMPLE 2.—What is the solid content of an oblate spheroid, the fixed axis B D = 30, and revolving axis A C = 50?

Here $A C^2 \times B D \times \cdot 5236 = 50^2 \times 30 \times \cdot 5236$
 $= 39270$, the solid content. = *Ans.*

PROBLEM XI.

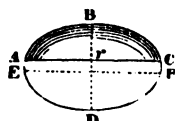
To find the solidity of the segment of a spheroid when the base is circular or parallel to the revolving axis.

RULE.—From triple the fixed axis take double the height of the segment; multiply the difference by the square of the height, and by $\cdot 5236$; then, as the square of the fixed axis is to the square of the revolving axis, so is the former product to the solidity.

* A spheroid is a solid body generated by an ellipse revolving about one of its axes; when it revolves about the transverse or major axis the solid thus generated is called a *prolate* spheroid, when the revolution revolves about the conjugate or minor axis it is called an *oblate* spheroid.

EXAMPLE 1.—Required the solid content of the segment A B C, whose height B r is 10; the revolving axis E F = 40, and fixed axis B D = 25.

Here $BD^2 : EF^2 :: (3 BD - 2 Br) \times Br^3 \times .5236 : [(3 BD - 2 Br) \times Br^3 \times .5236 \times EF^2] \div BD^2$
 $= [(3 \times 25 - 2 \times 10) \times 10^3 \times .5236 \times 40^2] \div 25^2 = [(75 - 20) \times 100 \times .5236 \times 1600] \div 625 = (55 \times 52.36 \times 64) \div 25 = (220 \times 52.36 \times 64) \div 100 = 22 \times 5.236 \times 64 = 7372.288. = Ans.$



EXAMPLE 2.—What is the solid content of the segment of a spheroid whose height B r = 20 inches, the revolving axis E F = 25 inches, and fixed axis B D = 50 inches?

Here $BD^2 : EF^2 :: (3 BD - 2 Br) \times Br^3 \times .5236 : [(3 BD - 2 Br) \times Br^3 \times .5236 \times EF^2] \div BD^2$
 $= [(3 \times 50 - 2 \times 20) \times 20^3 \times .5236 \times 25^2] \div 50^2 = [(150 - 40) \times 400 \times .5236 \times 625] \div 2500 = (110 \times 52.36 \times 2500) \div 2500 = 110 \times 52.36 = 5759.6 \text{ cubic inches, the solid content.} = Ans.$

PROBLEM XII.

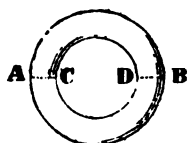
To find the convex surface and solid content of a cylindric ring.

RULE 1.—To the thickness of the ring, add the inner diameter, multiply the sum by the thickness, and by 9.8696, and the product will be the convex surface.

RULE 2.—To the thickness of the ring add the inner diameter; multiply that sum by the square of the thickness and by 2.4674, and the product will be the solid content.

EXAMPLE 1.—The thickness of a cylindric ring **A C** or **D B** = 2 inches, and inner diameter = 18 inches, required the convex superficies.

Here $(A C + C D) \times A C \times 9.8696 = (2 + 18) \times 2 \times 9.8696$
 $= 20 \times 2 \times 9.8696 = 394.784$
 square inches, which \div by 144 =
 2.742 superficial feet. = *Ans.*



EXAMPLE 2.—Required the solid content of the ring as above.

Here $(A C + C D) \times A C^2 \times 2.4674 = (18 + 2) \times 2^2 \times 2.4674 = 20 \times 4 \times 2.4674 = 197.392$ cubic inches, which \div 1728 = .114 cubic feet. = *Ans.*

NOTE.—A cubic foot is equal to 1728 cubic inches;
 or 2200 cylindrical inches;
 or 3300 spherical inches;
 or 6600 conical inches.

Also, the cubic foot being considered unity, or 1,
 A cylinder 1 foot diameter, and 1 foot in length = .7854
 A sphere 1 foot in diameter = .5236
 And a cone 1 foot diameter at the base and 1
 foot in height = .2618.

OF TIMBER MEASURE.

Timber is chiefly estimated by the square or superficial foot of 144 inches, or cubic foot of 1728; the calculation of which is performed by duodecimals; that is, the foot or inch, &c., divided into 12 parts or divisions, thus :—

12 fourths make	1 third.
12 thirds	„	.	.	.	1 second.
12 seconds	„	.	.	.	1 inch.
12 inches	„	.	.	.	1 foot.

And the several values arising are :—

Feet multiplied by feet give feet,
 Feet multiplied by inches give inches,
 Feet multiplied by seconds give seconds,
 Inches multiplied by inches give seconds,
 Inches multiplied by seconds give thirds,
 Seconds multiplied by seconds give fourths, &c.

But this rule is more commonly called *Cross Multiplication*, on account of commencing with the left hand figure of the multiplier.

To find the superficial content of timber.

RULE 1.—Place the multiplier under the multiplicand, feet under feet, inches under inches, seconds under seconds, &c.

2.—Multiply each denomination of the length by the feet of the breadth, beginning at the lowest, and place each product under that denomination of the multiplicand from which it arises, always carrying one for every 12.

3.—Multiply by the inches, and set each product one place farther to the right hand.

4.—Then multiply by the seconds, and set each product another place toward the right hand, &c.

Thus proceed in like manner with all the other denominations, and their sum will be the content.

EXAMPLE 1.—Required the superficial content of a board 12 feet 6 inches long and 1 foot $5\frac{1}{2}$ inches broad.

$$\begin{array}{r}
 \text{ft. in.} \\
 12 \ . \ 6 \\
 \text{Multiplied by } 1 \ . \ 5 \ . \ 6 \\
 \hline
 12 \ . \ 6 \\
 5 \ . \ 2 \ . \ 6 \\
 6 \ . \ 3 \\
 \hline
 \text{Ans.} = 18 \ . \ 2 \ . \ 9
 \end{array}$$

When the two ends of a board or plank are of different breadths, add the two breadths together, and multiply the length by half the sum.

EXAMPLE 2.—A plank is 1 foot 4 inches broad at one end, $11\frac{1}{2}$ inches broad at the other, and 18 feet 9 inches long, what is its superficial content?

Here $(16 + 11\frac{1}{2}) \div 2 = 27\frac{1}{2} \div 2 = 13\frac{3}{4}$ inches.

$$\begin{array}{r}
 \text{ft. in.} \\
 \text{Then } 18 \ . \ 9 \\
 13\frac{3}{4} \text{ inches} = 1 \ . \ 1 \ . \ 9 \\
 \hline
 18 \ . \ 9 \\
 1 \ . \ 6 \ . \ 9 \\
 1 \ . \ 2 \ . \ 0 \ . \ 9 \\
 \hline
 \text{Ans.} = 21 \ . \ 5 \ . \ 9 \ . \ 9
 \end{array}$$

Superficial measure by the Engineer's Slide Rule.

When the length is given in feet, and the breadth in inches, the gauge point is 12 ; but if the dimensions are all inches, the gauge point is 144.

RULE.—Set the breadth upon B to the gauge point upon A, and against the length upon A is the content in square feet upon B.

EXAMPLE 1.—Required the number of square feet contained in a board $11\frac{1}{2}$ inches broad and 18 feet long.

Set 11·5 upon B to 12 upon A ; and against 18 upon A is 17·3 feet upon B.

The content of one board being found, the content of any number of the same dimensions may be found by setting 1 upon B to the content of the one found upon A; and against any number of boards upon B is the whole content upon A.

Find the content of 8 boards, each being 17·3 square feet.

Set 1 upon B to 17·3 upon A; and against 8 upon B is 138·4 feet upon A.

EXAMPLE 2.—If a board is 10 inches broad at one end, and 7 inches at the other, what must be its length to make a square foot?

Here $(10 + 7) \div 2 = 17 \div 2 = 8\frac{1}{2}$ inches. Set 8·5 upon B to 144 upon A; and against 1 upon B is 16·9 inches long upon A.

To find the solidity of timber.

The solid content of timber (according to custom) is found by multiplying the length by the square of the $\frac{1}{4}$ girth.

EXAMPLE.—Required the content of a tree in cubic feet, whose girth in the middle is 84 inches, and length 25 feet 6 inches.

Here $84 \div 4 = 21$ inches = $\frac{1}{4}$ girth.

$$\begin{array}{r}
 \text{ft. in.} \\
 \text{and 21 inches} = 1 \cdot 9 \\
 \text{Multiplied by } 1 \cdot 9 \\
 \hline
 1 \cdot 9 \\
 1 \cdot 8 \cdot 1 \\
 \hline
 \text{Ans.} = 3 \cdot 0 \cdot 9
 \end{array}$$

$$\begin{array}{r}
 \text{ft. in.} \\
 \text{Then } 25 \cdot 6 \\
 \text{Multiplied by } 3 \cdot 0 \cdot 9 \\
 \hline
 76 \cdot 6 \\
 1 \cdot 7 \cdot 1 \cdot 6 \\
 \hline
 \text{Ans.} = 78 \cdot 1 \cdot 1 \cdot 6
 \end{array}$$

But a more expeditious method is obtained by means of the following

TABLE.

$\frac{1}{4}$ Girth in Inches.	Area in Feet.	$\frac{1}{4}$ Girth in Inches.	Area in Feet.	$\frac{1}{4}$ Girth in Inches.	Area in Feet.
6	25000	12 $\frac{1}{2}$	104210	19	250694
6 $\frac{1}{2}$	27127	12 $\frac{3}{4}$	108507	19 $\frac{1}{2}$	264063
6 $\frac{3}{4}$	29340	12 $\frac{7}{8}$	112891	20	277777
6 $\frac{7}{8}$	31641	13	117361	20 $\frac{1}{2}$	291840
7	34028	13 $\frac{1}{8}$	121918	21	306250
7 $\frac{1}{8}$	36502	13 $\frac{1}{4}$	126563	21 $\frac{1}{2}$	321007
7 $\frac{1}{4}$	39063	13 $\frac{3}{8}$	131293	22	336111
7 $\frac{3}{8}$	41710	14	136111	22 $\frac{1}{2}$	351563
8	44444	14 $\frac{1}{8}$	141016	23	367361
8 $\frac{1}{8}$	47266	14 $\frac{1}{4}$	146007	23 $\frac{1}{2}$	383507
8 $\frac{1}{4}$	50174	14 $\frac{3}{8}$	151085	24	400000
8 $\frac{3}{8}$	53168	15	156250	24 $\frac{1}{2}$	416840
9	56250	15 $\frac{1}{8}$	161502	25	434028
9 $\frac{1}{8}$	59418	15 $\frac{1}{4}$	166840	25 $\frac{1}{2}$	451563
9 $\frac{1}{4}$	62647	15 $\frac{3}{8}$	172266	26	469444
9 $\frac{3}{8}$	66016	16	177777	26 $\frac{1}{2}$	487674
10	69444	16 $\frac{1}{8}$	183377	27	506250
10 $\frac{1}{8}$	72960	16 $\frac{1}{4}$	189063	27 $\frac{1}{2}$	525174
10 $\frac{1}{4}$	76563	16 $\frac{3}{8}$	194835	28	544444
10 $\frac{3}{8}$	80252	17	200694	28 $\frac{1}{2}$	564063
11	84028	17 $\frac{1}{8}$	206641	29	584028
11 $\frac{1}{8}$	87891	17 $\frac{1}{4}$	212674	29 $\frac{1}{2}$	604340
11 $\frac{1}{4}$	91840	17 $\frac{3}{8}$	218793	30	625000
11 $\frac{3}{8}$	95877	18	225000	31	6467361
12	100000	18 $\frac{1}{2}$	237674	32	711111

RULE.—Multiply the area corresponding to the $\frac{1}{4}$ girth in inches by the length of the timber in feet; and the product is the solidity in feet and decimal parts.

EXAMPLE.—A piece of timber 18 feet long and 14 inches square, how many cubic feet does it contain?

Here $1'361' \times 18 = 24.5$ cubic feet.

By the Slide Rule.

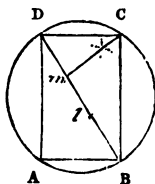
Set the length in feet upon B to 144 upon A; and against the square, or $\frac{1}{4}$ girth upon D, is the solid content in feet upon C.

EXAMPLE.—How many cubic feet is contained in a tree 28 feet long and 16 inches $\frac{1}{4}$ girth?

Set 28 upon B to 144 upon A; and against 16 upon D is 49.8 feet upon C.

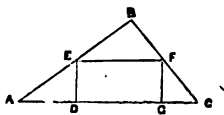
To find the transverse section of the strongest beam that can possibly be cut out of a round piece of timber.

Let the circle A B C D be the transverse section of the piece of timber given, draw the diameter B D, and divide it into three equal parts, as B l m D, erect the perpendicular m C, meeting the circle in C, draw D C and C B; then draw the chord A B parallel to D C, join A D, and the rectangle A B C D will be a section of the beam as required.



To determine the greatest rectangle that can possibly be obtained in a given triangle.

Let A B C be a given triangle, bisect any two of its sides, as E F; join E F, and to each end of which draw lines at right angles with the other side A C, and D E F G will be the rectangle required.



ON THE STRENGTH OF MATERIALS.

A knowledge of the strength of materials is one of the most important, and at the same time one of the most difficult subjects that the practical mechanic has to contend with, owing chiefly to the very different qualities of bodies of the same name; hence arise some doubts in selecting experiments whereon to build the data, there being scarcely two experiments made producing the same results. However, the following tables and rules are founded upon a mean of Messrs. Rennie, Barlow, and Telford's experiments, having found them to agree the best with practice, and my own experiments on similar bodies.

ON THE COHESIVE STRENGTH OF BODIES.

The cohesive strength of a body is that force with which it resists separation in the direction of its length, as in the case of ropes, &c.; and no reason can be assigned why the strength should not vary directly as the section of fracture, and is totally independent of the length in position, except so far as the weight of the body may increase the force applied: neglecting this, and supposing the body uniform in all its parts, the strength of bodies exposed to strains in the direction of their length is directly proportionate to their transverse area, whatever be their figure, length, or position.

The following Table contains the result of experiments on the cohesive strength of various bodies in avoirdupois pounds; also one-third of the ultimate strength of each body, this being considered sufficient, in most cases, for a permanent load.

Names of Bodies.	Square or rectangular Bar.	One-Third.	Round Bar.	One-Third.
WOODS.	<i>lbs.</i>	<i>lbs.</i>	<i>lbs.</i>	<i>lbs.</i>
Boxwood	20000	6667	15708	5236
Ash	17000	5667	13357	4452
Teak	15000	5000	11781	3927
Fir	12000	4000	9424	3141
Beech	11500	3833	9032	3011
Oak	11000	3667	8639	2880
METALS.				
Cast iron	18656	6219	14652	4884
English wrought iron	55872	18624	43881	14627
Swedish ditto	72064	24021	56599	18866
Blistered steel.....	133152	44384	104577	34859
Shear ditto.....	124400	41467	97703	32568
Cast ditto.....	134256	44752	106454	35151
Cast copper	19072	6357	14979	4993
Wrought ditto.....	33792	11264	26540	8847
Yellow brass	17968	5989	14112	4704
Cast tin	4736	1579	3719	1240
Cast lead	1824	608	1432	477

PROBLEM I.

To find the ultimate cohesive strength of square, round, and rectangular bars, of any of the various bodies, as specified in the table.

RULE.—Multiply the strength of an inch bar (as in the table), of the body required, by the cross sectional area of square and rectangular bars, or by the square of the diameter of round bars; and the product will be the ultimate cohesive strength.

EXAMPLE 1.—A bar of cast iron being $1\frac{1}{2}$ inches square, required its cohesive power.

Here $1.5 \times 1.5 \times 18656 \text{ lbs.} = 41976 \text{ lbs.} = \text{Ans.}$

EXAMPLE 2.—Required the cohesive force of a bar of English wrought iron, 2 inches broad, and $\frac{3}{8}$ of an inch in thickness.

Here $2 \times .375 \times 55872 \text{ lbs.} = 41904 \text{ lbs.} = \text{Ans.}$

EXAMPLE 3.—Required the ultimate cohesive strength of a round bar of wrought copper, $\frac{3}{4}$ of an inch in diameter.

Here $.75^3 \times 26540 \text{ lbs.} = 14928.75 \text{ lbs.} = \text{Ans.}$

PROBLEM II.

The weight of a body being given, to find the cross sectional dimensions of a bar or rod capable of sustaining that weight.

RULE.—For square and round bars,—Divide the weight given by one-third of the cohesive strength of an inch bar, (as specified in the table,) and the square root of the quotient will be the side of the square, or diameter of the bar in inches.

And if rectangular, divide the quotient by the breadth, and the result will be the thickness.

EXAMPLE 1.—What must be the side of a square bar of Swedish iron to sustain a permanent weight of 18000 lbs.?

Here $\sqrt{\frac{18000}{24021}} = \sqrt{.7493} = .87$, or nearly $\frac{7}{8}$ of an inch square. = *Ans.*

EXAMPLE 2.—Required the diameter of a round rod of cast copper to carry a weight of 6800 lbs.

Here $\sqrt{\frac{6800}{4993}} = \sqrt{1.3619} = 1.16$ inches diameter. = *Ans.*

EXAMPLE 3.—A bar of English wrought iron is to be applied to carry a weight of 2760 lbs.; required the thickness, the breadth being two inches.

$\frac{1}{2} \left(\frac{2760}{18624} \right) = \frac{1}{2} \left(\frac{115}{776} \right) = \frac{57.5}{776} = .074$ of an inch in thickness. = *Ans.*

A TABLE

Showing the circumference of a rope equal to a chain made of iron of a given diameter, and the weight in tons that each is proved to carry; also the weight of a foot of chain made from iron of that dimension.

<i>Ropes. Circumference in inches.</i>	<i>Chains. Diameter in inches.</i>	<i>Proved to carry in tons.</i>	<i>Weight of a lineal foot in lbs. Av.</i>
3	$\frac{1}{4} + \frac{1}{16}$	1	1.08
4	$\frac{1}{2} + \frac{1}{16}$	2	1.5
4 $\frac{1}{2}$	$\frac{3}{4} + \frac{1}{16}$	3	2
5 $\frac{1}{2}$	$\frac{7}{8} + \frac{1}{16}$	4	2.7
6	$1 + \frac{1}{16}$	5	3.3
6 $\frac{1}{2}$	$1\frac{1}{8} + \frac{1}{16}$	6	4
7	$1\frac{1}{4} + \frac{1}{16}$	8	4.6
7 $\frac{1}{2}$	$1\frac{3}{8} + \frac{1}{16}$	9 $\frac{1}{2}$	5.5
8	$1\frac{1}{2} + \frac{1}{16}$	11 $\frac{1}{2}$	6.1
9	$1\frac{3}{4} + \frac{1}{16}$	13	7.2
9 $\frac{1}{2}$	$1\frac{7}{8} + \frac{1}{16}$	15	8.4
10 $\frac{1}{2}$	1 inch.	18	9.4

ON THE TRANSVERSE STRENGTH OF BODIES.

The *transverse strength* of a body is that power which it exerts in opposing any force acting in a perpendicular direction to its length, as in the case of beams, levers, &c., for the fundamental principles of which observe the following:—

That the transverse strength of beams, &c., is inversely as their lengths, and directly as their breadths and square of their depths, and, if cylindrical, as the cubes of their diameters; that is, if a beam 6 feet long, 2 inches broad, and 4 inches deep, can carry 2000 *lbs.*, another beam of the same material, 12 feet long, 2 inches broad, and 4 inches deep, will only carry 1000, being inversely as their lengths. Again, if a beam 6 feet long, 2 inches broad, and 4 inches deep, can sup-

port a weight of 2000 *lbs.*, another beam of the same material, 6 feet long, 4 inches broad, and 4 inches deep, will support double that weight, being directly as their breadths;—but a beam of that material, 6 feet long, 2 inches broad, and 8 inches deep, will sustain a weight of 8000 *lbs.*; being as the square of their depths.

From a mean of experiments made, to ascertain the transverse strength of various bodies, it appears that the ultimate strength of an inch square, and an inch round bar of each, 1 foot long, loaded in the middle, and lying loose at both ends, is nearly as follows, in *lbs.* avoirdupois.

<i>Names of Bodies.</i>	<i>Square Bar.</i>	<i>One-Third.</i>	<i>Round Bar.</i>	<i>One-Third.</i>
Oak	800	267	628	209
Ash	1137	379	893	298
Elm	569	190	447	149
Pitch pine	916	305	719	240
Deal	566	189	444	148
Cast iron	2580	860	2026	675
Wrought iron	4013	1338	3152	1051

PROBLEM I.

To find the ultimate transverse strength of any rectangular beam, supported at both ends, and loaded in the middle; or supported in the middle, and loaded at both ends; also, when the weight is between the middle and the end; likewise, when fixed at one end and loaded at the other.

RULE.—Multiply the strength of an inch square bar, 1 foot long (as in the table), by the breadth, and square of the depth in inches, and divide the product by the length in feet; the quotient will be the weight in *lbs.* avoirdupois.

EXAMPLE 1.—What weight will break a beam of oak 4 inches broad, 8 inches deep, and 20 feet between the supports?

$$\text{Here } \frac{800 \times 4 \times 8^3}{20} = 40 \times 4 \times 64 = 10240 \text{ lbs.}$$

= *Ans.*

NOTE.—When a beam is supported in the middle, and loaded at each end, it will bear the same weight as when supported at both ends and loaded in the middle; that is, each end will bear half the weight.

When the weight is not situated in the middle of the beam, but placed somewhere between the middle and the end,—Divide four times the product of the two distances of the weight from the two ends by the whole length of the beam. The quotient will be the effective length.

EXAMPLE 2.—Required the ultimate transverse strength of a pitch pine plank, 24 feet long, 3 inches broad, 7 inches deep, and the weight placed 8 feet from one end.

$$\text{Here } \frac{4 \times 16 \times 8}{24} = \frac{64}{3} = 21.3' \text{ effective length.}$$

$$\text{And } \frac{916 \times 3 \times 7^3}{21.3'} = \frac{916 \times 9 \times 49}{64} = \frac{229 \times 9 \times 49}{16}$$

$$= \frac{100989}{16} = 6312 \text{ lbs.} = \text{Ans.}$$

Again, when a beam is fixed at one end and loaded at the other, it will only bear $\frac{1}{4}$ of the weight as when supported at both ends and loaded in the middle.

EXAMPLE 3.—What is the weight requisite to break a deal beam 6 inches broad, 9 inches deep, and projecting 12 feet from the wall?

$$\text{Here } \frac{566 \times 6 \times 9^3}{12 \times 4} = 283 \times 20\frac{1}{4} = 5730\frac{3}{4} \text{ lbs.}$$

= *Ans.*

The same rules apply as well to beams of a cylindrical form, with this exception, that the strength of a round bar (as in the table) is multiplied by the cube of the diameter, in place of the breadth and square of the depth.

EXAMPLE 4.—Required the ultimate transverse strength of a solid cylinder of cast iron, 12 feet long and 5 inches diameter.

$$\text{Here } \frac{2026 \times 5^3}{12} = \frac{253250}{12} = 21104 \text{ lbs.} = \text{Ans.}$$

EXAMPLE 5.—What is the ultimate transverse strength of a hollow shaft of cast iron, 12 feet long, 8 inches diameter outside, and containing the same cross sectional area as a solid cylinder 5 inches diameter?

$$\begin{aligned} \text{Here } \frac{\{8^2 - \sqrt{(8^2 - 5^2)}\} \times 2026}{12} &= \frac{(512 - \sqrt{39^2}) \times 2026}{12} \\ &= \frac{(512 - \sqrt{59319}) \times 2026}{12} = \frac{(512 - 243.555) \times 1013}{6} \\ &= \frac{268.445 \times 1013}{6} = \frac{271934.785}{6} = 45322 \text{ lbs.} = \end{aligned}$$

Ans.

NOTE.—When a beam is fixed at both ends, and loaded in the middle, it will bear one-half more than it will when loose at both ends.

And if a beam is loose at both ends, and the weight laid uniformly along its length, it will bear double; but if fixed at both ends, and the weight laid uniformly along its length, it will bear triple the weight.

PROBLEM II.

To find the breadth or depth of beams intended to support a permanent weight.

RULE.—Multiply the length between the supports, in feet, by the weight to be supported in *lbs.*, and divide the product by one-third of the ultimate strength of an inch bar (*as in the table*), multiplied by the square of the depth; the quotient will be the breadth, or, multiplied by the breadth, the quotient will be the square of the depth, both in inches.

EXAMPLE 1.—Required the breadth of a cast iron beam, 16 feet long, 7 inches deep, to support a weight of 4 tons in the middle.

Here 4 tons = 8960 lbs.

$$\text{Then } \frac{8960 \times 16}{860 \times 7^3} = \frac{1024}{301} = 3.4 \text{ inches.} = \text{Ans.}$$

EXAMPLE 2.—What must be the depth of a cast iron beam 3.4 inches broad, 16 feet long, to bear a permanent weight of four tons in the middle?

$$\begin{aligned} \text{Here } \sqrt{\frac{8960 \times 16}{860 \times 3.4}} &= 4 \times \sqrt{\frac{8960}{86 \times 34}} = 4 \times \\ \sqrt{\frac{2240}{43 \times 17}} &= 4 \times \sqrt{\frac{2240}{731}} = 4 \times \sqrt{3.07} = 4 \\ \times 1.75 &= 7 \text{ inches.} = \text{Ans.} \end{aligned}$$

NOTE 1.—When a beam is fixed at both ends, the divisor must be multiplied by 1.5, on account of its being capable of bearing one-half more.

2.—When a beam is loaded uniformly throughout, and loose at both ends, the divisor must be multiplied by 2, because it will bear double the weight.

3.—If a beam is fast at both ends, and loaded uniformly throughout, the divisor must be multiplied by 3, on account that it will bear triple the weight.

EXAMPLE 3.—Required the breadth of an oak beam, 20 feet long, 12 inches deep, made fast at both ends, to be capable of supporting a weight of 12 tons in the middle.

Here 12 tons = 26880 lbs.

$$\begin{aligned} \text{Then } \frac{26880 \times 20}{266.6' \times 12^3 \times 1.5} &= \frac{2688 \times 6 \times 2}{8 \times 144 \times 3} = \frac{28}{3} = \\ 9\frac{1}{3} \text{ inches.} &= \text{Ans.} \end{aligned}$$

Again, when a beam is fixed at one end and loaded at the other, the dividend must be multiplied by 4; because it will only bear one-fourth of the weight.

EXAMPLE 4.—Required the depth of a beam of ash, 6 inches broad, 9 feet projecting from the wall, to carry a weight of 47 cwt.

Here 47 cwt. = 5264 lbs.

$$\begin{aligned}\text{Then } \sqrt{\frac{5264 \times 9 \times 4}{379 \times 6}} &= \sqrt{\frac{329 \times 4^2 \times 3^2 \times 2^2}{2274}} \\ &= 24 \times \sqrt{\frac{329}{2274}} = 24 \times \sqrt{.1447} = 24 \times .38 = \\ &9.12 \text{ inches deep.} = \text{Ans.}\end{aligned}$$

And when the weight is not placed in the middle of a beam, the effective length must be found as in *Problem I*.

EXAMPLE 5.—Required the depth of a deal beam 20 feet long, to support a weight of 63 cwt. 6 feet from one end.

Here $\frac{4 \times 14 \times 6}{20} = \frac{84}{5} = 16.8$, effective length of beam,

And 63 cwt. = 7056 lbs.

$$\begin{aligned}\text{Hence } \sqrt{\frac{7056 \times 16.8}{189 \times 6}} &= \sqrt{\frac{784 \times 2}{15}} = \frac{28}{15} \times \\ \sqrt{30} &= \frac{28}{15} \times 5.477 = 10.22 \text{ inches deep.} = \text{Ans.}\end{aligned}$$

Beams or shafts exposed to lateral pressure are subject to all the foregoing rules; but in the case of water-wheel shafts, &c., some allowance must be made for wear, then the divisor may be changed from 675 to 600 for cast iron.

EXAMPLE 6.—Required the diameter of bearings for a water-wheel shaft 12 feet long, to carry a weight of 10 tons in the middle.

Here 10 tons = 22400 lbs.

Then $\sqrt[3]{\frac{12 \times 22400}{600}} = \sqrt[3]{448} = 7.65$ inches diameter. = *Ans.*

And when the weight is equally distributed along its length, the cube root of half the quotient will be the diameter, thus :

Here $\sqrt[3]{\frac{448}{2}} = \sqrt[3]{224} = 6.07$ inches diameter. = *Ans.*

EXAMPLE 7.—Required the diameter of a solid cylinder of cast iron, for the shaft of a crane, to be capable of sustaining a weight of 10 tons; one end of the shaft to be made fast in the ground, the other to project $6\frac{1}{2}$ feet; and the effective leverage of the jib as $1\frac{1}{2}$ to 1.

Here 10 tons = 22400 lbs.

Then $\frac{22400 \times 6.5 \times 1.75}{675 \times .25} = \frac{2 \times 224 \times 13 \times 7}{27}$
= 1510.

And $\sqrt[3]{1510} = 11.47$ inches diameter. = *Ans.*

The strength of cast iron to wrought iron, in this direction, is as 9 is to 14 nearly; hence, if wrought iron is taken in place of cast iron in the last example, what must be its diameter?

$\sqrt[3]{\frac{1510 \times 9}{14}} = \sqrt[3]{970.7} = 9.9$ inches diameter. = *Ans.*

ON TORSION OR TWISTING.

The strength of bodies to resist *torsion*, or wrenching asunder, is directly as the cubes of their diameters; or, if square, as the cube of one side; and inversely as the force applied multiplied into the length of the lever.

Hence the rule.—1. Multiply the strength of an inch bar, by experiment (*as in the following table*), by the

cube of the diameter, or of one side in inches; and divide by the radius of the wheel, or length of the lever also in inches; and the quotient will be the ultimate strength of the shaft or bar, in *lbs. avoirdupois*.

2.—Multiply the force applied in pounds by the length of the lever in inches, and divide the product by one-third of the ultimate strength of an inch bar (as in the table), and the cube root of the quotient will be the diameter, or side of a square bar in inches; that is, capable of resisting that force permanently.

The following Table contains the result of experiments on inch bars, of various metals, in lbs. avoirdupois.

<i>Names of Bodies.</i>	<i>Round Bar.</i>	<i>One-Third.</i>	<i>Square Bar.</i>	<i>One-Third.</i>
Cast iron	11943	3981	15206	5069
English wrought iron	12063	4021	15360	5120
Swedish ditto	11400	3800	14592	4864
Blistered steel	20025	6675	25497	8499
Shear ditto	20508	6836	26112	8704
Cast ditto	21111	7037	26880	8960
Yellow brass	5549	1850	7065	2355
Cast copper	4825	1608	6144	2048
Tin	1688	563	2150	717
Lead	1206	402	1536	512

EXAMPLE 1.—What weight, applied on the end of a 5 feet lever, will wrench asunder a 3 inch round bar of cast iron?

$$\text{Here } \frac{11943 \times 3^3}{60} = \frac{107487}{20} = 5374 \text{ lbs. avoirdupois.}$$

= *Ans.*

EXAMPLE 2.—Required the side of a square bar of wrought iron, capable of resisting the twist of 600 *lbs.* on the end of a lever 8 feet long.

$$\sqrt[3]{\frac{600 \times 96}{5120}} = \sqrt[3]{\frac{15 \times 3}{4}} = \sqrt[3]{\frac{90}{8}} = \sqrt[3]{\frac{90}{2}} = 2\frac{1}{4} \text{ inches.} = \text{Ans.}$$

In the case of revolving shafts for machinery, &c., the strength is directly as the cubes of their diameters and revolutions, and inversely as the resistance they have to overcome; hence,

From *practice*, we find that a 40-horse power steam-engine, making 25 revolutions per minute, requires a shaft (*if made of wrought iron*) to be 8 inches diameter: now, the cube of 8, multiplied by 25, and divided by 40 = 320; which serves as a constant multiplier for all others in the same proportion.

EXAMPLE 3.—What must be the diameter of a wrought iron shaft for an engine of 65-horse power, making 23 revolutions per minute?

$$\text{Here } \sqrt[3]{\frac{65 \times 320}{23}} = \sqrt[3]{\frac{20800}{23}} = \sqrt[3]{904.4} = 9.67$$

inches diameter. = *Ans.*

Mr. Robertson Buchanan, in his Essay on Shafts, gives 400 as a constant multiplier for cast iron shafts that are intended for first movers in machinery;

200 for second movers; and

100 for shafts connecting smaller machinery, &c.

EXAMPLE 1.—The velocity of a 30-horse power steam-engine is intended to be 19 revolutions per minute. Required the diameter of bearings for the fly wheel shaft.

$$\text{Here } \sqrt[3]{\frac{400 \times 30}{19}} = \sqrt[3]{\frac{12000}{19}} = \sqrt[3]{631.6} = 8.58$$

inches diameter. = *Ans.*

EXAMPLE 2.—Required the diameter of the bearings of shafts, as second movers for a 30-horse engine; their velocity being 36 revolutions per minute.

$$\text{Here } \sqrt[3]{\frac{200 \times 30}{36}} = \sqrt[3]{\frac{1000}{6}} = 10 \times \sqrt[3]{\frac{1}{6}} =$$

$$\frac{10}{6} \sqrt[3]{36} = \frac{10}{6} \times 3.3 = 5.5 \text{ inches diameter.} = \text{Ans.}$$

NOTE.—When shafting is intended to be of wrought iron, use 160 as the multiplier for second movers; and 80 for shafts connecting smaller machinery.

TABLE

Of the proportionate length of bearings, or journals for shafts of various diameters.

<i>Diameter in Inches.</i>	<i>Length in Inches.</i>	<i>Diameter in Inches.</i>	<i>Length in Inches.</i>
1	1½	6½	8½
1½	2½	7	9½
2	3	7½	10
2½	3½	8	10½
2½	3½	8½	11½
3	4½	9	12
3½	4½	9½	12½
4	5½	10	13½
4½	6½	10½	14
5	6½	11	14½
5½	7½	11½	15½
6	8½	12	16

OF THE MECHANICAL POWERS.

When power is applied to overcome weight, or force to overcome resistance, the machines employed are called mechanic powers ; and the application of such, the science of mechanics.

The power and weight are said to balance each other, or to be in equilibrio, when the effort of the one to produce motion in one direction is equal to the effort of the other to produce it in an opposite direction ; or when the weight opposes that degree of resistance which is precisely required to destroy the action of the power.

The momentum or quantity of force of any moving body is the result of the quantity of matter multiplied by the velocity with which it is moved ; and when the product arising from the multiplication of the particular quantities of matter in any two bodies by their respective velocities are equal, their momentums will be so too.

And it holds universally true, that when two bodies are suspended upon any machine, so as to act contrary to each other, if the machine be put in motion, and the perpendicular ascent of one body, multiplied into its weight, be equal to the perpendicular descent of the other multiplied into its weight, those bodies, however unequal they may be in weight, will balance each other in all situations ; for, as the whole ascent of the one is performed in the same time as the whole descent of the other, their respective velocities must be as the spaces they move through ; and the excess of weight in the one is compensated by the excess of velocity in the other. Upon this principle it is easy to compute the power of any machine, either simple or compound ; for it is only finding how much swifter the power moves than the weight, and just so much is the power increased by the help of the machine.

The simple machines, usually called mechanic powers, are six in number, namely, the Lever, the Wheel and Axle, the Pulley, the Inclined Plane, the Wedge, and the Screw.

There are three kinds of levers, caused by the different situations of the weights, props, and powers.

1.—When the weight is at one end, the power at the other, and the prop somewhere between.

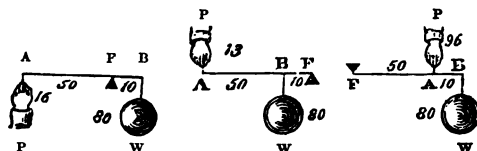
2.—When the prop is at one end, the power at the other, and the weight between.

3.—When the prop is at one end, the weight at the other, and the power between.

1.

2.

3.



That on any kind of lever there may be a balance between the power and weight, their intensities must be inversely as their distances from the fulcrum, or prop, at which they act, that is, $P \times A F = W \times B F$; therefore,

Multiply the weight or power given by its distance from the prop, and divide by the distance of the prop from the power or weight; the quotient will be the power or weight required.

EXAMPLES 1, 2, and 3.

Required the power necessary to counterpoise a weight of 80 *lbs.* on each of the three levers, whose lengths are 60 inches, and in the first and second 10 inches from weight to prop, the third being 10 inches from weight to power.

$$\text{First} \dots \frac{80 \times 10}{50} = 16 \text{ lbs. power.} = 1^{\text{st}} \text{ Ans.}$$

$$\text{Second} \dots \frac{80 \times 10}{60} = 13.33 \text{ lbs. power.} = 2\text{nd Ans.}$$

$$\text{Third} \dots \frac{80 \times 60}{50} = 96 \text{ lbs. power.} = 3\text{rd Ans.}$$

EXAMPLE 4.—What power is necessary to raise a weight of 620 *lbs.* by a lever of the first order, 72 inches long, and the prop placed 12 inches from the weight?

Here $72 - 12 = 60$ inches, the distance of prop to power.

$$\text{Then } \frac{620 \times 12}{60} = 124 \text{ lbs.} = \text{Ans.}$$

NOTE.—In a lever of the first kind, when its length is given together with the weight and power, multiply the weight or power by the length of the lever, and divide by the sum of the weight and power, the quotient is the distance of the power or weight from the prop.

EXAMPLE 5.—A weight of 620 *lbs.* is to be lifted by a power of 124 *lbs.* applied to the end of a lever of the first order, 72 inches long; required at what distance from the weight the prop must be placed.

$$\text{Here } \frac{124 \times 72}{620 + 124} = \frac{124 \times 72}{744} = 12 \text{ inches.} = \text{Ans.}$$

EXAMPLE 6.—A beam 20 feet long, and supported at both ends, bears a weight of 73 *cwt.* 4 feet 6 inches from one end; required the proportion of weight upon each support.

$$\text{Here } \frac{\text{whole weight} \times \text{dist. from nearest support}}{\text{whole length}} =$$

$$\frac{73 \times 4.5}{20} = 16.425 \text{ cwt. on the furthest support.} =$$

1st Ans.

$$\text{And } 73 - 16.425 = 56.575 \text{ cwt. on the nearest support.} = 2\text{nd Ans.}$$

EXAMPLE 7.—A weight of 300 *lbs.* is fixed on the end of a lever 6 feet long; required the power, applied $2\frac{1}{2}$ feet from the prop, to raise the weight.

$$\text{Here } \frac{300 \times 6}{2.5} = 720 \text{ lbs. power.} = \text{Ans.}$$

WHEEL AND AXLE.

Here the velocity of the power is to the velocity of the weight as the diameter of the wheel is to the diameter of the axle; hence, divide the velocity of the power by the velocity of the weight, and the quotient is the weight that the power is equal to.

EXAMPLE 1.—A power equal to 30 lbs. is applied to the winch of a crane whose length is 15 inches; the pinion contains 10 teeth, the wheel 120, and the barrel is 9 inches diameter; required the weight raised.

Here $15 \times 2 = 30$, the diameter of the circle described by the winch, or handle.

Next $120 \div 10 = 12$ revolutions of the pinion for one of the wheel.

$$\text{Then } \frac{30 \times 12 \times 30}{9} = 1200 \text{ lbs. raised by this crane.} \\ = \text{Ans.}$$

EXAMPLE 2.—What would be the increase of power, in the last example, if a wheel of 150 teeth, and a pinion of 15, were added to the crane?

Here $150 \div 15 = 10$, that is, the velocity of the weight is diminished, while the velocity of the power is the same.

$$\text{Then } \frac{30 \times 12 \times 10 \times 30}{9} = 12000 \text{ lbs. raised.} = \\ \text{Ans., the power here being increased ten times.}$$

EXAMPLE 3.—What power is requisite to raise 42 tons 60 feet high in 10 minutes, the velocity of the power being 20 feet per minute?

$$\text{Here } 60 \div 10 = 6, \text{ and } \frac{42 \times 6}{20} = 12.6 \text{ tons power.} \\ = \text{Ans.}$$

TO CALCULATE FOR THE DIFFERENT PARTS OF A CRANE,
AS RESPECTS MECHANICAL ADVANTAGE.

1.—*The number of revolutions of the pinion to one of the wheel, the length of the handle, and the force applied given, to find the diameter of the barrel.*

RULE.—Multiply the diameter of the circle described by the winch, or handle, in inches, by the power applied in *lbs.*, and by the number of revolutions of the pinion to one of the wheel; divide the product by the weight to be raised in *lbs.*, and the quotient is the barrel's diameter in inches.

EXAMPLE.—Suppose that two men were required to raise a weight of one ton, by a crane, and each man to exert a constant force of $33\frac{1}{2}$ *lbs.* on a handle 16 inches long, the pinion making seven revolutions for one of the wheel, what must be the barrel's diameter?

Here $16 \times 2 = 32$ inches, diameter of the circle described by the handle, and $33\frac{1}{2} \times 2 = 67$ *lbs.* constant force.

$$\text{Then } \frac{32 \times 67 \times 7}{2240} = 6.7 \text{ inches.} = \text{Ans.}$$

2.—*The diameter of the barrel, the length of the handle, and force applied given, to find the number of revolutions of the pinion to one of the wheel.*

RULE.—Multiply the weight to be raised in *lbs.* by the diameter of the barrel in inches, and divide the product by the diameter of the circle described by the handle in inches, multiplied by the power applied in *lbs.*, and the quotient is the revolutions of the pinion to one of the wheel.

EXAMPLE.—What must be the number of revolutions of the pinion to one of the wheel, when the power applied is 67 *lbs.*, the length of the handle 16 inches,

and the barrel 6·7 inches diameter, to counterpoise a weight of one ton, or 2240 lbs. ?

Here $\frac{2240 \times 6\cdot7}{32 \times 67} = 7$ revolutions to one of the wheel. = *Ans.*

3.—*The diameter of the barrel, the number of revolutions of the pinion to one of the wheel, and the power applied given, to find the length of the handles.*

RULE.—Multiply the weight to be raised in lbs. by the barrel's diameter in inches, and divide the product by the power applied in lbs., multiplied by the number of revolutions of the pinion to one of the wheel, and half the quotient is the length of the handles.

EXAMPLE.—It is estimated that the united effort of two men at the handles of a crane is 67 lbs. nearly ; now a crane having a barrel of 6·7 inches diameter, and a pinion 7 to 1 of the wheel, what must be the length of handles to raise a weight of 1 ton ?

Here $\frac{2240 \times 6\cdot7}{67 \times 7 \times 2} = \frac{32}{2} = 16$ inches. = *Ans.*

4.—*The diameter of the barrel, the revolutions of the pinion to one of the wheel, and length of handles given, to find the power required.*

RULE.—Multiply the weight to be raised in lbs. by the diameter of the barrel in inches, and divide the product by the diameter of the circle described by the handle, multiplied by the revolutions of the pinion to one of the wheel, and the quotient is the power required.

EXAMPLE.—What power will be required to raise one ton by a crane, whose barrel is 6·7 inches diameter, the pinion 7 to 1 of the wheel, and each handle 16 inches long ?

Here $\frac{2240 \times 6.7}{32 \times 7} = 67 \text{ lbs. power.} = \text{Ans.}$

NOTE.—The handles of a crane ought not to be less than 2 feet 11 inches, or 3 feet from the ground, and the jib to stand at an angle of about 45 degrees.

To find the thickness of cast iron for a crane post, when fixed at one end, and loaded at the other.

RULE.—Multiply the weight that the crane is to lift in *lbs.* by the leverage of the jib to one of the post, and by the length of the post in feet; divide the product by 168, then subtract the quotient from the cube of the outside diameter, and the cube root of the difference is the inside diameter.

EXAMPLE.—What thickness must the metal be for a crane post to carry a weight of 10 tons, the diameter of the post being 16 inches, and projecting 6 feet from the ground, the leverage of the jib being as $3\frac{1}{2}$ to 1 of the post?

Here 10 tons = 22400 *lbs.*

Then $\sqrt[3]{\left(16^3 - \frac{22400 \times 3.5 \times 6}{168}\right)} = \sqrt[3]{(4098 - 2800)} = \sqrt[3]{1298} = 10.9$ the inside diameter.

And $\frac{16 - 10.9}{2} = \frac{5.1}{2} = 2.6$ inches in thickness. = *Ans.*

THE PULLEY.

A single pulley, that only turns on its axis, and does not move out of its place, serves only to change the direction of the power, but gives no mechanical advantage. The advantage gained is always as twice the number of moveable pulleys, without taking any notice of the fixed pulleys necessary to compose the system of pulleys; hence, divide the weight to be raised by twice the number of moveable pulleys, and the quotient is the power required to raise the weight, in terms of the same name.

EXAMPLE 1.—What power is requisite to raise 250 *lbs.* with a pair of four-shieved blocks, the one block moveable and the other fixed?

$$\text{Here } \frac{250}{4 \times 2} = \frac{125}{4} = 31.25 \text{ lbs. power.} = \text{Ans.}$$

EXAMPLE 2.—What weight will a power of 120 *lbs.* raise, when applied to a three and four-shieved block, the three being moveable and the other fixed?

$$\text{Here } 3 \times 2 \times 120 = 720 \text{ lbs. raised.} = \text{Ans.}$$

THE INCLINED PLANE.

In the inclined plane, when the power acts parallel thereto, the weight it will support is to the power applied as the length of the plane to its perpendicular height; hence, multiply the weight by the perpendicular height of the plane, and divide by its length, the quotient is the power that will support that weight upon the plane.

EXAMPLE 1.—Required the power, or equivalent weight, capable of supporting a load of 300 *lbs.* upon an inclined plane 50 feet long and 16 feet high.

Here 50 is to 16 as 300 *lbs.* is to 96 *lbs.*, the power.
= *Ans.*

Or, $\frac{300 \text{ lbs.} \times 16}{50} = 96 \text{ lbs. the power.} = \text{Ans. as before.}$

EXAMPLE 2.—A power of 120 *lbs.*, with a velocity of 50 feet per minute, is to be applied to move a weight up an inclined plane at the rate of 30 feet per minute; the plane is 25 feet long and 8 feet high; required the weight that the power is equal to.

Here $\frac{30 \times 8}{25} = \frac{48}{5} = \text{perpendicular velocity of the weight.}$

Then, as $\frac{48}{5} : 50 :: 120 \text{ lbs.} : 625 \text{ lbs.} = \text{Ans.}$

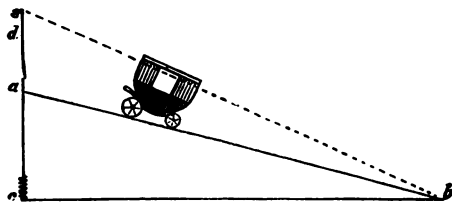
The weight multiplied by the length of the base, and divided by the length of the plane, equals the pressure on the plane.

The space which a body describes upon an inclined plane, when descending by the force of gravity, is to the space which it would fall freely in the same time, as the height is to the length of the plane, and the spaces being the same, the times will be inversely in this proportion.

Again, if two bodies descend from rest down two planes, equally inclined to the horizon, and then, without any loss of velocity, proceed to descend down two other inclined planes, also equally inclined to the horizon, the lengths of which are to each other in the same proportion as the lengths of the first two planes, the squares of the times of their whole motion will be in the same proportion as the lengths of the planes.

Means of ascertaining practically the effect produced by inclined planes.

Provide a board or box, $a b$, capable of holding pebbles, sand, &c., and which, by a screw, c , can be easily raised at one end, as $a d s$, &c.



When $a b$ lies flat on $c b$, a carriage placed upon $a b$ will be at rest; but by the screw at c raising $a b$ leisurely, the carriage will, at a certain height, set off by itself, and run down the plane. Then are we in possession of a triangle that solves what force is necessary to drag any load of any kind on a road or level ground; for the

hypotenuse ab represents the weight of the carriage, and the perpendicular ac what portion of that weight is necessary to draw the carriage on level ground; thus,

Suppose the carriage . . . 12 *cwt.*

The line ab 24 *feet.*

Height ca 3 *feet.*

The declivity, then, is as 3 to 24, or $\frac{1}{8}$. In this case it will be found that $\frac{1}{8}$ of the weight of the carriage would drag it on such a road or level ground, namely, $1\frac{1}{2}$ *cwt.*; but if the road were very deep and rough, it might require to be raised perhaps as high as d or s , before the carriage would set off. Now, if cs were half the length of sb , then it would require one-half the weight of the carriage to drag it on level ground, or, in the above case 6 *cwt.*

This rule is universal, and has been proved by carriages at large, on roads of every description.

In estimating the draft up hill, the draft on the level must be added to it. Suppose the hill rises 1 foot in 4, then $\frac{1}{4}$ part of the weight must be added to the draft on level ground.

If the weight be, as before, 12 *cwt.*, then $\frac{1}{4}$ would be 3 *cwt.*; and if its draft on a level were $1\frac{1}{2}$ *cwt.*, then $4\frac{1}{2}$ *cwt.* would be the real draft necessary to draw 12 *cwt.* up a hill rising 1 foot in 4, &c.

EXAMPLE.—Suppose I find that, on an edge railway, a loaded carriage will just move by itself when there is a descent of $3\frac{1}{2}$ inches per chain, or about one perpendicular for 224 horizontal, which is, (reckoning the carriage to weigh 1 ton) $10\text{ lbs.} = \frac{1\text{ ton}}{224}$ required to move

it on a level. Now, from the above data, what force will be required to drag the same weight up a similar road ascending 1 inch per yard, or $\frac{1}{36}$? Here $\frac{1}{36}$ of a ton is 62.2 *lbs.*, which added to 10 *lbs.* as above, amounts to 72.2 *lbs.*, the weight required to drag it up an ascent of $\frac{1}{36}$; and allowing the strength of an ordinary horse

to be 140 *lbs.*, he will only be able to drag 1·9, or say 2 tons up an ascending plane of 1 in 36.

THE WEDGE.

As the wedge is seldom used without being driven, the force of the blow is not easily ascertained; of course, in practice it is not worth taking into account with respect to calculation.

THE SCREW.

The advantage gained by the screw amounts to this, that the circumference of the circle described by the lever or handle is to the pitch of the screw (*viz.* the distance between two consecutive threads) as the weight to the power.

EXAMPLE.—What power is necessary to raise a weight of 6000 *lbs.*, the length of the lever being 20 inches and the screw $\frac{3}{4}$ pitch?

Here as $20 \times 2 \times 3.1416 = 125.7 : .75 :: 6000 \text{ lbs.} : 35.8 \text{ lbs.}$, power required.

NOTE.—There are few machines but what, on account of the friction of the parts against one another, will require a third part more power to work them, when loaded, than is requisite to constitute a balance between power and weight.

The following Table shows the estimated power of man or horse as applied to machinery.

Application of the Power.	Lbs. Avr. at the rate of 220 feet per minute.	Or Lbs. Avr. at the rate of one foot per minute.
A man is supposed to be capable of lifting or carrying	27-273	or 6000
A man is supposed to be capable of turning the winch of a crane with a force equal to	28-636	or 6300
When the united efforts of two men are applied to the winch of a crane, the handles being at right angles, each man exerts a force equal to	33-409	or 7350
A man is supposed to exert a power in pumping equal to	17-336	or 3814
In ringing, a man exerts a force equal to.....	38-955	or 8570
And in rowing.....	40-955	or 9010
The power of a horse equal to	150	or 33000

OF FALLING BODIES.

In bodies falling freely by their own weight, their velocities are as the times, and the spaces as the square of the times; therefore, if the times be as the numbers1, 2, 3, 4, &c. The velocities will be also1, 2, 3, 4, &c. The spaces passed through1, 4, 9, 16, &c. And the spaces for each time, as the odd

numbers1, 3, 5, 7, &c.

It has been ascertained by experiment that a body falling freely from rest will descend through $16\frac{1}{10}$ feet in the first second of time, and will then have acquired a velocity which, being continued uniformly, will carry it through $32\frac{2}{5}$ feet in the next second, consequently, if the first series of numbers be expressed in seconds,

	1"	2"	3"	
Velocities in feet will be	$32\frac{1}{5}$	$64\frac{2}{5}$	$96\frac{3}{5}$	&c.
Spaces in the whole times	$16\frac{1}{10}$	$64\frac{2}{5}$	$144\frac{9}{10}$	&c.
And the spaces for each second .	$16\frac{1}{10}$	$48\frac{3}{10}$	$80\frac{1}{2}$	&c.

To find the velocity a falling body will acquire in any given time.

RULE.—Multiply the time in seconds by $32\cdot19$, and the product will be the velocity acquired in feet per second.

EXAMPLE.—Required the velocity in 7 seconds.

Here $32\cdot19 \times 7 = 225\cdot33$ feet, velocity acquired. =
Ans.

To find the velocity a body will acquire by falling from any given height.

RULE.—Multiply the space in feet by $64\cdot4$, and the square root of the product will be the velocity acquired in feet per second.

EXAMPLE.—Required the velocity a ball will acquire in descending through 201 feet.

Here $\sqrt{(64 \cdot 4 \times 201)} = \sqrt{12944 \cdot 4} = 113 \cdot 8$ feet. =
Ans.

To find the space through which a body will fall in any given time.

RULE.—Multiply the square of the time in seconds by 16·095, and the product will be the space in feet.

Here $16 \cdot 095 \times 7^2 = 16 \cdot 095 \times 49 = 788 \cdot 655$ feet.
= *Ans.*

EXAMPLE.—Required the space fallen through in 7 seconds.

NOTE.—The velocity acquired by a body in falling from rest, through a given height, is the same whether it fall freely or descend through a plane any way inclined.

The diameter of a circle perpendicular to the horizon, and any chord terminating at either extremity of that diameter, are described by a falling body in the same length of time.

And the velocities which bodies acquire by descending along chords of the same circle, are as the lengths of those chords.

TABLE

Of accelerated motion of falling bodies.

Time in seconds of the body's fall.	Space fallen through during each second, in feet.	Whole space fallen through, in feet.	Velocity acquired at the end of the time.
1	16·095	16·095	32·19
2	48·286	64·381	64·38
3	80·476	144·857	96·57
4	112·667	257·524	128·76
5	144·857	402·381	160·95
6	177·048	579·429	193·14
7	209·238	788·668	225·33
8	241·429	1030·097	257·52
9	273·619	1303·716	289·71
10	305·810	1609·526	321·91

ON PENDULUMS.

The length of a pendulum that vibrates seconds, or 60, in the latitude of Königsberg ($54^{\circ} 42' 50''$ N.) is, according to Bessel, 440·8179 lines of his toise, which is shorter by $\frac{1}{1278}$ of a line than the toise of Peru.

The French metre = 513074 of the toise of Peru = 39·370089 English inches, with which we get the length of the second's pendulum at Königsberg = 39·14997 English inches, and at London (Lat. $51^{\circ} 31' 8\cdot4''$, N.) = 39·13907 English inches; and $\sqrt{39\cdot1391} \times 60 = 375\cdot367$, serves as a constant number for other pendulums; thus, 375·367 divided by the square root of the pendulum's length, gives the number of vibrations per minute; and divided by the vibrations per minute, gives the square root of the length of the pendulum.

EXAMPLE 1.—Required the number of vibrations a pendulum 25 inches long will make per minute.

Here $\frac{375\cdot367}{\sqrt{25}} = \frac{375\cdot367}{5} = 75\cdot073$ vibrations per minute. = *Ans.*

EXAMPLE 2.—Required the length of a pendulum to make 80 vibrations per minute.

Here $\left(\frac{375\cdot367}{80}\right)^2 = 4\cdot6921^2 = 22\cdot0157$ inches long. = *Ans.*

Table containing the length of pendulums, in English inches, to vibrate seconds in various parts of the world. From the Ency. Metrop., Art. "Figure of the Earth," by G. B. Airy, Esq., Astronomer Royal.

At Sierra Leone	39·01997 in.	At New York ..	39·10120 in.
„ Trinidad	39·01688 „	„ Bordeaux....	39·11296 „
„ Madras.....	39·02630 „	„ Paris.....	39·12877 „
„ Jamaica	39·03503 „	„ Leith Fort ..	39·15546 „
„ Rio Janeiro ..	39·04350 „	„ Greenland ..	39·20335 „

A pendulum vibrating half seconds in the latitude of London is 9·8 inches in length; and for quarter seconds, 2·5 inches.

ON THE VELOCITY OF WHEELS, DRUMS, PULLEYS, &c.

When wheels are applied to communicate motion from one part of a machine to another, their teeth act alternately on each other; consequently, if one wheel contains 60 teeth and another 20, the one containing 20 teeth will make three revolutions, while the other makes but one; and if drums or pulleys are taken in place of wheels, the result will be the same; because their circumferences, describing equal spaces, render their revolutions unequal: from this the rule is derived, namely,

Multiply the velocity of the driver by the number of teeth it contains, and divide by the velocity of the driven; the quotient will be the number of teeth it ought to contain. Or, multiply the velocity of the driver by its diameter, and divide by the velocity of the driven; the quotient will be the diameter of the driven.

If the velocities of driver and driven are given with the distance of their centres,

Then the sum of the velocities : $\left\{ \begin{array}{l} \text{velocity of driver} \\ \text{velocity of driven} \end{array} \right\} ::$

distance of centres : $\left\{ \begin{array}{l} \text{radius of driven.} \\ \text{radius of driver.} \end{array} \right\}$

EXAMPLE 1.—If a wheel that contains 75 teeth makes 16 revolutions per minute, required the number of teeth in another to work in it, and make 24 revolutions in the same time.

$$\text{Here } \frac{75 \times 16}{24} = 50 \text{ teeth.} = \text{Ans.}$$

EXAMPLE 2.—A wheel, 64 inches diameter, and making 42 revolutions per minute, is to give motion to

a shaft at the rate of 77 revolutions in the same time : required the diameter of a wheel suitable for that purpose.

$$\text{Here } \frac{64 \times 42}{77} = 34.9 \text{ inches.} = \text{Ans.}$$

EXAMPLE 3.—Required the number of revolutions per minute made by a wheel or pulley 20 inches diameter, when driven by another of 4 feet diameter, and making 46 revolutions per minute.

$$\text{Here } \frac{48 \times 46}{20} = 110.4 \text{ revolutions.} = \text{Ans.}$$

EXAMPLE 4.—A shaft, at the rate of 22 revolutions per minute, is to give motion, by a pair of wheels, to another shaft at the rate of $15\frac{1}{2}$; the distance of the shafts from centre to centre is $45\frac{1}{2}$ inches; the diameters of the wheels at the pitch lines are required.

$$\text{Here } 22 + 15.5 : 22 :: 45.5 \text{ in.} : \frac{22 \times 45.5}{22 + 15.5} = 26.69 \text{ in.}$$

the radius of the driven wheel; which, doubled, gives 53.38 in., the diameter. = 1st Ans.

Therefore $45.5 \text{ in.} - 26.69 \text{ in.} = 18.81 \text{ in.}$ the radius of the driver; which, doubled, gives 37.62 in., the diameter. = 2nd Ans.

EXAMPLE 5.—Suppose a drum to make 20 revolutions per minute, required the diameter of another to make 58 revolutions in the same time.

Here $58 \div 20 = 2.9$, that is, their diameters must be as 2.9 to 1; thus, if the one making 20 revolutions be called 30 inches, the other will be $30 \div 2.9 = 10.345$ inches diameter.

EXAMPLE 6.—Required the diameter of a pulley, to make $12\frac{1}{2}$ revolutions in the same time as one of 32 inches making 26.

$$\text{Here } \frac{32 \times 26}{12.5} = 66.56 \text{ inches diameter.}$$

EXAMPLE 7.—A shaft, at the rate of 16 revolutions per minute, is to give motion to a piece of machinery at the rate of 81 revolutions in the same time; the motion is to be communicated by means of two gearing wheels and two pulleys with an intermediate shaft; the driving wheel contains 54 teeth, and the driving pulley on the axis of the driven wheel is 25 inches diameter; required the number of teeth in the other wheel, and the diameter of the other pulley.

Let the driven wheel have a velocity of 36, a mean proportional between the extreme velocities 16 and 81;

then, $\frac{16 \times 54}{36} = 24$, the number of teeth in the driven

wheel. = *1st Ans.*

And $\frac{36 \times 25}{81} = 11.11$ inches, diameter of the driven pulley. = *2nd Ans.*

EXAMPLE 8.—Suppose in the last example the revolutions of one of the wheels to be given, the number of teeth in both, and likewise the diameter of each pulley, to find the revolutions of the last pulley.

Here $\frac{16 \times 54}{24} = 36$, velocity of the intermediate shaft. = *Ans.*

Also $\frac{36 \times 25}{11.11} = 81$, the velocity of the machine.

TABLE

For finding the radius of a wheel when the pitch is given, or the pitch of a wheel when the radius is given, that shall contain from 10 to 150 teeth, and any pitch required.

Num. of Teeth.	Radius.	Num. of Teeth.	Radius.	Num. of Teeth.	Radius.	Num. of Teeth.	Radius.
10	1·618	46	7·327	81	12·893	116	18·464
11	1·775	47	7·486	82	13·051	117	18·623
12	1·932	48	7·645	83	13·210	118	18·782
13	2·089	49	7·804	84	13·370	119	18·941
14	2·247	50	7·963	85	13·531	120	19·101
15	2·405	51	8·122	86	13·691	121	19·260
16	2·563	52	8·281	87	13·849	122	19·419
17	2·721	53	8·440	88	14·008	123	19·578
18	2·879	54	8·599	89	14·168	124	19·737
19	3·038	55	8·758	90	14·327	125	19·896
20	3·196	56	8·917	91	14·486	126	20·055
21	3·355	57	9·076	92	14·645	127	20·214
22	3·513	58	9·235	93	14·804	128	20·374
23	3·672	59	9·395	94	14·963	129	20·533
24	3·831	60	9·554	95	15·122	130	20·692
25	3·989	61	9·713	96	15·281	131	20·851
26	4·148	62	9·872	97	15·440	132	21·010
27	4·307	63	10·031	98	15·600	133	21·169
28	4·466	64	10·190	99	15·759	134	21·328
29	4·625	65	10·349	100	15·918	135	21·488
30	4·783	66	10·508	101	16·077	136	21·647
31	4·942	67	10·667	102	16·236	137	21·806
32	5·101	68	10·826	103	16·395	138	21·965
33	5·260	69	10·985	104	16·554	139	22·124
34	5·419	70	11·144	105	16·713	140	22·283
35	5·578	71	11·303	106	16·873	141	22·442
36	5·737	72	11·463	107	17·032	142	22·602
37	5·896	73	11·622	108	17·191	143	22·761
38	6·055	74	11·781	109	17·350	144	22·920
39	6·214	75	11·940	110	17·509	145	23·079
40	6·373	76	12·099	111	17·668	146	23·238
41	6·532	77	12·258	112	17·827	147	23·397
42	6·691	78	12·417	113	17·987	148	23·556
43	6·850	79	12·576	114	18·146	149	23·716
44	7·009	80	12·735	115	18·305	150	23·875
45	7·168						

RULE.—Multiply the radius in the table by the pitch given, and the product will be the radius of the wheel required.

Or, divide the radius of the wheel by the radius in the table, and the quotient will be the pitch of the wheel required.

EXAMPLE 1.—Required the radius of a wheel to contain 64 teeth, of 3 inch pitch.

Here $10.19 \times 3 = 30.57$ inches. = *Ans.*

EXAMPLE 2.—What is the pitch of a wheel to contain 80 teeth, when the radius is 25.47 inches?

Here $25.47 \div 12.735 = 2$ inch pitch.

Or, set off upon a straight line seven times the pitch given, divide that, or another exactly the same length, into eleven equal parts; call each of those divisions four, or each of those divisions will be equal to four teeth upon the radius.

EXAMPLE.—Were it required to find the diameter of a wheel to contain 21 teeth, the construction would be as follows:—

1	2	3	4	5	6	7
1	2	3	4	5	6	7
8	9	10	11			

<..4..8..12..16..20>

Thus, 5 divisions and $\frac{1}{2}$ of another equal the radius of the wheel.

Regular approved proportions for wheels with flat arms in the middle of the ring, and ribs, or feathers, on each side.

The length of the teeth = $\frac{6}{9}$ the pitch, besides clearance, or $\frac{5}{7}$ the pitch, clearance included.

Thickness of the teeth.....	$\frac{4}{9}$ the pitch.
Breadth on the face.....	$2\frac{1}{2}$ „
Edge of the rim	$\frac{4}{9}$ „
Rib projecting inside the rim	$\frac{4}{9}$ „
Thickness of the flat arms	$\frac{4}{9}$ „

Breadth of the arms at the points = 2 teeth and $\frac{1}{4}$ the pitch, getting broader towards the centre of the wheel in the proportion of $\frac{1}{2}$ inch to every foot in length.

Thickness of the ribs, or feathers, $\frac{1}{4}$ the pitch.

Thickness of metal round the eye, or centre, $\frac{7}{8}$ the pitch.

Wheels made with plain arms, the teeth are in the same proportion as above; the ring and the arms are each equal to one cog or tooth in thickness, and the metal round the eye same as above, in feathered wheels.

To find the power that a cast iron wheel is capable of transmitting at any given velocity.

RULE.—Multiply the breadth of the teeth, or face of the wheel in inches, by the square of the thickness of one tooth, and divide the product by the length of the teeth, the quotient is the strength in horses' power at a velocity of 136 feet per minute.

EXAMPLE.—Required the power that a wheel of the following dimensions ought to transmit with safety, namely,

Breadth of teeth..... $7\frac{1}{2}$ inches.

Thickness 1.4 „

And length..... 2 „

$$\text{Here } \frac{7.5 \times 1.4^2}{2} = \frac{7.5 \times 1.96}{2} = 7.35 \text{ horses' power.}$$

The strength at any other velocity is found by multiplying the power so obtained by any other required velocity, and by .0044, the product is the power at that velocity.

Suppose the wheel as above, at a velocity of 320 feet per minute.

$$\text{Then } 7.35 \times 320 \times .0044 = 10.3488 \text{ horses' power.}$$

ON THE MAXIMUM VELOCITY AND POWER OF WATER WHEELS.

Since publishing the first edition of this work, I have endeavoured, as far as possible, to acquire the most improved practical principles of water wheels as a moving power; and

1.—*Of undershot wheels.*

The term "undershot" is applied to a wheel when the water strikes at, or below, the centre. And the greatest effect is produced when the periphery of the wheels moves with a velocity of $\cdot 57$ that of the water;—then, to find the velocity of the water, multiply the square root or the perpendicular height of the fall in feet by 8, and the product is the velocity in feet per second.

EXAMPLE.—Required the maximum velocity of an undershot wheel, when propelled by a fall of water 6 feet in height.

Here $8\sqrt{6} = 2.45 \times 8 = 19.6$ feet velocity of water.

And $19.6 \times \cdot 57 = 11.17$ feet per second for the wheel. = *Ans.*

2.—*Of breast and overshot wheels.*

Wheels that have the water applied between the centre and the vertex are styled breast wheels, and overshot when the water is brought over the wheel and laid on the opposite side; however, in either case the maximum velocity is $\frac{2}{3}$ that of the water; then, to find the head of water proper for a wheel at any velocity, say,

As the square of 16.095, or 259, is to 4, so is the

square of the velocity of the water in feet per second, to the head * of water required.

EXAMPLE.—Required the head of water necessary for a wheel of 24 feet diameter, moving with a velocity of 5 feet per second.

Here $5 \div \frac{2}{3} = 5 \times \frac{3}{2} = 7.5$, velocity of the water in feet.

And $259 : 4 :: 7.5^2 : .87$ feet, head of water required. = *Ans.*

But for wheels of from 15 to 20 feet diameter inclusive, add one-tenth of the diameter minus 1 foot. And for wheels of from 20 to 30 feet diameter inclusive, add one-twentieth of the diameter.

This additional head is intended to compensate for the friction of water in the aperture of the sluice to keep the velocity as 3 to 2 of the wheel; thus, in place of .87 feet head for a 24 feet wheel, it will be $.87 + \frac{2.4}{20} = .87 + 1.2 = 2.07$ feet head of water.

If the water flow from under the sluice, multiply the square root of the depth in feet by 5.4, and by the area of the orifice also in feet, and the product is the quantity discharged in cubic feet per second.

Again, if the water flow over the sluice, multiply the square root of the depth in feet by 3.6; and the product multiplied by the length and depth, also in feet, gives the number of cubic feet discharged per second, nearly.

EXAMPLE 1.—Required the number of cubic feet per second that will issue from the orifice of a sluice 5 feet long, 9 inches wide, and 4 feet from the surface of the water.

Here $\sqrt{4} \times 5.4 \times 5 \times .75 = 2 \times 5.4 \times 5 \times .75 = 40.5$ cubic feet per second. = *Ans.*

* By head is understood the distance between the aperture of the sluice and where the water strikes upon the wheel.

EXAMPLE 2.—What quantity of water per second will be expended over a wear, dam, or sluice, whose length is 10 feet, and depth 6 inches?

Here $\sqrt{.5 \times 3.6 \times 10 \times .5} = .707107 \times 3.6 \times 5 = 12.7279$ cubic feet per second. = *Ans.*

In estimating the power of water wheels, half the head must be added to the whole fall, because 1 foot of fall is equal to 2 feet of head; call this the effective perpendicular descent; multiply the weight of the water per second by the effective perpendicular descent and by 60; divide the product by 33000, and the quotient is the effect expressed in horses' power.

EXAMPLE 1.—Given 16 cubic feet of water per second, to be applied to an *undershot* wheel, the head being 12 feet (and the fall = 0), required the power produced.

$$\text{Here } 12 \div 2 = 6 \text{ and } \frac{6 \times 16 \times 62.5 \times 60}{33000} =$$

10.9 horses' power, nearly.

EXAMPLE 2.—Given 16 cubic feet of water per second, to be applied to a high breast or an *overshot* wheel, with 2 feet head and 10 feet fall; required the power.

$$\text{Here } 2 \div 2 = 1 \text{ and } \frac{(1 + 10) \times 16 \times 62.5 \times 60}{33000}$$

= 20 horses' power.

N.B.—Only about two-thirds of the above results can be taken as real communicative power to machinery.

OF THE CIRCLE OF GYRATION IN WATER WHEELS.

The centre or circle of gyration is that point or circle in a revolving body into which, if the whole quantity of matter were collected or placed, the same moving force would generate the same angular velocity, which renders

it of the utmost importance in the erection of water wheels, and the motion ought always to be communicated from that centre when it is possible.

To find the circle of gyration.

RULE.—Add together twice the weight of the shrouding, buckets, &c., $\frac{2}{3}$ of the weight of the arms, and the weight of the water; multiply the sum by the square of the radius; divide the product by twice the sum of the weight of the shrouding, arms, &c., added to the weight of the water, and the square root of the quotient is the radius of the circle of gyration from the centre of suspension, nearly.

EXAMPLE.—Required the distance of the centre of gyration from the centre of suspension in a water wheel 22 feet diameter, shrouding, buckets, &c., = 18 tons, arms = 12 tons, and water = 10 tons.

Here $\sqrt{\{(2 \times 18 + \frac{2}{3} \times 12 + 10) \times 11^2 \div [(18 + 12) \times 2 + 10]\}} = \sqrt{\{(36 + 8 + 10) \times 121 \div (30 \times 2 + 10)\}} = \sqrt{\{54 \times 121\} \div 70} = \sqrt{6534 \div 70} = \sqrt{93.34} = 9.7$ feet from the centre of suspension, nearly.

Table of angles for windmill sails.

The radius is supposed to be divided into six equal parts, and $\frac{1}{6}$ from the centre is called 1, the extremity being denoted by 6.

No.	Angle with the Plane of Motion.	
1	18°	24°
2	19	21
3	18	18
4	16	14
5	12½	9
6	7	3 extremity.

The first column contains the angles according to Smeaton ; but experience has taught us that the angles in the second column are preferable.

THE VELOCITY OF THRESHING MACHINES, MILLSTONES,
CUTTERS FOR BORING IRON, &c.

The drum or beaters of a threshing machine ought to move with a velocity of about 3000 feet per minute ; hence, divide 11460 by the diameter of the drum in inches ; or 955 by the diameter of the drum in feet ; and the quotient is the number of revolutions required per minute. And

The feeding rollers must make half the revolutions of the drum, when their diameters are about $3\frac{1}{2}$ inches.

If the machine is driven by horses, their velocity ought to be from $2\frac{1}{2}$ to 3 times round a 24 feet ring per minute.

The velocity of millstones ought to be from about 1550 to 1600 feet per minute ; hence, divide 500 by the diameter of a millstone, in feet, or 6000 by the diameter in inches, and the quotient is the number of revolutions required per minute.

In boring cast iron, the cutters ought to have a velocity of about 110 inches per minute ; hence, divide 35 by the diameter in inches, the quotient is the number of revolutions of the cutter or boring head per minute.

And divide 100 by the diameter in inches, the quotient is the number of revolutions per minute for turning wrought iron in general, which is about 300 feet per minute, and about half that velocity for cast iron.

OF PUMPS AND PUMPING ENGINES.

Pumps are chiefly designated by the names of "lifting" and "force" pumps. Lifting pumps are applied to wells, &c., where the height of the bucket, from the surface of the water, must not exceed 33 feet; this being nearly equal to the pressure of the atmosphere, or the height to which water would be forced up into a vacuum by the pressure of the atmosphere. Force pumps are applicable on all other occasions, as raising water to any required height, supplying boilers against the force of the steam, hydrostatic presses, &c.

The power required to raise water to any height is as the weight and velocity of the water with an addition of about $\frac{1}{3}$ of the whole power for friction; hence the rule,—Multiply the perpendicular height of the water, in feet, by the velocity, also in feet, and by the square of the pump's diameter in inches, and again by 2.046; divide the product by 165000, and the quotient will be the number of horses' power required.

EXAMPLE.—Required the power necessary to overcome the resistance and friction of a column of water 4 inches diameter, 60 feet high, and flowing with a velocity of 130 feet per minute.

$$\begin{aligned} \text{Here } \frac{60 \times 130 \times 4^2 \times 2.046}{165000} &= \frac{6 \times 13 \times 16 \times 31}{25000} \\ &= \frac{6 \times 13 \times 16 \times 124}{100000} = \frac{154752}{100000} = 1.55 \text{ horses' power,} \end{aligned}$$

nearly.

NOTE.—Hot liquor pumps, or pumps to be employed in raising any fluid where steam is generated, require to be placed in the fluid, or as low as the bottom of it, on account of the steam filling the pipes, and acting as a counterpoise to the atmosphere; and the diameter of the pipes to and from a pump ought not to be less than $\frac{1}{3}$ of the pump's diameter.

The diameter of a pump and velocity of the water given, to find the quantity discharged in gallons, or cubic feet, in any given time.

RULE.—Multiply the velocity of the water, in feet per minute, by the number of minutes in the given time, by the square of the pump's diameter in inches, and by '034 for imperial gallons; or, '0005456 for cubic feet, and the product will be the number of gallons or cubic feet discharged in the given time, nearly.

EXAMPLE.—What is the number of imperial gallons of water discharged per hour by a pump 4 inches diameter, the water flowing at the rate of 130 feet per minute?

Here $130 \times 60 \times 4^2 \times '034 = 13 \times 6 \times 16 \times 3'4 = 4243'2$ gallons. = *Ans.*

The length of stroke and number of strokes given, to find the diameter of a pump, and number of horses' power that will discharge a given quantity of water in a given time.

RULE 1.—Multiply the number of imperial gallons required in the given time, by 353, or the number of cubic feet by 2201, and divide the result by the product of the velocity of the water in inches, and the number of minutes in the given time, the square root of the quotient will be the pump's diameter, in inches.

2.—Multiply the number of gallons in the given time by 60, or the number of cubic feet by 375, and by the perpendicular height of the water in feet, divide the result by the product of 165000 and the number of minutes in the given time, then will the quotient be the number of horses' power required.

EXAMPLE.—Required the diameter of a pump, and number of horses' power, capable of filling a cistern 20

feet long, 12 feet wide, and $6\frac{1}{2}$ feet deep, in 45 minutes, whose perpendicular height is 53 feet; the pump to have an effective stroke of 26 inches, and make 30 strokes per minute.

Here $20 \times 12 \times 6.5 = 1560$ cubic feet; and $26 \times 30 = 780$, the velocity in inches per minute.

$$\begin{aligned} \text{Then, } \sqrt{\frac{1560 \times 2201}{780 \times 45}} &= \sqrt{\frac{2 \times 2201}{45}} = \\ \sqrt{\frac{4402}{45}} &= \sqrt{\frac{880.4}{9}} = \frac{\sqrt{880.4}}{3} = \frac{29.67}{3} = 9.89 \text{ inches} \end{aligned}$$

diameter of pump. = 1st Ans.

$$\begin{aligned} \text{Also, } \frac{1560 \times 375 \times 53}{165000 \times 45} &= \frac{13 \times 53}{165} = \frac{689}{165} = \\ 4.18 \text{ horses' power.} &= 2nd \text{ Ans.} \end{aligned}$$

To find the time a cistern will take in filling, when a known quantity of water is going in, and a known portion of that water is going out, in a given time.

RULE.—Divide the content of the cistern, in gallons, by the difference of the quantity going in, and the quantity going out, and the quotient is the time in hours and parts that the cistern will take in filling.

EXAMPLE.—If 30 gallons per hour run in and $22\frac{1}{2}$ gallons per hour run out of a cistern capable of containing 200 gallons, in what time will the cistern be filled?

$$\text{Here } \frac{200}{30 - 22.5} = \frac{200}{7.5} = \frac{800}{30} = 26.666, \text{ number of}$$

hours = 26 hours and 40 minutes. = Ans.

To find the time a vessel will take in emptying itself of water.

Mr. Banks ascertained, from very accurate experiments, that a vessel, 3·166 feet long and 2·705 inches diameter, would empty itself in 3 minutes and 16 seconds, through an orifice in the bottom, whose area is ·0141 inches; and another 6·458 feet long, the diameter and orifice, as before, would do the same in 4 minutes and 40 seconds; hence, from these experiments a rule is obtained, namely,

Multiply the square root of the depth in feet by the area of the falling surface in inches, divide the product by the area of the orifice, multiplied by 3·7, and the quotient is the time required in seconds, nearly.

EXAMPLE.—How long will it require to empty a vessel of water, 9 feet high, and 20 inches diameter, through a hole $\frac{3}{4}$ inch in diameter?

Here $\sqrt{9} = 3$, the square root of the depth,

And $20^2 \times \cdot 7854 = 400 \times \cdot 7854$ square inches, area of the falling surface,

And $\cdot 75^2 \times \cdot 7854 = \cdot 5625 \times \cdot 7854$ square inches, area of the orifice;

$$\text{Then, } \frac{400 \times \cdot 7854 \times 3}{\cdot 5625 \times \cdot 7854 \times 3 \cdot 7} = \frac{400 \times 3}{\cdot 5625 \times 3 \cdot 7} =$$

$$\frac{16 \times 400 \times 3}{9 \times 3 \cdot 7} = \frac{6400}{11 \cdot 1} = 576 \cdot 6 \text{ seconds; or,}$$

9 minutes and 37 seconds, *very nearly.* = *Ans.*

On the pressure of fluids.

The side of any vessel containing a fluid sustains a pressure equal to the area of the side, multiplied by half the depth; thus,

Suppose each side of a vessel to be 12 feet long and 5 feet deep, when filled with water, what pressure is upon each side?

$12 \times 5 = 60$ feet, the area of the side,

$2\cdot5$ feet = half the depth, and

$62\cdot5$ lbs. = the weight of a cubic foot of water.

Then, $60 \times 2\cdot5 \times 62\cdot5 = 9375$ lbs. = *Ans.*

To find the number of imperial gallons contained in a yard of circular pipe, of any given diameter.

RULE.—Multiply the square of the diameter of the circular pipe in inches by $\cdot102$, the product will be the content of the pipe in imperial gallons nearly.

EXAMPLE 1.—Required the number of imperial gallons contained in each yard of a $6\frac{1}{4}$ inch circular pipe.

Here $6\cdot25^2 \times \cdot102 = 39\cdot0625 \times \cdot102 = 3\cdot984375$ imperial gallons. = *Ans.*

EXAMPLE 2.—Required the content of a yard of 4 inch circular pipe in imperial gallons.

Here $4^2 \times \cdot102 = 16 \times \cdot102 = 1\cdot632$ imperial gallons. = *Ans.*

To find the weight that a given power can raise by one of Bramah's pumps, or hydrostatic presses.

RULE.—Multiply the square of the diameter of the ram in inches by the power applied in lbs., and by the effective leverage of the pump handle; divide the product by the square of the pump's diameter, also in inches, and the quotient is the weight that the power is equal to.

EXAMPLE.—What weight will a power of 50 lbs. raise by means of an hydrostatic press, whose ram is

7 inches diameter, pump $\frac{7}{8}$, and the effective leverage of the pump handle being as 6 to 1?

$$\text{Here } \frac{7^2 \times 50 \times 6}{.875^2} = \frac{7^2 \times 8^2 \times 50 \times 6}{7^2} = 64 \times$$

300 = 19200 lbs., or 8 tons 11 cwt. = *Ans.*

In the following rules for pumping engines the boiler is supposed to be loaded with about $2\frac{1}{2}$ lbs. per square inch, and the barometer attached to the condenser indicating 26 inches on an average, or 13 lbs., making altogether $15\frac{1}{2}$ lbs., from which deducting $\frac{1}{2}$ for friction, leaves a pressure of 10 lbs. nearly upon each square inch of the piston.

To find the diameter of a cylinder to work a pump of a given diameter for a given depth.

RULE.—Multiply the pump's diameter in inches by the square root of $\frac{1}{3}$ of the depth of the pit in fathoms, and the product will be the cylinder's diameter in inches.

EXAMPLE.—Required the diameter of a cylinder to work a pump 12 inches diameter for a depth of 27 fathoms.

Here $12 \times \sqrt{\frac{27}{3}} = 12 \times \sqrt{9} = 12 \times 3 = 36$
inches diameter. = *Ans.*

To find the diameter of a pump that a cylinder of a given diameter can work at a given depth.

RULE.—Multiply the cylinder's diameter in inches by the square root of 3 times the depth of the pit in fathoms, and divide the product by the depth of the pit in fathoms, the quotient will be the pump's diameter in inches.

EXAMPLE.—What diameter of a pump will a 36 inch cylinder be capable of working 27 fathoms deep?

$$\text{Here } \frac{36 \times \sqrt{(3 \times 27)}}{27} = \frac{36 \times \sqrt{81}}{27} = \frac{4 \times 9}{3} =$$

$$4 \times 3 = 12 \text{ inches diameter.} = \textit{Ans.}$$

To find the depth from which a pump of a given diameter will work by means of a cylinder of a given diameter.

RULE.—Divide three times the square of the cylinder's diameter in inches by the square of the pump's diameter also in inches, and the quotient will be the depth of the pit in fathoms.

EXAMPLE.—Required the depth that a cylinder of 36 inches diameter will work a pump of 12 inches diameter.

$$\text{Here } \frac{36^2 \times 3}{12^2} = 3^2 \times 3 = 27 \text{ fathoms.} = \textit{Ans.}$$

APPROXIMATE RULES FOR CALCULATING LIQUIDS.

To find the number of imperial gallons contained in any square or rectangular cistern.

RULE.—Multiply the content of the cistern in cubic feet by 6·232, or the content in cubic inches by ·003607, and the product is the number of gallons nearly.

EXAMPLE 1.—A cistern is 8 feet long, $4\frac{1}{2}$ feet wide, and 3 feet deep, required its contents in imperial gallons.

Here $8 \times 4\cdot5 \times 3 = 108$ cubic feet,

And $108 \times 6\cdot232 = 673\cdot056$ gallons. = *Ans.*

Or, 8 feet = 96 inches; $4\frac{1}{2}$ feet = 54 inches; and 3 feet = 36 inches;

Then, $96 \times 54 \times 36 = 186624$ cubic inches,

And $186624 \times \cdot003607 = 673\cdot153$ gallons. = *Ans.*
as before, nearly.

Any two dimensions of a square or rectangular cistern being given, to find the third, that shall contain any number of imperial gallons required.

RULE.—Multiply the number of gallons that the cistern is required to contain by ·16046 for feet, or by 277·274 for inches, according as the dimensions are given in feet or inches, divide this product by the products of the two given dimensions, and the quotient will be the third dimension of the cistern nearly.

EXAMPLE.—Required the depth of a cistern to contain 800 imperial gallons, the length being $6\frac{1}{2}$ feet, and width $4\frac{3}{4}$ feet.

$$\text{Here } \frac{800 \times \cdot16046}{6\frac{1}{2} \times 4\frac{3}{4}} = \frac{8 \times 128\cdot368}{13 \times 19} = \frac{1026\cdot944}{247} =$$

4·16 feet deep. = *Ans.*

To find the content of a cylinder in imperial gallons.

RULE.—Multiply the square of the diameter in feet by the length of the cylinder, also in feet, and by 4·895 ;

Or, multiply the square of the diameter in inches by the length in feet and by ·034 ;

Or, multiply the square of the diameter in inches by the length, also in inches, and by ·00283', and the product will be the content in gallons, nearly.

EXAMPLE.—How many imperial gallons are contained in a circular well $22\frac{1}{2}$ feet deep, and $3\frac{1}{2}$ feet diameter ?

Here $3\cdot5^2 \times 22\cdot5 \times 4\cdot895 = 1349$ gallons. = *Ans.*

Or, $3\frac{1}{2}$ feet = 42 inches,

And, $42^2 \times 22\cdot5 \times \cdot034 = 1349$ gallons. = *Ans.*
as before.

Also, $22\frac{1}{2}$ feet = 270 inches,

And $42^2 \times 270 \times \cdot00283' = 1349$ gallons. = *Ans.*
as before.

The length of a cylinder given, to find the diameter, or the diameter given, to find the length that shall contain any number of imperial gallons required.

RULE.—Multiply the number of gallons that the cylinder is required to contain by ·2043, divide the product by the length in feet, and the square root of the quotient is the diameter in feet, and parts of a foot ;

Or, multiply the number of gallons by ·2043, and divide the product by the square of the diameter in feet, and the quotient is the length in feet and parts of a foot ;—and

If the dimensions are in inches in place of feet, use 353 in place of ·2043.

EXAMPLE.—What must be the diameter of a cylinder to contain 5 imperial gallons, when the length is 20 inches ?

$$\text{Here } \sqrt{\left(\frac{353 \times 5}{20}\right)} = \sqrt{\frac{353}{4}} = \frac{1}{2} \times \sqrt{353} =$$

$$\frac{18.8}{2} = 9.4 \text{ inches diameter.} = \text{Ans.}$$

The cube of the diameter of a sphere in feet, multiplied by 3.263 = imperial gallons;

Or, the cube of the diameter of a sphere in inches, multiplied by .001888 = imperial gallons.

NOTE.—The weight of a cubic foot of water = 62.5 *lbs.* avoirdupois.

Weight of a cubic inch = .03617 *lbs.* avoirdupois.

Weight of a column of water 12 inches high and 1 inch square = .434 *lbs.* avoirdupois.

Weight of a cylindrical foot of water = 49.1 *lbs.* avoirdupois.

Weight of a cylindrical inch = .02842 *lbs.* avoirdupois.

Weight of a cylindrical column of water 12 inches high and 1 inch diameter = .341 *lbs.* avoirdupois.

Take, for example, a cylindrical column of water 11 inches diameter and 15 feet high, required its weight.

Here $11^2 \times 15 \times .341 = 618.915$ *lbs.* avoirdupois.

11.2 imperial gallons of water = 1 *cwt.*

224 imperial gallons of water = 1 *ton.*

1.8 cubic feet of water..... = 1 *cwt.*

35.84 ditto = 1 *ton.*

1 ditto = 6.238 imperial gallons.

1 cylindrical foot of ditto.... = 4.895 imperial gallons.

OF STEAM AND THE STEAM ENGINE.

Steam is the visible moist vapour which arises from all bodies that contain juices easily expelled from them by heats not sufficient for their combustion.

But steam, as applicable at present to the steam engine, is highly rarified water, the particles of which are expanded by the absorption of caloric, or the matter of heat.

Water rises in vapour at all temperatures, but is confined to the surface of the fluid acted upon until it has attained 212° Fahrenheit, called the boiling point; at that heat steam ascends through it, preventing its elevation to a higher temperature by carrying the heat off in a latent form.

The latent heat of steam at the common pressure of the atmosphere, according to very accurate experiments, is found to be 1000° ; and we know that the sensible, or thermometric heat $= 212^{\circ}$. Now $212^{\circ} - 32^{\circ} = 180^{\circ}$ and $1000^{\circ} + 180^{\circ} = 1180^{\circ}$; therefore, steam at 212° is highly rarified water, containing 1180° of heat; hence, to find the latent heat of steam at any other temperature, subtract the sensible heat from 1180° , and the difference $+ 32^{\circ} =$ the latent heat.

EXAMPLE.—Required the latent heat of steam whose sensible heat is 224° .

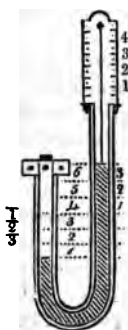
$$\text{Here } 1180^{\circ} - 224^{\circ} = 956^{\circ},$$

$$\text{And } 956^{\circ} + 32^{\circ} = 988^{\circ} \text{ latent heat.} = \text{Ans.}$$

One cubic inch of water produces about 1700 cubic inches of steam at 212° , or the common pressure of the atmosphere; but the boiling point varies considerably, according to the pressure on the surface of the fluid, and, of course, materially affects the density of the vapour produced; thus, in a vacuum, water boils at about 90° ; under common pressure, at 212° ; and when pressed with a column of mercury 5 inches in height, will not boil

until heated to 217° ; each inch of mercury producing by its pressure a rise of about 1° in the thermometer.

The pressure or force of steam in the boiler (less than the weight upon the safety valve) is generally indicated by a column of mercury in a bent iron tube, which causes the range of the float to be only half the range of the mercury, 2 inches of mercury being nearly equal to 1 lb. pressure of steam in the boiler, thus :—

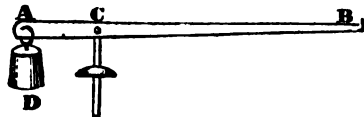


Each inch of the float indicates a pressure of 1 lb. nearly.

—Level of the mercury when there is no pressure of steam.

To calculate the effect of a lever and weight upon the safety valve of a steam boiler, &c.

The lever, in all cases, is supposed to be made, finished, and balanced, by a known weight or weights, on the short end, making that point where it rests, or is attached to the valve, the centre of motion; then that weight, added to the weight of the lever, is the effective weight upon the valve, independent of any other additional weight, thus :—



Then there are three different ways that it may be required to calculate the lever.

1.—*When a certain pressure may be required upon the valve, the distance of the weight upon the lever, and distance of the valve from the centre of motion given, to find what weight will be required upon the lever at that distance.*

RULE.—From the required pressure on the valve in *lbs.* subtract the weight of the valve, plus the effective weight of the lever, multiply the remainder by the distance between the fulcrum and the valve, divide the product by the distance between the fulcrum and the weight, and the quotient is the weight in *lbs.* required to be placed upon the lever at that distance.

2.—*When a certain pressure upon the valve is required, the weight upon the lever and distance of valve from the centre of motion given, to find where that weight must be placed.*

RULE.—From the required weight upon the valve in *lbs.* subtract the weight of the valve, plus the effective weight of the lever, multiply the remainder by the distance between the fulcrum and the valve, divide the product by the weight in *lbs.* upon the lever, and the quotient is the distance in inches from the fulcrum that the weight must be placed.

3.—*When the distance of weight, distance of valve from the centre of motion, and weight upon the lever given, to find what pressure is upon that valve.*

RULE.—Multiply the weight in *lbs.* upon the lever by the distance in inches to the fulcrum, divide the product by the distance between the fulcrum and the valve, and the quotient, plus the weight of the valve and effective weight of the lever, equal the weight upon the valve in *lbs.*

EXAMPLE 1.—Suppose the lever A B (as above) to be 24 inches in length, and the valve C placed 5 inches

from the centre of motion A, what weight must be placed upon the lever 20 inches from A, to equal 80 lbs., on the valve C, the weight of the lever being 2 lbs., the weight D, which balances the lever, $4\frac{1}{2}$ lbs. and the weight of the valve 3 lbs.

Here 2.0 lbs. weight of the lever.

4.5 lbs. to balance ditto.

3.0 lbs. weight of the valve.

$$\text{Sum} = 9.5 \text{ lbs. then } \frac{(80 - 9.5) \times 5}{20} = \frac{70.5 \times 5}{20}$$

$$\frac{70.5}{4} = 17.625 \text{ lbs.} = \text{Ans.}$$

EXAMPLE 2.—Suppose, as in the last example, the weight upon the lever equal 17.625 lbs., it is required at what distance from A the weight must be placed equal 80 lbs. at C.

$$\text{Here } \frac{(80 - 9.5) \times 5}{17.625} = \frac{70.5 \times 5}{17.625} = \frac{70.5 \times 20}{70.5}$$

20 inches. = Ans.

EXAMPLE 3.—Suppose, as before, that a weight 17.625 lbs. is placed upon the lever 20 inches from the pressure at C, the distance from the centre of motion being 5 inches, and the effective weight the lever at that point equal $6\frac{1}{2}$ lbs., also the weight the valve 3 lbs.

$$\text{Here } \frac{17.625 \times 20}{5} = 17.625 \times 4 = 70.5 \text{ lbs. to which}$$

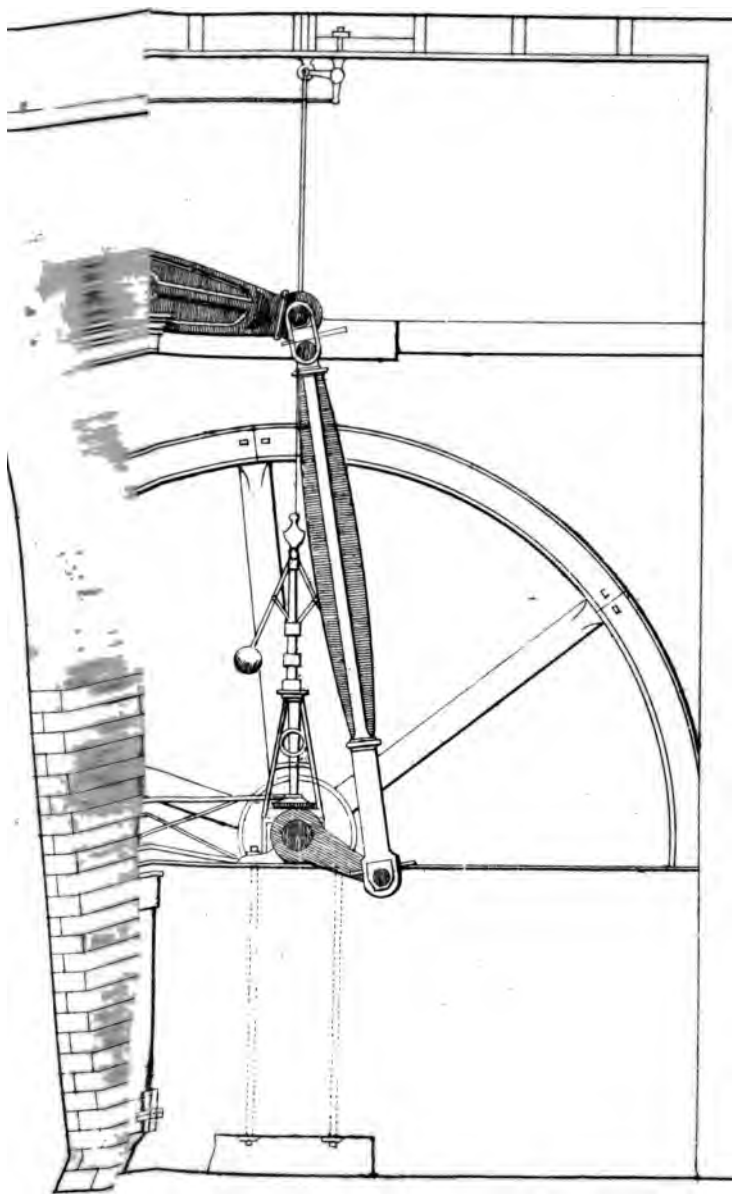
6.5 lbs. } add
3.0 lbs. }

$$\text{Sum} = 80 \text{ lbs.} = \text{Ans.}$$

To find the proper diameter for a safety valve.

RULE.—Multiply the bottom surface of the boiler, surface immediately exposed to the action of the fire, feet, by the multiplier opposite to the pressure in lb on each square inch of the safety valve, and the square root of the product is the valve's diameter in inches





7 PRINCIPLE.

the narrowest part. If the boiler is to have two safety valves, then the square root of half the product equal the diameter of each.

Pressure in lbs. per square inch.	Multipliers.	Pressure in lbs. per square inch.	Multipliers.
3	·356	15	·315
4	·353	20	·305
5	·348	25	·293
6	·344	30	·289
7	·339	35	·282
8	·336	40	·275
10	·329	45	·270
12	·321	50	·264

Table of the elastic force of steam on a square inch.

Steam with a pressure of	lbs. on a square inch, equal	lbs. on a circular inch ; and to maintain that pressure requires to be kept at a uniform temperature of	° F	and will support a column of mercury	inches in height.
2½		1·963	220		5·15
3		2·356	222		6·18
3½		2·749	223½		7·21
4		3·142	225½		8·24
4½		3·534	227		9·27
5		3·927	228½		10·3
5½		4·320	230		11·3
6		4·712	231½		12·3
6½		5·105	233		13·4
7		5·498	234		14·4
7½		5·888	235		15·4
8		6·283	236		16·5
8½		6·676	237½		17·5
9		7·068	239		18·5
9½		7·461	240		19·6
10		7·854	241		20·6
10½		8·247	242		21·6
11		8·639	243		22·6
11½		9·032	244		23·7
12		9·424	245½		24·7
15		11·78	252		30·9
20		15·71	261		41·2
25		19·63	269		51·5
30		23·56	276		61·8
35		27·49	283		72·1
40		31·42	289		82·4
45		35·34	294½		92·7
50		39·27	300		103

Multiply the degrees of heat in either this or the following table by '06, and the product will be the superficial feet of flue plate exposed to the action of the fire for each horse power.

And multiply the degrees of heat by '41, and the product will be the areal inches of furnace bar for each horse power.

Table of the elastic force of steam on a circular inch.

Steam with a pressure of	<i>Lbs.</i> on a circular inch, equal	<i>Lbs.</i> on a square inch ; and to maintain that pressure requires to be kept at a uniform temperature of	° F	and will support a column of mercury	inches in height.
2½	3.183		222½		6.56
3	3.820		224½		7.87
3½	4.456		226½		9.18
4	5.093		228½		10.5
4½	5.729		230½		11.8
5	6.366		232		13.1
5½	7.002		234		14.4
6	7.639		235½		15.7
6½	8.276		236½		17.0
7	8.912		238½		18.3
7½	9.549		239½		19.7
8	10.19		241		21.0
8½	10.82		242½		22.3
9	11.46		244		23.6
9½	12.09		245½		24.9
10	12.73		247		26.2
10½	13.36		248½		27.5
11	14.00		250		28.9
11½	14.64		251		30.1
12	15.28		252½		31.5
15	19.10		259		39.3
20	25.46		270		52.5
25	31.83		278½		65.6
30	38.20		287		78.7
35	44.56		294		91.8
40	50.93		300½		105
45	57.29		305		118
50	63.66		309		131

Proportions of fuel.

The proportion that various substances bear to each other in producing heats sufficient to raise equal quan-

tities of water to equal temperatures are nearly as follows :

Coke.....	0·375	Culm or Slack...	1·875
Coal	1·000	Wood	2·875

Hence, multiply the degrees of heat in either of the preceding tables by the following numbers opposite the material by which the steam is to be produced, and the product will be the weight in *lbs.* avoirdupois that is required on an average per hour for each horse power :

Coke.....	·024	Slack	·118
Coal	·063	Wood	·180

To find the height of a column of water to supply a steam boiler against any pressure of steam required.

RULE.—Multiply the pressure in pounds (upon a square inch of the boiler) by 2·5, and the product will be the height in feet above the surface of water in the boiler.

EXAMPLE.—Required the length of feed pipe capable of supplying a boiler with water when the pressure of steam is 4 pounds per square inch.

Here $2\cdot5 \times 4 = 10$ feet above the surface of the water in the boiler. = *Ans.*

STEAM ENGINE is the name of a machine which derives its moving powers from the elasticity and condensibility of steam.

Steam, to produce a maximum of useful effect as a moving power, requires to be reduced to a certain determined velocity, and although this maximum velocity has been exhibited to the public by various eminent writers upon the steam engine, still discrepancies exist amongst practical engineers ; and no universally acknowledged rules have as yet been established : however, the following tables may be relied upon as exhibiting the results deduced from the most celebrated

rules, and tested by many engines doing the greatest amount of duty, as proved by accurate trials with indicators of the most recent and approved construction.

Length of Stroke in ft. and in.	Number per Minute.	Velocity in Feet per Minute.	Length of Stroke in ft. and in.	Number per Minute.	Velocity in Feet per Minute.
2 0	43	172	4 6	24½	218½
2 6	38	190	5 0	22	220
3 0	34	204	6 0	19	228
3 6	30	210	7 0	17½	245
4 0	27	216	8 0	16	256

N.B.—These are to be considered as the velocities of land engines, or engines whose connecting rods are not less than three times the length of stroke; but marine engines, being generally confined to connecting rods of not more than 2 or 2½ times the length of stroke, have their maximum velocities considerably reduced. Hence, the subjoined table will be found pretty correct when the periphery of the wheels moves with a velocity of about 1300 feet per minute, and the floats or paddle boards calculated by the following rules, which I have found, in practice, to produce the greatest satisfaction; namely, economizing of fuel, a steady supply of steam, without waste, and the vessel propelled quicker than when the surface of the floats was less, and moving at a greater velocity.

Table of velocities for marine engines.

Length of Stroke in ft. and in.	Number per Minute.	Velocity in Feet per Minute.	Length of Stroke in ft. and in.	Number per Minute.	Velocity in Feet per Minute.
2 0	42	168	4 0	24	192
2 3	39½	177½	4 6	21½	193½
2 6	36	180	5 0	20	200
2 9	33	181½	5 6	19	209
3 0	31	186	6 0	18	216
3 6	27	189	7 0	15½	220½

To find the surface of the floats or paddle boards.

RULE 1.—Multiply the number of horses' power that the engine is equal to by $3\frac{3}{4}$, divide the product by the diameter of the wheel in feet, and the quotient is the area of each float, or paddle board.

RULE 2.—Multiply the number of horses' power by $2\frac{1}{10}$, divide this product by the diameter, and the quotient is the length in feet.

RULE 3.—The breadth is 1 foot 10 inches.

EXAMPLE.—Required the area, length, and breadth of each paddle board, for a steam vessel with two engines of 80-horse power each, and wheels of 20 feet diameter.

$$\text{Here } \frac{80 \times 3\frac{3}{4}}{20} = 3\frac{3}{4} \times 4 = 15 \text{ feet area.} = 1^{\text{st}}$$

Ans.

$$\text{Then } \frac{80 \times 2\frac{1}{10}}{20} = 4 \times 2\frac{1}{10} = 8\frac{1}{10} = \text{length of}$$

each board. = *2nd Ans.*

And 1 foot 10 inches the breadth. = *3rd Ans.*

And when there is only one engine in the vessel, take $\frac{2}{3}$ of the area, and length found as above.

Each wheel, from 12 to 14 feet diameter, ought to have 12 floats; from 14 to 16 feet diameter, 14 floats; from 16 to 18 feet diameter, 16 floats; and from 18 to 22 feet diameter, 18 floats, &c. (which is nearly 1 float to each foot of the diameter).

Principles upon which the rule is founded for calculating the power of a steam engine.

Hitherto it has been customary, in estimating the power of condensing engines, to reckon the force of

the steam at a constant quantity, namely, $2\frac{1}{2}$ lbs. per circular inch, totally disregarding any extra pressure in the boiler, or increased weight upon the safety valve.

Hence, in order to form a rule whereby to approximate more nearly to the real effective power of the engine, it was necessary first to ascertain the effective force of the steam,—And,

To determine this, I recently made a series of experiments upon engines without any extra lap upon the valves, whereby to work expansively, when I found that, on account of the nature of the valve's motion, only about three-fourths of the stroke was performed by steam at, or near, the density of the steam in the boiler, the stroke, of course, being terminated expansively; hence, the whole effective force of the steam thus applied can only be taken at about four-fifths of its original pressure.

The benefit arising from the condenser is on an average equal to 26 inches of mercury, or about 13 lbs. per square inch, consequently, 13 plus four-fifths of the pressure on each square inch of the safety valve, equal the whole effective force, on each square inch of the piston's area.

Then about $8\frac{1}{4}$ lbs. is expended in overcoming the resistance and friction of a condensing engine, and may be thus estimated: 13 minus $8\frac{1}{4}$ equal $4\frac{3}{4}$, and $4\frac{3}{4}$ plus $\frac{4}{5}$ ths of the weight upon each square inch of the safety valve equal the whole amount of useful effect in giving motion to machinery.

The process of calculation may be simplified thus: $4\frac{3}{4}$ lbs. per square inch = $3\cdot73$ lbs. per circular inch, by which means the circle only requires to be squared, and the labour of multiplying by $\cdot7854$ is dispensed with.

GENERAL RULES.

1.—Multiply the square of the cylinder's diameter in inches by $3\cdot73$ plus $\frac{4}{5}$ ths the pressure on each circular

inch of the safety valve, and by the velocity of the piston in feet per minute; divide the product by 33000, and the quotient is the effect of the engine expressed in horses' power.

EXAMPLE.—Suppose a cylinder $24\frac{1}{2}$ inches diameter, stroke 4 feet, or 200 feet velocity per minute, and the weight upon the safety valve $3\cdot5$ lbs. per circular inch, required the effective power.

Here $\frac{4}{3}$ ths of $3\cdot5 = 2\cdot8$, and $3\cdot73 + 2\cdot8 = 6\cdot53$ lbs. effective force.

$$\begin{aligned} \text{Then, } \frac{24\cdot5^2 \times 6\cdot53 \times 200}{33000} &= \frac{49^2 \times 6\cdot53 \times 2}{2^2 \times 330} = \\ \frac{2401 \times 6\cdot53}{660} &= \frac{2401 \times 6\cdot53}{66} = \frac{1568}{66} = 24 \text{ horses' } \\ \text{power.} &= \text{Ans.} \end{aligned}$$

2.—Multiply 33000 by the number of horses' power required, and divide the product by the velocity of the piston in feet per minute, multiplied by $3\cdot73$ plus $\frac{4}{3}$ ths the pressure on each circular inch of the safety valve, and the square root of the quotient is the cylinder's diameter in inches.

EXAMPLE.—Required the diameter of a cylinder for an engine of 30 horses' power, with a 6 feet stroke, or 228 feet per minute, and steam at $2\frac{1}{2}$ lbs. per circular inch.

Here $\frac{4}{3}$ ths of $2\cdot5 = 2$; and $3\cdot73 + 2 = 5\cdot73$ lbs. effective force.

$$\begin{aligned} \text{Hence } \sqrt{\frac{33000 \times 30}{228 \times 5\cdot73}} &= \sqrt{\frac{990000}{228 \times 5\cdot73}} = \\ \sqrt{\frac{247500}{57 \times 5\cdot73}} &= \sqrt{\frac{247500}{326\cdot61}} = \sqrt{758} = 27\cdot53 \text{ inches } \\ \text{diameter.} &= \text{Ans.} \end{aligned}$$

NOTE.—To obtain four-fifths of the pressure of steam, multiply the original pressure by 4 and divide by 5, the quotient is the pressure required.

Or, subtract from the original pressure its fifth part.

The above are to be taken as general practical rules for engines not working expansively further than what is compulsory from the nature of the slide valve; but where engines are worked more expansively, and greater accuracy required, recourse must be had to the following rules for obtaining the uniform force of the steam.

RULE 1.—Divide the length of the stroke in inches by the distance (also in inches) that the piston moves before the steam is shut off, and divide the pressure on the boiler in *lbs.* by the quotient:—

2.—Add 1 to the hyperbolic logarithm of the first quotient, which is the number of times to which the steam is expanded, and multiply the sum by the second quotient, which is the number of *lbs.* to which the steam is expanded, and the product is the uniform force of the steam acting throughout the whole stroke.

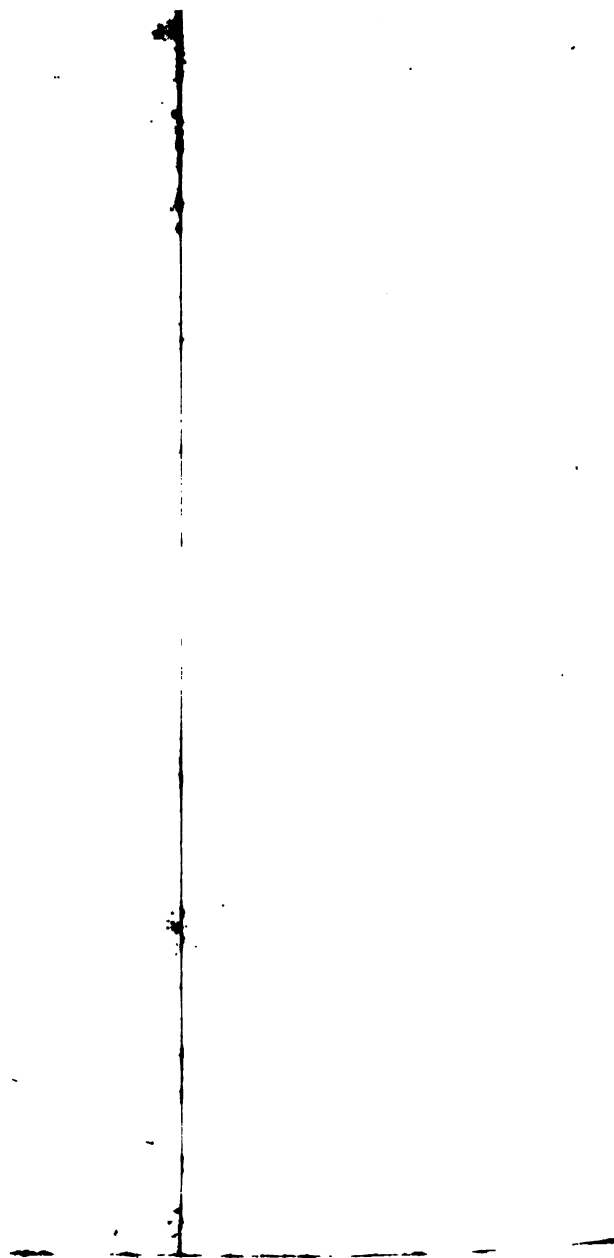
EXAMPLE.—Let the steam in the boiler of an engine equal 45 *lbs.* per inch, the length of stroke 4 feet, and the steam to be shut off after the piston has moved 16 inches; required an equivalent force of steam in the cylinder.

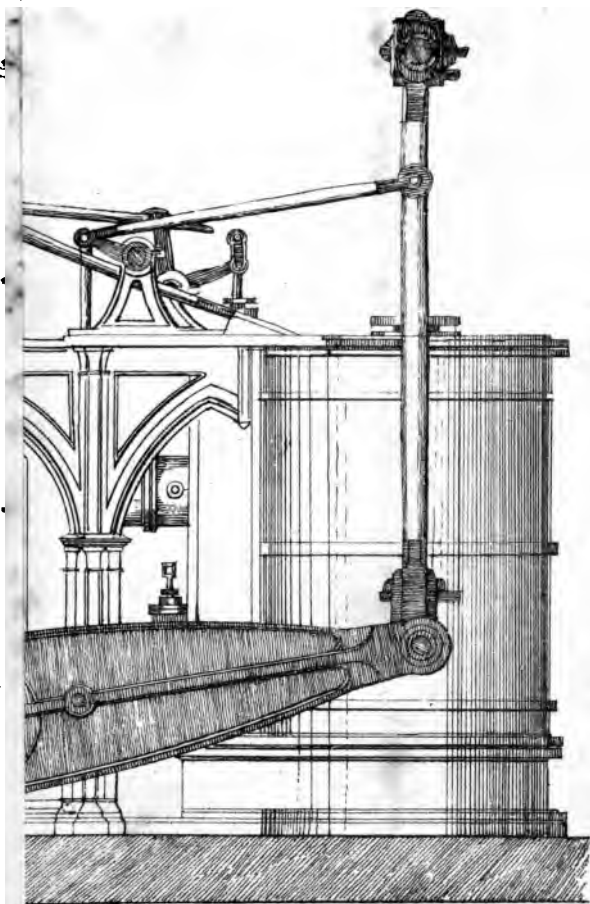
Here 4 feet = 48 inches, and $48 \div 16 = 3$, the first quotient.

Then, $45 \div 3 = 15$ *lbs.*, the second quotient.

And $1 + \text{hyp. log. } 3 = 1 + 1.0986123 = 2.0986123$.

Hence 2.0986123×15 *lbs.* = 31.4791845 *lbs.* uniform force of the steam. = *Ans.*





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HYPERBOLIC LOGARITHMS.

No.	Logarithms.	No.	Logarithms.	No.	Logarithms.	No.	Logarithms.
1½	·2231436	3½	1·1786550	5½	1·6582281	7½	1·9810015
1½	·4054651	3½	1·2527630	5½	1·7047481	7½	2·0149030
1½	·5596158	3½	1·3217558	5½	1·7491999	7½	2·0476928
2	·6931472	4	1·3862944	6	1·7917595	8	2·0794415
2½	·8109302	4½	1·4469190	6½	1·8325815	8½	2·1400662
2½	·9162907	4½	1·5040774	6½	1·8718022	9	2·1972246
2½	1·0116009	4½	1·5581446	6½	1·9095425	9½	2·2512918
3	1·0986123	5	1·6094379	7	1·9459101	10	2·3025851

THE STEAM WAY.

Multiply the square of the cylinder's diameter by ·08, and divide the product by 3, the quotient will be the area of port or steam way.

EXAMPLE.—What area of port or steam way is necessary for a cylinder 36 inches diameter.

$$\text{Here } \frac{36^2 \times \cdot 08}{3} = 36 \times 12 \times \cdot 08 = 432 \times \cdot 08 =$$

34·6 inches area of steam way. = *Ans.*

OF THE SLIDE VALVE.

When the valve is at the middle of its stroke, the faces ought to cover the apertures on the exhausting side about $\frac{1}{10}$ of an inch; the cover on the steam side being for the purpose of cutting off the steam at any part of the stroke, is, therefore, at the entire discretionary judgment of the engineer. However, we find from practice, that high-pressure engines with short strokes, as locomotives, &c., require no more than will cover the apertures properly; whereas condensing engines, with steam of $2\frac{1}{2}$ to 3 lbs. per square inch, will work well with $\frac{1}{8}$ of an inch cover on the steam side; and marine engines give great satisfaction with $1\frac{3}{8}$ inches cover, when the steam is $4\frac{1}{2}$ lbs. to 5 lbs. per square inch.

Again, the lead of the valve (as it is termed amongst engineers) is a certain distance that the extremity of the eccentric must be in advance of the crank, so that the valve may be open as required when the piston is at the top or bottom of the cylinder,—for this reason, that at the return of the stroke, the steam in the cylinder may be of, or nearly, an equal density with the steam in the boiler; consequently, the nearer that the length of the aperture is to the area of the cylinder, the less lead is required. Thus,

Suppose a cylinder of 48 inches diameter, with an aperture 16 inches long, $\frac{48^2}{16} = 144$; and another 24 inches diameter, with an aperture 8 inches long $\frac{24^2}{8} = 72$; then $\frac{144}{72} = 2$. Hence it is evident that, although both apertures bear the same ratio to the diameter of the cylinder, and both valves move the same distance, the 48 inch cylinder would be twice the time in filling with steam to that of the 24 inch, for a cylinder twice the diameter is four times the area; but scarcely two engineers agree upon this point. However, the following is an approximate rule to a number of celebrated working engines, namely: multiply the square of the cylinder's diameter in inches by .002, and divide the product by the length of the aperture, also in inches; the quotient will be the width that the valve must be open when the piston is exactly at the top or bottom of the cylinder.

EXAMPLE.—Let a cylinder be 30 inches diameter, with an aperture 12 inches long.

$$\text{Here } \frac{30^2 \times .002}{12} = \frac{900 \times .001}{6} = \frac{.9}{6} = .15 \text{ parts}$$

of an inch for the aperture to be open at the return of the stroke. = *Ans.*

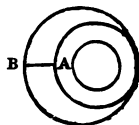
THE ECCENTRIC.

1.—The length of the levers on the weight or traverse shaft given, to find the required throw of the eccentric.

RULE.—Multiply the distance that the valve is to travel by the length of the lever on the weigh shaft for the eccentric rod; divide the product by the length of the lever for working the valve, and the quotient is the throw of the eccentric.

EXAMPLE.—Let a valve be required to travel 6 inches, the lever on the weigh shaft for working the valve 12 inches in length, and the lever for the eccentric rod 10 inches; required the throw of that eccentric.

$$\text{Here } \frac{6 \times 10}{12} = 5 \text{ inches throw.} = \text{Ans.}$$



The throw of the eccentric is the distance between A and B on that eccentric.

2.—The throw of the eccentric and the stroke of the valve, also one of the levers on the weigh shaft given, to find the other.

RULE.—Multiply the throw of the eccentric by the length of the lever to work the valve, and divide by the distance the valve is to travel; the quotient will be the length of the other lever. *Or,*

Multiply the travel of the valve by the length of the lever on the weigh shaft, for the eccentric rod, and divide by the throw of the eccentric; the quotient will be the length of the lever for working the valve.

EXAMPLE.—Suppose a valve be required to travel 6 inches, the throw of the eccentric 5, and the length of the lever on the weigh shaft for working equal 12 inches; required the length of the other.

$$\text{Here } \frac{5 \times 12}{6} = 10 \text{ inches ;}$$

$$\text{Or, } \frac{6 \times 10}{5} = 12 \text{ inches.}$$

THE COLD WATER PUMP.

Taking practice for the data whereby to determine the quantity of water sufficient for condensation in all ordinary cases of the steam engine, I find that, at the common temperature of the atmosphere, four imperial gallons of water to each horse power are quite capable of condensing steam at 220° Faht. to water at 80°; but if the temperature of the steam be raised, the quantity of water must be augmented, according to the result of the following

RULE.—Multiply the temperature of the steam in the boiler by .019, and the product will be the quantity required in imperial gallons per minute to each horse power. Hence, to find the diameter of the pump, when the stroke is given, or the stroke of the pump, when the diameter is given,

Multiply the quantity required in gallons for each horse power by the number of horses' power that the engine is equal to, and by 353; divide the product by the intended stroke of the pump in inches multiplied by the number of strokes per minute, and the square root of the quotient is the pump's diameter in inches. And, when the pump's diameter is given and the length of the stroke required, divide the product by the number of strokes per minute multiplied by the square of the pump's diameter, and the quotient is the length of the stroke.

EXAMPLE.—Let an engine of 25 horses' power be propelled by steam at 7 lbs. per square inch, what must be the pump's diameter when the stroke is 23 inches, and making 22 strokes per minute?

Here 7 *lbs.* per square inch (see Table, *p.* 127) = 234°;
And $234 \times .019 = 4.446$ gallons to each horse power.

$$\text{Then, } \sqrt{\frac{4.446 \times 25 \times 353}{23 \times 22}} = \sqrt{\frac{111.15 \times 353}{506}} \\ = \sqrt{77.54} = 8.8 \text{ inches diameter, nearly.} = \text{Ans.}$$

And when the pump's diameter is given, as above, we have

$$\frac{4.446 \times 25 \times 353}{77.54 \times 22} = 23 \text{ inches, length of stroke.}$$

NOTE.—The diameter of the injection cock ought to be at least equal to $\frac{1}{15}$ th of the cylinder's diameter.

To find the necessary quantity of water for a boiler.

RULE.—Add 15 to the pressure on each square inch of the boiler in *lbs.*, divide the sum by 90, and the quotient is the quantity in imperial gallons per minute for each horse power. Hence, the rule for the cold water pump is also applicable for the hot water pump.

EXAMPLE.—Suppose a 6-horse engine to be propelled by steam at 30 *lbs.* per square inch, stroke of pump 9 inches, and making 45 strokes per minute, required the pump's diameter.

Here $\frac{30 + 15}{90} = \frac{45}{90} = \frac{1}{2}$ a gallon per minute to each horse power.

$$\text{Then, } \sqrt{\frac{\frac{1}{2} \times 6 \times 353}{45 \times 9}} = \frac{1}{3} \sqrt{\frac{353}{15}} = \frac{1}{3 \times 15} \sqrt{(353 \times 15)} \\ = \frac{\sqrt{5295}}{45} = \frac{72.8}{45} = 1.6 \text{ inches diameter.} \\ = \text{Ans.}$$

And when the pump's diameter is given, as above, we have

$$\frac{\frac{1}{2} \times 6 \times 353}{45 \times 1.6^2} = \frac{353}{15 \times 2.56} = \frac{353}{38.4} = 9 \text{ inches length of stroke.}$$

THE AIR PUMP.

The *Air Pump* for a land engine generally requires to be larger in proportion to the cylinder than the air pump for a marine engine, on account of having frequently to condense with water at a higher temperature; hence, when the stroke of the bucket is half the stroke of the piston, multiply the cylinder's diameter in inches by $\cdot 67$, and the product is the diameter of air pump. —Again, multiply the diameter of the cylinder of a marine engine, in inches, by $\cdot 575$, or $\frac{23}{40}$, and the product is the diameter of air pump.

EXAMPLE.—What diameter of air pump is requisite for an engine whose cylinder is 28 inches diameter?

Here $28 \times \cdot 67 = 18\cdot76$ inches diameter. = *Ans.*

When the stroke of the bucket is either more or less than half the stroke of the piston, the pump's diameter will then be obtained by the following

RULE.—Square the given diameter, multiply by the length, and divide by the length proposed, extract the square root, and the product will be the diameter.

Or, multiply the given diameter by the square root of the quotient of the first length by the altered length.

EXAMPLE.—Suppose an engine with a 4-feet stroke required an air pump 26 inches diameter with a 2-feet stroke, but necessity requires it to be 6 inches nearer the end of the beam, what must be the diameter of air pump, the beam being 11 feet long?

Here radius of beam = 66 inches.

Then 4 feet : 2 feet :: 66 inches : 33 inches, first distance of the air pump.

And $33 + 6 = 39$ inches, the altered distance of the air pump.

And $66 : 48 :: 39 : 28\cdot36$ inches, length of stroke.

And $\sqrt{\frac{26^2 \times 24}{28\cdot36}} = 26 \sqrt{\frac{24}{28\cdot36}} = 260 \sqrt{\frac{24}{2836}}$

$$= 260 \sqrt{\frac{6}{709}} = \frac{260 \sqrt{(6 \times 709)}}{709} = \frac{260 \sqrt{4254}}{709} =$$

$$\frac{260 \times 65.22}{709} = \frac{16957.2}{709} = 24 \text{ inches, diameter of pump,}$$

nearly. = *Ans.*

The *Condenser* ought to be a little more in capacity than the air pump; but in the case of marine engines, where the bottom of the condenser and bottom of the cylinder are nearly on a level, care must be taken to make the passage between the valves and condenser large enough to contain the condensing water required for one stroke of the piston, besides leaving a proper communication, otherwise the connexion between the cylinder and condenser will be cut off by water of nearly 100° of heat, on account of the cylinder being twice filled with steam for each effective stroke of the air pump.

One-fourth of the area of air pump will give the area of foot and discharging valves; thus, 24 inches diameter gives 452.39 inches area, one-fourth of which = 113.0975 inches, area of valves.

The piston rod is about $\frac{1}{10}$ of the cylinder's diameter; the air pump rod in the same ratio, unless it be made of copper, and then it may be about $\frac{1}{8}$ of the pump's diameter.

THE BEAM.

When a beam is applied to an engine its length ought not to be less than three times the length of the stroke, and its breadth half the stroke, or, in high pressure engines, $\frac{2}{3}$ of the stroke; also its best form is a parabola.

To find the thickness of a beam when its length and breadth, together with the diameter of the cylinder, are given.

RULE.—Multiply the whole pressure of steam on the piston, in *lbs.*, by half the length of the beam in feet, and divide the product by 70 times the square of the breadth, in inches, and the quotient will be the thickness in inches, nearly.

EXAMPLE.—What thickness of beam is requisite for an engine whose cylinder is 25 inches diameter, the length of the beam being 15 feet, length of stroke 5 feet, and the effective pressure on each square inch of the piston = 15 lbs.

Here area of piston = $25^2 \times .7854 = 490.875$ square inches.

And length of stroke = 60 inches, which divided by 2, gives 30 inches, the breadth of the beam.

Whence
$$\frac{490.875 \times 15 \times 7.5}{30^2 \times 70} = \frac{490.875 \times 15 \times 15}{900 \times 140}$$

$$= \frac{490.875}{20 \times 28} = \frac{14.025}{16} = .8765625, \text{ or } \frac{7}{8} \text{ of an inch in}$$

 thickness, nearly. = *Ans.*

To find the versed sine of the arc described by the beam of an engine.

RULE.—Divide the square of half the length of the stroke, in inches, by the length of the beam, also in inches, and the quotient is the versed sine.

EXAMPLE.—Required the versed sine of the arc described by an engine beam 12 feet in length, the chord of the arc, or length of the stroke, being 4 feet.

Here
$$\frac{24^2}{144} = \frac{24^2}{12^2} = 2^2 = 4 \text{ inches, the versed sine.}$$

 = *Ans.*

NOTE.—When the beam is not equal lengths at each end from the centre on which it vibrates, the length is then to be taken equal to twice the radius of that end of which the versed sine is required.

THE PARALLEL MOTION.

The beam being given, to find the length of the radius rods.

RULE.—Divide the square of the distance between A and B, on the beam, by the distance between B and C, and the quotient is the length of the radius rod $d x$.



IONS.

to
es

FIG 3rd A motion that is chiefly used
in marine Engines.

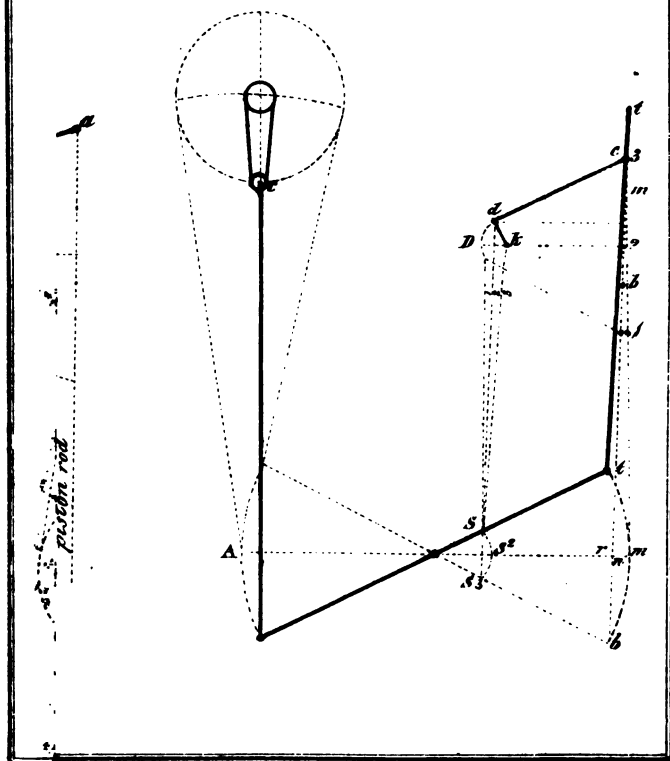


FIG. 1, EXAMPLE.—Suppose a beam 12 feet long, and the stud for the back links 39 inches from the centre, required the length of radius rods.

Here radius of beam = 72 inches, and $72 - 39 = 33$ in.;
then $\frac{39^2}{33} = \frac{39 \times 13}{11} = \frac{507}{11} = 46.09$ inches. = *Ans.*

NOTE.—The length of the front and back links equal half the length of the stroke.

FIG. 2, EXAMPLE.—Suppose $b d = 32\frac{1}{4}$, and $d a = 35\frac{1}{4}$, to find $d f$.

Here $\frac{32.25^2}{35.25} = \frac{129^2}{564} = 29.5$ inches, nearly. = *Ans.*

FIG. 3.—As the calculation of this motion is rather tedious, on account of the various angles formed by the side rods, it is considered better to lay it down in the following geometrical form :—

Upon the line $A m$, with the radius of the beam, describe the arc $b m t$; from m , with half the length of stroke, cut the arc in b and t , draw the line $b t$ and $r m$ equal the versed sine described by the beam; bisect $r m$ in n , and erect a perpendicular line for the centre of the cylinder. Again, from $b m t$, with the length of the side rods, cut the perpendicular line; at the bottom, middle, and top stroke of the cross-head, draw the lines $b b$, $m m$, $t t$; from the end of the cross-head, or top of the side rods, with any convenient distance, set off the pin or stud in the side rod for the end of the parallel bar 1, 2, 3, from which, with the distance $s t$, describe arcs at $d D d$; draw the lines $d 1$, $D 2$, &c. Then the length of the crank may be found either by the seventh problem in Geometry, page 31, or the eleventh problem in Mensuration, page 52.

THE CONNECTING ROD.

The length of connecting rod is in general three times the length of stroke, which determines the per-

pendicular distance between the centre of the beam and centre of fly-wheel shaft. Or, if the engine is erected, the length of connecting rod is the perpendicular distance between the centre of the fly-wheel shaft, and centre of the beam.

THE FLY-WHEEL.

To find the weight of the rim or ring of a fly-wheel proper for a steam engine.

RULE.—Multiply the constant number, 1368, by the number of horses' power that the engine is equal to; divide the product by the diameter of the wheel, in feet, multiplied by the number of revolutions per minute; and the quotient is the weight of the ring in *cwts.*, nearly.

EXAMPLE.—Required the weight of the rim of a fly-wheel proper for an engine of 20 horses' power, the wheel to be 16 feet diameter, and make 21 revolutions per minute.

$$\frac{1368 \times 20}{16 \times 21} = \frac{114 \times 5}{7} = \frac{570}{7} = 81.4 \text{ cwt., nearly.}$$

= Ans.

NOTE.—The fly-wheel of an engine for a corn or flour mill ought to be of such a diameter that the velocity of the periphery of the wheel may exceed the velocity of the periphery of the stones, to prevent, as much as possible, any tendency to back lash, as it is termed.

The necessary weight and diameter of the wheel being found, suppose a breadth of rim, and the thickness to make the weight in cast iron will be found by the following

RULE.—Multiply the required weight in *lbs.* by 4.841, and divide the result by the product of the sum and difference of the diameters, in inches, and the quotient is the thickness of the ring in inches.

EXAMPLE.—What thickness must a ring be to equal 81·4 *cwts.*, when the outer diameter is 16 feet, and the inner diameter 14 feet 8 inches ?

Here 81·4 *cwts.* = 9116·8 *lbs.*

$$\begin{aligned}\text{Then, } \frac{9116 \cdot 8 \times 4 \cdot 841}{(192 + 176) \times (192 - 176)} &= \frac{9116 \cdot 8 \times 4 \cdot 841}{368 \times 16} \\ &= \frac{284 \cdot 9 \times 4 \cdot 841}{184} = \frac{1379 \cdot 2}{184} = 7 \cdot 496 \text{ inches, nearly.} \\ &= \text{Ans.}\end{aligned}$$

And if the ring is to be of a cylindrical form, find the diameter of a circle (by Problem XIII. *p.* 55, in Mensuration) having the same area as the cross-section of the ring found.

Thus, suppose the ring, in the last example, be required to be cylindrical,—Required its cross-sectional diameter to equal 81·4 *cwts.*, the diameter of the wheel being 16 feet.

Here $7 \cdot 496 \times 8 = 59 \cdot 968$ inches cross-sectional area of the ring found.

$$\begin{aligned}\text{And } \sqrt{\frac{59 \cdot 968 \times 452}{355}} &= \sqrt{\frac{27105 \cdot 536}{355}} = \sqrt{76 \cdot 35} \\ &= 8 \cdot 74 \text{ inches diameter, nearly.} = \text{Ans.}\end{aligned}$$

Or, as an approximation, multiply the required weight in *lbs.*, by 1·62; divide the product by the diameter of the wheel, in inches, and the square root of the quotient will be the diameter of the cross-section of the ring, in inches, nearly.

$$\begin{aligned}\text{Thus } \sqrt{\left(\frac{9116 \cdot 8 \times 1 \cdot 62}{16 \times 12}\right)} &= \sqrt{(284 \cdot 9 \times \cdot 27)} = \\ \sqrt{76 \cdot 923} &= 8 \cdot 78 \text{ inches.} = \text{Ans. as before, nearly.}\end{aligned}$$

Sometimes (for various reasons) it is necessary to have the fly-wheel upon a second mover; for instance,

there is a 6-horse engine making 50 revolutions per minute, having a fly-wheel of 7 ft. diameter, and 9 cwt., but, by the rule, it ought to be 23·46 cwt. Now, a larger wheel cannot be got in, but the same may be put upon a second mover,—required the velocity that will increase its momentum equal to 23·46 cwt. on the first mover.

Here 7 feet (diameter) \times 3·1416 = 21·9912 feet circumference, and 21·9912 \times 50 revolutions = 1099·56 feet velocity.

<i>Cwt.</i>	<i>Cwt.</i>	<i>Velocity.</i>	<i>Velocity.</i>
Then, as 9 :	23·46 ::	1099·56 :	2866·1864,
and 2866·1864 \div	21·9912 =	130 revolutions per minute nearly. = <i>Ans.</i>	

To find the centrifugal force of a fly-wheel.

RULE.—Multiply ·6132* by the diameter of the wheel in feet and its weight, and divide the product by the square of the time of one revolution; the quotient will be the centrifugal force.

EXAMPLE.—Required the centrifugal force of a fly-wheel 15 feet diameter, and making 40 revolutions per minute, the weight of the ring being 3 tons.

Here $60 \div 40 = 1·5$, time of one revolution.

And $\frac{·6132 \times 15 \times 3}{1·5^2} = \frac{·6132}{·05} = \frac{61·32}{5} = 12·264$

tons, the centrifugal force. = *Ans.*

The centre of percussion in a fly-wheel, or wheels in general, is $\frac{3}{4}$ distant from the centre of suspension nearly.

* This number is $= \frac{3·1416^2}{16·095} = \frac{24}{\text{length of seconds pendulum}} = \frac{24}{39·139}$ (see page 101).

NOTE.—The centrifugal force is that power or tendency which all revolving bodies have to burst, or fly asunder in a direct line.

And the centre of percussion in a revolving body is that point where the whole force or motion is collected, or that point which would strike any obstacle with the greatest effect.

THE GOVERNOR OR REGULATOR.

The length of pendulums given, to find the number of revolutions per minute.

RULE.—Divide 375 (*see page 101*) by the square root of the pendulum's length, and half the quotient will be the velocity required.

EXAMPLE.—What number of revolutions ought a governor to make per minute whose pendulums are 24 inches long?

$$\begin{aligned} \text{Here } \frac{375}{\sqrt{24}} &= \frac{375 \times \sqrt{24}}{24} = \frac{3000 \times \sqrt{24}}{192} = \\ \frac{1000}{64} \times 2 \times \sqrt{6} &= \frac{1000}{32} \times \sqrt{6} = \frac{1000}{32} \times 2.449 = \\ \frac{2449}{32} &= 76, \text{ then, } 76 \div 2 = 38 \text{ revolutions per minute.} \\ &= \text{Ans.} \end{aligned}$$

The revolutions per minute of a governor given, to find the length of pendulums.

RULE.—Divide 375 by twice the number of revolutions per minute, and the square of the quotient will be the length required.

EXAMPLE.—When the velocity of a governor is 38 revolutions per minute, what ought to be the length of pendulums?

$$\begin{aligned} \text{Here } 38 \times 2 &= 76, \text{ and } \left(\frac{375}{76}\right)^2 = 4.93^2 = 24.3049 \\ \text{inches.} &= \text{Ans.} \end{aligned}$$

OF HIGH-PRESSURE STEAM ENGINES.

High-pressure engines, in general (if in good condition), will work when the force of the steam is about 4 *lbs.* per circular inch,—that is, 4 *lbs.* on each circular inch of the piston will overcome the resistance and friction of the engine itself, divested of machinery, &c. Hence the rule.

1. From the pressure in *lbs.* on each circular inch of the boiler deduct 4 *lbs.*; multiply the remainder by the square of the cylinder's diameter in inches, and by the velocity of the piston in feet per minute; divide the product by 33000, and the quotient will be the force of the engine expressed in horses' power.

EXAMPLE.—Suppose a cylinder 8 inches diameter, stroke 2 feet, making 45 revolutions per minute, or 180 feet, and steam 23·5 *lbs.* per circular inch, required the power.

$$\text{Here } \frac{(23\cdot5 - 4) \times 8^2 \times 180}{33000} = \frac{19\cdot5 \times 64 \times 6}{1100} = \frac{7488}{1100} = 6\cdot8 \text{ horses' power, nearly.} = \text{Ans.}$$

2. Multiply 33000 by the number of horses' power required, and divide the product by the velocity of the piston in feet per minute, multiply by the force of the steam in *lbs.* on each circular inch of the boiler, minus 4 *lbs.*, and the square root of the quotient is the cylinder's diameter in inches.

EXAMPLE.—Required the diameter of the cylinder for an engine of 6·8 horses' power, when the stroke is 2 feet, and making 45 strokes per minute, the force of the steam being 23·5 *lbs.* per circular inch.

$$\begin{aligned} \text{Here } \sqrt{\frac{33000 \times 6.8}{180 \times (23.5 - 4)}} &= \sqrt{\frac{1100 \times 6.8}{6 \times 19.5}} = \\ \sqrt{\frac{7480}{117}} &= \sqrt{64} = 8 \text{ inches diameter.} = \text{Ans.} \end{aligned}$$

NOTE.—There is always a resistance of steam on the piston of a high-pressure or non-condensing engine equal to the pressure of the atmosphere, but this cannot be taken into account, unless we also take into account the pressure of the atmosphere upon the boiler.

MISCELLANIES.

Approximate rules for finding the weight of round, square, and rectangular beams, bars, &c., of cast and wrought iron.

RULE 1. For cylinders.—Multiply the square of the diameter in inches by the length in feet, and by 2·65 for wrought iron, or 2·48 for cast iron, and the product will be the weight in pounds avoirdupois, *nearly*.

2, or generally.—Multiply the area of the cross section in inches by the length in feet, and by 3·38 for wrought iron, or 3·16 for cast iron, and the product will be the weight in pounds avoirdupois, *nearly*.

EXAMPLE 1.—Required the weight of a round bar of wrought iron 14 feet long and $2\frac{1}{2}$ inches diameter.

Here $2\cdot5^2 \times 14 \times 2\cdot65 \text{ lbs.} = 232 \text{ lbs.} = \text{Ans.}$

Or, if the area of the cross section, $2\cdot5^2 \times \cdot7854 = 4\cdot91$, had been given, then by the second or general rule $4\cdot91 \times 14 \times 3\cdot38 \text{ lbs.} = 232 \text{ lbs.} = \text{Ans., as before.}$

EXAMPLE 2.—The length of a piece of cast iron is $9\frac{1}{2}$ feet, its breadth 7 inches, and thickness $2\frac{1}{4}$, required its weight.

Here $9\frac{1}{2} \times 7 \times 2\frac{1}{4} \times 3\cdot16 = 472\cdot815 \text{ lbs.} = \text{Ans.}$

The dimensions of a cast iron ring being given, to find its weight, nearly.

RULE.—Multiply the breadth of the ring added to the inner diameter by ·0074, and that again by the breadth and by the thickness, and the product will be its weight in *cwt.*s., *nearly*.

EXAMPLE.—Required the weight of a ring whose dimensions are 8 feet 4 inches interior diameter, 5 inches broad and 4 inches thick.

Inches.

Here 8 ft. 4 in. = 100; then $(100 + 5) \times 5 \times 4$
 $\times \cdot 0074 = 105 \times 20 \times \cdot 0074 = 15\cdot54$ *cwts.*, nearly.
 = *Ans.*

To find the weight of any cast iron ball whose diameter is given.

RULE.—Multiply the cube of the diameter in inches by $\cdot 1377$, and the product will be the weight in avoirdupois pounds, nearly.

EXAMPLE.—Required the weight of a ball 7 inches diameter.

Here $7^3 \times \cdot 1377 = 343 \times \cdot 1377 = 47\cdot2311$ *lbs.*
 = *Ans.*

To find the diameter of a cast iron ball when the weight is given.

RULE.—Multiply the cube root of the weight in pounds by $1\cdot9365$, and the product will be the diameter in inches, nearly.

EXAMPLE.—Required the diameter of a ball that will weigh 64 pounds.

Here $\sqrt[3]{64} \times 1\cdot9365 = 4 \times 1\cdot9365 = 7\cdot746$ inches diameter. = *Ans.*

TABLE

Containing the weight of a square foot of copper and lead, in lbs. avoirdupois, from $\frac{1}{32}$ to $\frac{1}{2}$ an inch in thickness, advancing by $\frac{1}{32}$.

Thickness.	Copper.	Lead.
$\frac{1}{32}$	1.45	1.85
$\frac{1}{16}$	2.90	3.70
$\frac{3}{32}$	4.35	5.54
$\frac{1}{8}$	5.80	7.39
$\frac{5}{32} + \frac{1}{32}$	7.26	9.24
$\frac{3}{16} + \frac{1}{16}$	8.71	11.08
$\frac{7}{32} + \frac{1}{32}$	10.16	12.93
$\frac{1}{2}$	11.61	14.77
$\frac{1}{2} + \frac{1}{32}$	13.07	16.62
$\frac{1}{2} + \frac{1}{16}$	14.52	18.47
$\frac{1}{2} + \frac{1}{8}$	15.97	20.31
$\frac{1}{2}$	17.42	22.16
$\frac{1}{2} + \frac{1}{32}$	18.87	24.00
$\frac{1}{2} + \frac{1}{16}$	20.32	25.85
$\frac{1}{2} + \frac{1}{8}$	21.77	27.70
$\frac{1}{2}$	23.22	29.55

TABLE

Of the weight of a square foot of sheet iron in lbs. avoirdupois, the thickness being the number on the wire gauge.—No. 1 is $\frac{8}{16}$ of an inch; No. 4, $\frac{1}{4}$; No. 11, $\frac{1}{8}$, &c.

No. on wire gauge. ..	1	2	3	4	5	6	7	8	9	10	11
Pounds avoird. ..	12.5	12	11	10	9	8	7.5	7	6	5.68	5
No. on wire gauge. ..	12	13	14	15	16	17	18	19	20	21	22
Pounds avoird. ..	4.62	4.31	4	3.95	3	2.5	2.18	1.93	1.62	1.5	1.37

TABLE

Of the weight of a square foot of boiler plate iron, from $\frac{1}{8}$ to 1 inch thick, in lbs. avoirdupois.

$\frac{1}{8}$	$\frac{3}{16}$	$\frac{1}{4}$	$\frac{5}{16}$	$\frac{3}{8}$	$\frac{7}{16}$	$\frac{1}{2}$	$\frac{9}{16}$	$\frac{5}{8}$	$\frac{11}{16}$	$\frac{3}{4}$	$\frac{13}{16}$	$\frac{7}{8}$	$\frac{15}{16}$	1 in.
5	7.5	10	12.5	15	17.5	20	22.5	25	27.5	30	32.5	35	37.5	40

TABLE

Of the weight of solid cylinders of cast iron, 12 inches long, in lbs. avoirdupois.

Dmr. Inch.	Weight, in lbs.	Dmr. Inch.	Weight, in lbs.	Dmr. Inch.	Weight, in lbs.	Dmr. Inch.	Weight, in lbs.	Dmr. Inch.	Weight, in lbs.
$\frac{3}{4}$	1·394	$2\frac{1}{8}$	11·193	$3\frac{1}{2}$	30·364	$5\frac{1}{2}$	81·953	9	200·776
$\frac{7}{8}$	1·898	$2\frac{1}{4}$	12·548	$3\frac{3}{4}$	32·572	6	89·234	$9\frac{1}{4}$	223·705
1 in.	2·479	$2\frac{3}{8}$	13·981	$3\frac{7}{8}$	34·857	$6\frac{1}{2}$	96·825	10	247·872
$1\frac{1}{8}$	3·137	$2\frac{1}{2}$	15·492	4	37·219	$6\frac{3}{4}$	104·726	$10\frac{1}{2}$	273·279
$1\frac{1}{4}$	3·873	$2\frac{7}{8}$	17·080	$4\frac{1}{4}$	39·660	$6\frac{7}{8}$	112·936	11	299·925
$1\frac{3}{8}$	4·686	3	18·745	$4\frac{3}{4}$	44·772	7	121·457	$11\frac{1}{4}$	327·811
$1\frac{1}{2}$	5·577	$3\frac{1}{8}$	20·488	$4\frac{7}{8}$	50·194	$7\frac{1}{4}$	130·288	12	356·935
$1\frac{5}{8}$	6·545	$3\frac{1}{4}$	22·308	$4\frac{7}{8}$	55·926	$7\frac{1}{2}$	139·428	13	418·903
$1\frac{3}{4}$	7·591	$3\frac{3}{8}$	24·206	5	61·968	$7\frac{3}{4}$	148·878	14	485·829
$1\frac{7}{8}$	8·714	$3\frac{1}{2}$	26·181	$5\frac{1}{4}$	68·320	8	158·638	15	557·712
2	9·915	$3\frac{3}{4}$	28·234	$5\frac{1}{2}$	74·981	$8\frac{1}{4}$	179·087	16	634·552

Cubic inches of cast iron multiplied by ·263 = lbs. avoirdupois.

Circular inches of cast iron multiplied by ·2066 = lbs. avoirdupois.

TABLE

For finding the weight of malleable iron, copper, and lead pipes, 12 inches long, of various thicknesses, and any diameter required.

Thickness.	Mall. Iron.	Copper.	Lead.
$\frac{1}{32}$ of an in.	·104	·1210	·1539
$\frac{1}{16}$	·207	·2419	·3078
$\frac{3}{32}$	·3108	·3629	·4616
$\frac{1}{8}$	·414	·4839	·6155
$\frac{1}{8}$ + $\frac{1}{32}$	·518	·6049	·7694
$\frac{1}{8}$ + $\frac{1}{16}$	·621	·7258	·9232
$\frac{1}{8}$ + $\frac{3}{32}$	·725	·8468	1·0771
$\frac{1}{4}$	·828	·9678	1·2310

RULE.—Multiply the circumference of the pipe in inches by the numbers opposite the thickness required, and by the length in feet; the product will be the weight in avoirdupois lbs. nearly.

EXAMPLE.—Required the weight of a copper pipe 12 feet long, 15 inches in circumference, $\frac{1}{8}$ and $\frac{1}{16}$ of an inch in thickness.

Here $7258 \text{ lbs.} \times 15 \times 12 = 130644 \text{ lbs. nearly.} = \text{Ans.}$

TABLE

Containing the weight of wrought iron bars 12 inches long in lbs. avoirdupois.

Inch.	Round	Square.	Inch.	Round.	Square.
$\frac{1}{8}$	·166	·211	$2\frac{1}{8}$	16·59	21·13
$\frac{1}{4}$	·373	·475	$2\frac{1}{4}$	18·30	23·29
$\frac{3}{8}$	·664	·845	$2\frac{3}{8}$	20·08	25·56
$\frac{1}{2}$	1·04	1·32	$2\frac{1}{2}$	21·94	27·94
$\frac{5}{8}$	1·60	1·90	3	23·96	30·42
$\frac{3}{4}$	2·03	2·59	$3\frac{1}{4}$	28·04	35·70
1	2·65	3·38	$3\frac{1}{2}$	32·52	41·41
$1\frac{1}{8}$	3·36	4·28	$3\frac{3}{4}$	37·34	47·53
$1\frac{1}{4}$	4·15	5·28	4	42·48	54·08
$1\frac{3}{8}$	5·02	6·39	$4\frac{1}{2}$	47·96	61·05
$1\frac{1}{2}$	5·99	7·60	$4\frac{3}{4}$	53·77	68·45
$1\frac{3}{4}$	7·01	8·92	$4\frac{1}{2}$	59·91	76·27
$1\frac{7}{8}$	8·13	10·35	5	66·38	84·51
$1\frac{1}{2}$	9·33	11·88	$5\frac{1}{2}$	73·18	93·17
2	10·62	13·52	$5\frac{1}{2}$	80·32	102·25
$2\frac{1}{8}$	11·99	15·26	$5\frac{3}{4}$	87·78	111·76
$2\frac{1}{4}$	13·44	17·11	6	95·58	121·69
$2\frac{3}{8}$	14·98	19·07	7	130·10	165·63

TABLE

Of the proportional dimensions of 6 sided nuts for bolts, from $\frac{1}{4}$ to $2\frac{1}{2}$ inches diameter.

Diameter of bolts	$\frac{1}{4}$	$\frac{3}{8}$	$\frac{1}{2}$	$\frac{5}{8}$	$\frac{3}{4}$	$\frac{7}{8}$	1	$1\frac{1}{8}$	$1\frac{1}{4}$
Breadth of nuts..	$\frac{11}{16}$	$\frac{13}{16}$	1	$1\frac{1}{16}$	$1\frac{1}{8}$	$1\frac{1}{16}$	$1\frac{1}{4}$	$1\frac{11}{16}$	$2\frac{1}{8}$
Breadth over the angles. . . }	$\frac{13}{16}$	$\frac{15}{16}$	$1\frac{1}{8}$	$1\frac{3}{8}$	$1\frac{1}{16}$	$1\frac{11}{16}$	2	$2\frac{1}{4}$	$2\frac{7}{8}$
Thickness	$\frac{5}{16}$	$\frac{7}{16}$	$\frac{9}{16}$	$\frac{3}{4}$	$\frac{7}{8}$	1	$1\frac{1}{8}$	$1\frac{1}{4}$	$1\frac{7}{8}$
Diameter of bolts	$1\frac{1}{8}$	$1\frac{1}{2}$	$1\frac{3}{8}$	$1\frac{1}{4}$	$1\frac{7}{8}$	2	$2\frac{1}{4}$	$2\frac{1}{2}$	
Breadth of nuts..	$2\frac{1}{16}$	$2\frac{1}{2}$	$2\frac{11}{16}$	$2\frac{1}{8}$	$3\frac{1}{16}$	$3\frac{1}{4}$	$3\frac{3}{8}$	4	
Breadth over the angles . . }	$2\frac{11}{16}$	$2\frac{7}{8}$	$3\frac{1}{8}$	$3\frac{1}{16}$	$3\frac{1}{2}$	$3\frac{3}{4}$	$4\frac{1}{16}$	$4\frac{5}{8}$	
Thickness	$1\frac{1}{16}$	$1\frac{11}{16}$	$1\frac{13}{16}$	2	$2\frac{1}{8}$	$2\frac{1}{4}$	$2\frac{1}{2}$	$2\frac{3}{4}$	

TABLE

Of the weight of flat bar iron, 12 inches long, in lbs. avoirdupois.

Thickness.	$\frac{1}{8}$	$\frac{3}{16}$	$\frac{1}{4}$	$\frac{3}{8}$	$\frac{1}{2}$	$\frac{5}{8}$	$\frac{3}{4}$	$\frac{7}{8}$	1 inch.
Breadth, in inches.	$\frac{1}{4}$.21	.31	.42	.63				
	$\frac{3}{8}$.32	.48	.63	.95	1.27	1.58		
	1	.42	.63	.84	1.26	1.69	2.11	2.53	
	$1\frac{1}{4}$.52	.79	1.05	1.58	2.11	2.64	3.16	3.70
	$1\frac{1}{2}$.58	.87	1.16	1.74	2.32	2.90	3.48	4.06
	$1\frac{3}{4}$.63	.95	1.27	1.90	2.53	3.17	3.80	4.44
	2	.74	1.11	1.48	2.21	2.95	3.70	4.43	5.19
	$2\frac{1}{4}$.84	1.27	1.69	2.53	3.38	4.22	5.07	5.92
	$2\frac{1}{2}$.95	1.42	1.90	2.85	3.80	4.75	5.70	6.65
	$2\frac{3}{4}$	1.06	1.58	2.11	3.17	4.22	5.28	6.33	7.40
	3	1.16	1.74	2.32	3.49	4.64	5.81	6.97	8.13
	$3\frac{1}{4}$	1.27	1.90	2.53	3.80	5.07	6.34	7.60	8.87
	$3\frac{1}{2}$	1.37	2.06	2.74	4.12	5.49	6.86	8.24	9.61
	$3\frac{3}{4}$	1.48	2.22	2.95	4.43	5.91	7.39	8.87	10.37
	4	1.58	2.38	3.17	4.75	6.34	7.92	9.51	11.15
	$4\frac{1}{4}$	1.69	2.53	3.38	5.07	6.76	8.45	10.14	11.83
	$4\frac{1}{2}$	1.90	2.85	3.80	5.70	7.60	9.50	11.41	13.31
	5	2.11	3.17	4.22	6.34	8.45	10.56	12.67	14.79
	$5\frac{1}{4}$	2.53	3.80	5.07	7.60	10.14	12.67	15.21	17.75
	6								20.28

Weight of a copper rod 12 inches long and 1 inch diameter = 3.039 lbs.

Weight of a brass rod 12 inches long and 1 inch diameter = 2.86 lbs.

TABLE

Of the specific gravity of water at different temperatures, that at 62° being taken as unity.

70° F		52° F	
68	.99913	50	1.00076
66	.99936	48	1.00087
64	.99958	46	1.00095
62	.99980	44	1.00102
60	1.	42	1.00107
58	1.00035	40	1.00111
56	1.00050	38	1.00113
54	1.00064		

The difference of temperatures between 62° and 38°, where water attains its greatest density, will vary the bulk of a gallon rather less than the third of a cubic inch.

TABLE

Of the weight of cast iron pipes 12 inches long, in lbs. avoirdupois.

Diam. of bore in inches.	THICKNESS IN INCHES.							
	$\frac{1}{8}$	$\frac{1}{4}$	$\frac{3}{8}$	$\frac{1}{2}$	$\frac{5}{8}$	1 in.	1 $\frac{1}{4}$	1 $\frac{1}{2}$
	lbs.	lbs.	lbs.	lbs.	lbs.	lbs.	lbs.	lbs.
1 $\frac{1}{2}$	6.9	9.9
2	8.8	12.4	16.3	20.4
2 $\frac{1}{2}$	10.7	14.9	19.4	24.1
3	12.5	17.4	22.4	27.8	33.6	39.7	46.0	..
3 $\frac{1}{2}$	14.4	19.9	25.5	31.5	37.9	44.7	51.6	..
4	16.2	22.3	28.6	35.2	42.3	49.6	57.1	65.1
4 $\frac{1}{2}$	18.1	24.8	31.7	38.9	46.7	54.6	62.7	71.2
5	20.0	27.3	34.9	42.7	51.0	59.5	68.3	77.4
5 $\frac{1}{2}$	21.9	29.8	38.0	46.4	55.3	64.5	73.9	83.6
6	23.7	32.2	41.1	50.1	59.6	69.4	79.4	89.8
6 $\frac{1}{2}$	25.6	34.7	44.2	53.8	63.9	74.4	85.0	96.0
7	27.4	37.2	47.3	57.5	68.3	79.3	90.6	102.2
7 $\frac{1}{2}$	29.3	39.7	40.4	61.2	72.6	84.3	96.2	108.4
8	31.1	42.2	53.5	65.0	77.0	89.3	101.8	114.6
8 $\frac{1}{2}$	33.0	44.7	56.6	68.7	81.3	94.3	107.4	120.8
9	34.9	47.1	59.6	72.4	85.7	99.2	112.9	127.0
9 $\frac{1}{2}$	36.8	49.6	62.7	75.1	80.0	104.2	118.5	133.2
10	38.6	52.1	65.8	79.9	94.3	109.1	124.1	139.4
10 $\frac{1}{2}$..	54.6	68.9	82.6	98.6	114.1	129.7	145.6
11	..	57.0	72.0	87.3	103.0	119.0	135.2	151.8
11 $\frac{1}{2}$..	59.6	75.1	90.0	107.3	124.0	140.8	158.0
12	..	62.0	78.2	94.7	111.7	128.9	146.4	164.2
13	84.4	102.2	120.4	138.8	157.5	176.6
14	90.6	109.6	129.0	148.8	168.7	189.0
15	96.8	117.0	137.7	158.7	179.8	201.4
16	124.5	146.4	168.6	181.0	213.8
17	131.9	155.1	178.5	202.1	224.2
18	139.4	163.7	188.4	213.3	238.5
19	172.4	198.3	224.4	250.9
20	181.1	208.2	235.6	263.3
21	218.2	246.7	275.7
22	228.1	257.9	288.1
23	238.0	269.0	300.5
24	247.9	280.2	312.9

NOTE.—The first column is the width of the pipes, expressed in inches and parts of an inch; and the remaining columns are the weights of the pipes, under the different thicknesses in which they are placed.

N.B.—Two flanges are generally reckoned equal to one foot of pipe.

TABLE

Of the weight of cast iron balls in lbs. avoirdupois, from 1 to 12 inches diameter, and advancing by an eighth.

Inches.	lbs. and parts.	Inches.	lbs. and parts.	Inches.	lbs. and parts.
1	·14	4½	14·76	8½	84·57
1½	·20	4¾	15·96	8¾	88·35
1½	·27	5	17·21	8¾	92·25
1½	·36	5½	18·54	8¾	96·26
1½	·47	5½	19·93	9	100·39
1½	·59	5½	21·38	9½	104·63
1½	·74	5½	22·91	9½	108·99
1½	·91	5½	24·51	9½	113·47
2	1·10	5½	26·18	9½	118·06
2½	1·32	5½	27·92	9½	122·77
2½	1·57	6	29·74	9½	127·63
2½	1·84	6½	31·64	9½	132·60
2½	2·15	6½	33·62	10	137·70
2½	2·49	6½	35·68	10½	142·93
2½	2·86	6½	37·81	10½	148·29
2½	3·27	6½	40·04	10½	153·78
3	3·72	6½	42·35	10½	159·40
3½	4·20	6½	44·75	10½	165·16
3½	4·73	7	47·23	10½	171·06
3½	5·29	7½	49·80	10½	177·10
3½	5·90	7½	52·47	11	183·28
3½	6·56	7½	55·23	11½	189·60
3½	7·26	7½	58·09	11½	196·06
3½	8·01	7½	60·04	11½	202·67
4	8·81	7½	64·09	11½	209·42
4½	9·67	7½	67·24	11½	216·33
4½	10·57	8	70·50	11½	223·38
4½	11·53	8½	73·86	11½	230·58
4½	12·55	8½	77·32	12	237·94
4½	13·62	8½	80·89		

TABLE

Of the weight of a square foot of millboard in lbs. avoirdupois.

Thickness, in inches..	½	¾	1	1½	2
Weight, in lbs.	·688	1·032	1·376	1·72	2·064

TABLE

Containing some of the properties of various bodies.

Names of bodies.	Melting and boiling points.	Contracts in cooling, in parts of an inch, for each foot in length.	Expands in heating from 32 to 212 deg. of Fahrenheit, the length being 1·00000.	Power of conducting heat.
Cast iron melts	17977°	·124	·00111	1·2
Wrought iron } welding hot }	12780	·137	·00122	1·1
Copper melts ..	4587	·193	·00172	1·0
Brass melts ..	3807	·210	·00187	1·0
Steel red hot ..	1077	·133	·00118	
Zinc melts	700	·329	·00294	
Mercury boils..	660		·01851	
Lead melts. ...	594	·319	·00286	2·5
Bismuth melts.	476	·156	·00139	
Tin melts	442	·278	·00248	1·7
Water boils ..	212		·04002	

TABLE

Showing the expansion of water by heat.

Temperature.	Expansion.	Temperature.	Expansion.
12° F.	1·00236	122° F.	1·01116
22	1·00090	132	1·01367
32	1·00022	142	1·01638
42	1·00000	152	1·01934
52	1·00021	162	1·02245
62	1·00083	172	1·02575
72	1·00180	182	1·02916
82	1·00312	192	1·03265
92	1·00477	202	1·03634
102	1·00672	212	1·04012
112	1·00880		

Proportions of cement for cast iron.

In mixing cement for cast iron, put one ounce of sal ammoniac to each hundred weight of borings, and use it without allowing it to heat. Multiply the length of any joint in feet by the breadth in inches, by the thickness in eighths, and by $\cdot 3$; the product will be the weight of dry borings, in *lbs.* avoirdupois, required to make cement to fill that joint *nearly*.

TABLE

Of boiling points of water holding various proportions of salt in solution.

	Parts of Salt.	Degrees of Fahrenheit.	Degrees of Reaumur.	Degrees of Centigrade.
Saturated solution	36·37	226·6	86·5	108·1
"	33·34	224·9	85·7	107·2
"	30·30	223·7	85·2	106·5
"	27·28	222·5	84·7	105·8
"	24·25	221·4	84·2	105·2
"	21·22	220·2	83·7	104·6
"	18·18	219·0	83·1	103·9
"	15·15	217·9	82·6	103·3
"	12·12	216·7	82·1	102·6
"	9·09	215·5	81·5	101·9
"	6·06	214·4	81·1	101·3
Sea water.....	3·03	213·2	80·5	100·7
Common water....	0·00	212·0	80·0	100·0

To reduce any number of degrees of temperature on Fahrenheit's scale to the number of degrees of an equal temperature on Reaumur's scale; and also to the number of degrees of an equal temperature on the Centigrade scale or otherwise.

The freezing points of water are 32° Fahrenheit, 0° Reaumur, 0° Centigrade; the boiling points of water are 212° F., 80° R., 100° C.; whence

FAHRENHEIT AND REAUMUR.

To convert degrees of Fahrenheit into degrees of Reaumur.

Multiply the difference between 32° and the given degrees of Fahrenheit, by 4, and divide by 9, the result will be degrees of Reaumur.

EXAMPLE 1.—Convert 77° of Fahrenheit into degrees of Reaumur.

Here $(77^{\circ} \infty 32^{\circ}) \times \frac{4}{9} = \frac{45^{\circ} \times 4}{9} = 20^{\circ}$ of Reaumur. = *Ans.*

EXAMPLE 2.—Turn 22° of Fahrenheit into degrees of Reaumur.

Here $(22^{\circ} \infty 32^{\circ}) \times \frac{4}{9} = \frac{10^{\circ} \times 4}{9} = 4\frac{4}{9}$ of Reaumur. = *Ans.*

FAHRENHEIT AND CENTIGRADE.

To convert degrees of Fahrenheit into Centigrade.

Multiply the difference between 32° and the given degrees of Fahrenheit, by 5, and divide by 9, the result is the number of degrees Centigrade.

EXAMPLE 3.—Convert 167° of Fahrenheit into Centigrade.

Here $(167^{\circ} \infty 32^{\circ}) \times \frac{5}{9} = \frac{135^{\circ} \times 5}{9} = 75^{\circ}$ Centigrade. = *Ans.*

EXAMPLE 4.—Convert 13° of Fahrenheit into Centigrade.

Here $(13^{\circ} \infty 32^{\circ}) \times \frac{5}{9} = \frac{19^{\circ} \times 5}{9} = 10\frac{5}{9}$ Centigrade. = *Ans.*

NOTE 1.—If the degrees of Fahrenheit are below 0° , take the sum instead of the difference between 32° and their quantity.

Thus (EXAMPLE 5) for 4° of Fahrenheit below 0° we have $(4^{\circ} + 32^{\circ}) \times \frac{4}{9} = \frac{36^{\circ} \times 4}{9} = 16^{\circ}$ of Reaumur. = *Ans.*, and

EXAMPLE 6. $(4^{\circ} + 32^{\circ}) \times \frac{5}{9} = \frac{36^{\circ} \times 5}{9} = 20^{\circ}$ Centigrade. = *Ans.*

REAUMUR, CENTIGRADE, AND FAHRENHEIT.

1. Above the freezing point.—Multiply the degrees of Reaumur by 9, and divide by 4; or multiply the Centigrade degrees by 9, and divide by 5; the quotient, added to 32° , gives the degrees of Fahrenheit.

EXAMPLE 7.—Convert 20° of Reaumur into degrees of Fahrenheit.

Here $32^{\circ} + \frac{20^{\circ} \times 9}{4} = 32^{\circ} + 45^{\circ} = 77^{\circ}$ of Fahrenheit. = *Ans.*

EXAMPLE 8.—Convert 75° Centigrade into degrees of Fahrenheit.

Here $32^{\circ} + \frac{75^{\circ} \times 9}{5} = 32^{\circ} + 135^{\circ} = 167^{\circ}$ of Fahrenheit. = *Ans.*

2. Below the freezing point.—Multiply and divide the given degrees as before if they are Reaumur's degrees and less than $14\frac{2}{3}^{\circ}$, or Centigrade and less than $17\frac{1}{2}^{\circ}$, subtract the quotient from 32° , the result will be degrees of Fahrenheit above 0° .

But if the given degrees are not less than $14\frac{2}{3}^{\circ}$ or $17\frac{1}{2}^{\circ}$ respectively, subtract 32° from the quotient, the remainder will be degrees of Fahrenheit below 0° .

EXAMPLE 9.—Convert $4\frac{4}{9}^{\circ}$ of Reaumur into Fahrenheit.

Here $32^{\circ} - \frac{4\frac{4}{9}^{\circ} \times 9}{4} = 32^{\circ} - 10^{\circ} = 22^{\circ}$ of Fahrenheit. = *Ans.*

EXAMPLE 10.—Convert $10\frac{4}{9}^{\circ}$ Centigrade into Fahrenheit degrees.

$$\text{Here } 32^{\circ} - \frac{10\frac{4}{9}^{\circ} \times 9}{5} = 32^{\circ} - 19^{\circ} = 13^{\circ} \text{ of}$$

Fahrenheit. = *Ans.*

EXAMPLE 11.—Turn 16° of Reaumur into degrees Fahrenheit.

$$\text{Here } \frac{16^{\circ} \times 9}{4} - 32^{\circ} = 36^{\circ} - 32^{\circ} = 4^{\circ} \text{ below } 0^{\circ}.$$

= *Ans.*

EXAMPLE 12.—Change 20° Centigrade into degrees Fahrenheit.

$$\text{Here } \frac{20^{\circ} \times 9}{5} - 32^{\circ} = 36^{\circ} - 32^{\circ} = 4^{\circ} \text{ of}$$

Fahrenheit below zero. = *Ans.*

NOTE 2.— 5° on the Centigrade scale = 4° on Reaumur's scale.

TABLE

Showing the quantity and weight of a superficial foot of brick work, from half a brick to two and a half bricks in thickness.

Thickness by number.	Thickness in inches.	Number of bricks.	Weight in lbs. avoird.
$\frac{1}{2}$ brick	$4\frac{1}{2}$	4.58	40.19
1	$9\frac{1}{2}$	9.15	80.37
$1\frac{1}{2}$	$14\frac{1}{2}$	13.72	120.56
2	$18\frac{1}{2}$	18.3	160.74
$2\frac{1}{2}$	$23\frac{1}{2}$	22.875	200.93

NOTE.—The weight is independent of mortar.

1 brick weighs 9 lbs. avoirdupois, nearly; $12\frac{1}{2} = 1$ cwt., and 250 = 1 ton.

TABLE

Of the specific gravities of those bodies chiefly used in machinery, building, &c., showing, in avoirdupois ounces and pounds, the weight of a cubic foot of each body also the weight of a cubic inch, and the number of cubic inches in a pound, with multipliers to each, for finding the weight when the dimensions are given.

Names of Bodies.	Weight of a Cubic Foot.		Weight of a Cubic Inch.		Number of Cubic Inches in a Pound.
	oz.	lbs.	oz.	lbs.	
Copper, cast	8788	549.25	5.086	3178	3.146
Copper, sheet	8915	557.19	5.159	3225	3.103
Brass, cast	8396	524.75	4.859	3037	3.293
Iron, cast	7271	454.44	4.208	263	3.802
Iron, bar	7788	486.75	4.507	282	3.550
Lead	11344	709.00	6.565	4103	2.437
Steel, soft	7833	489.56	4.533	2833	3.530
Steel, hard	7816	488.50	4.523	2827	3.537
Zinc, cast	7190	449.37	4.161	26	3.845
Tin, cast	7292	455.75	4.222	2636	3.792
Bismuth	9680	617.50	5.718	3574	2.798
Gun metal	8784	549.00	5.083	3177	3.148
Sand	1520	95.00	880	555	18.190
Coal	1250	78.12	723	452	22.118
Brick	2000	125.00	1.157	723	13.824
Stone, paving	2416	151.00	1.398	873	11.444
Slate	2672	167.00	1.546	967	10.347
Marble	2742	171.37	1.587	991	10.083
White Lead	3160	197.50	1.829	1143	8.749
Glass	2880	180.00	1.667	1042	9.600
Tallow	945	59.06	547	342	29.257
Cork	240	15.00	139	887	115.200
Larch	544	34.00	315	197	50.823
Elm	671	41.94	388	204	41.204
Pine, pitch	660	41.25	382	224	41.890
Beech	852	53.25	493	208	32.451
Teak	745	46.56	431	227	37.111
Ash	845	52.81	489	305	32.720
Mahogany	1063	66.44	615	384	26.009
Oak	970	60.62	561	351	28.503
Oil of Turpentine ..	870	54.37	503	315	31.779
Olive Oil	915	57.19	529	331	30.216
Linseed Oil	932	58.25	539	337	29.665
Spirits, proof	927	57.94	536	3352	29.825
Water, distilled	1000	62.50	579	3617	27.648
Water, sea	1028	64.25	595	372	26.895
Tar	1015	63.44	587	367	27.239
Vinegar	1026	64.12	594	37	26.947
Mercury	13568	848.00	7.852	4907	2.038

The 1st, 2nd, 3rd, and 4th columns require no further explanation than the titles they bear; the fifth column is to find the weight of any number of cubic inches, in avoirdupois pounds, of any of the different bodies required.

EXAMPLE 1.—Suppose a piece of cast iron to be $56\frac{3}{4}$ inches long, $16\frac{1}{2}$ inches broad, and $\frac{3}{4}$ of an inch in thickness, required its weight.

Here $56\cdot75 \times 16\cdot5 \times \cdot75 = 702\cdot28125$ cubic inches, which multiplied by $\cdot263 = 184\cdot7$ lbs., nearly. = *Ans.*

EXAMPLE 2.—Required the weight of an imperial gallon of proof spirits.

Here $277\cdot274 \times \cdot03352 = 9\cdot294$ lbs., nearly. = *Ans.*

EXAMPLE 3.—Required the thickness of metal for a concave copper ball, 8 inches diameter without, so as to sink to its centre in common water.

Here $8^3 \times \cdot5236 = 268\cdot0832$ cubic inches in the ball, the half of which = $134\cdot0416$ cubic inches to be immersed, or cubic inches of water to be removed,—Then $134\cdot0416 \times \cdot579$ (the weight of a cubic inch of water) = $77\cdot6100864$ ounces weight of water displaced, or, the weight of the copper ball; which divided by $5\cdot159$ (the weight of a cubic inch of copper) = $15\cdot0436$ cubic inches of copper in the ball.

Again, $8^2 \times \cdot7854 \times 4 = 201\cdot0624$ square inches, the superficies of the ball; and $15\cdot0436 \div 201\cdot0624 = \cdot0748$ inches, the required thickness of the copper, nearly. = *Ans.*

EXAMPLE 4.—Required the weight necessary to counterpoise a float of paving stone proper for a steam-engine boiler, &c., the float being 14 inches diameter and $2\frac{1}{2}$ inches thick.

Here $14^2 \times \cdot7854 \times 2\cdot5 = 384\cdot846$ cubic inches. Then $384\cdot846 \times \cdot0873 =$ the weight of the stone in lbs. And, $384\cdot846 \times \cdot03617 =$ the weight in lbs. of water displaced; then, $384\cdot846 \times (\cdot0873 - \cdot03617) = 384\cdot846 \times \cdot05113 = 19\cdot677$ lbs. the counterpoise required. = *Ans.*

RULES

FOR MAKING OR CORRECTING THE GAUGE POINTS ON
THE ENGINEER'S SLIDE RULE.

The engineer's slide rule is an instrument of extensive use to mechanics, and almost every one who is in possession of the rule, is also, or may be, in ample possession of instructions; but I am not aware that any information has been given in any other work, for either correcting the old gauge points, or obtaining new ones; hence the following may be found useful:—

And first, by making the third column on the rule (or that marked III) the first of our observations, the others are rendered very simple; thus,

The third column is the number of cubic inches contained in a pound, foot, gallon, &c.

The second column is the numbers in the third column expressed in the decimals of a foot, or multiplied by $\cdot 833$.

The first column is the third column divided by 1728.

The fifth column is the third column divided by $\cdot 7854$.

The fourth column is the fifth column expressed in the decimals of a foot, or multiplied by $\cdot 833$.

The seventh column is the third column divided by $\cdot 5236$. And,

The sixth column is the seventh column divided by 1728.

DECIMAL APPROXIMATIONS FOR FACILITATING
CALCULATIONS IN MENSURATION.

Lineal feet multiplied by	·000189	= miles.
" yards	·000568	= "
Square inches	·00694'	= square feet.
" yards	·0002066	= acres.
Circular inches	·005454	= square feet.
Cylindrical inches	·0004645	= cubic feet.
" feet	·02908'	= cubic yards.
Cubic inches	·0005787	= cubic feet.
" feet	·037'	= cubic yards.
" "	6·232	= imperial gallons.
" inches	·0036065	= " "
Cylindrical feet	4·895	= " "
" inches	·002832	= " "
Cubic inches	·2630	= lbs. avo. of cast iron.
" "	·2817	= " wrought do.
" "	·2833	= " steel.
" "	·3225	= " copper.
" "	·3037	= " brass.
" "	·2601	= " zinc.
" "	·4103	= " lead.
" "	·2636	= " tin.
" "	·4908	= " mercury.
Cylindrical inches	·2065	= " cast iron.
" "	·2212	= " wrought iron.
" "	·2225	= " steel.
" "	·2533	= " copper.
" "	·2385	= " brass.
" "	·2043	= " zinc.
" "	·3223	= " lead.
" "	·2071	= " tin.
" "	·3854	= " mercury.
Avoirdupois lbs.	·0089	= cwt.
" "	·000446	= tons.

**DECIMAL EQUIVALENTS TO FRACTIONAL PARTS
OF LINEAL MEASURES.**

One inch, the integer or whole number.					
·96875	are equal to	$+\frac{3}{32}$	·625	are equal to	$+\frac{3}{32}$
·9375		$+\frac{1}{16}$	·59375		$+\frac{3}{32}$
·90625		$+\frac{1}{32}$	·5625		$+\frac{1}{16}$
·875			·53125		$+\frac{1}{32}$
·84375		$+\frac{3}{32}$	·5		
·8125		$+\frac{1}{16}$	·46875		$+\frac{3}{32}$
·78125		$+\frac{1}{32}$	·4375		$+\frac{1}{16}$
·75			·40625		$+\frac{1}{32}$
·71875		$+\frac{3}{32}$	·375		
·6875		$+\frac{1}{16}$	·34375		$+\frac{1}{32}$
·65625		$+\frac{1}{32}$	·3125		$+\frac{1}{16}$
·28125		$+\frac{1}{32}$			
·25					
·21875		$+\frac{3}{32}$			
·1875		$+\frac{1}{16}$			
·15625		$+\frac{1}{32}$			
·125					
·09375		$+\frac{3}{32}$			
·0625		$+\frac{1}{16}$			
·03125		$+\frac{1}{32}$			
One foot, or 12 inches, the integer.					
·9166'	11 inches.		·4166'	5 inches.	
·8333'	10 "		·3333'	4 "	
·75	9 "		·25	3 "	
·6666'	8 "		·1666'	2 "	
·5833'	7 "		·0833'	1 "	
·5	6 "		·07292	$\frac{1}{16}$ "	
·0625					
·05208					
·04167					
·03125					
·02083					
·01042					
One yard, or 36 inches, the integer.					
·972'	35 inches.		·638'	23 inches.	
·944'	34 "		·611'	22 "	
·916'	33 "		·583'	21 "	
·888'	32 "		·555'	20 "	
·861'	31 "		·527'	19 "	
·833'	30 "		·5	18 "	
·805'	29 "		·472'	17 "	
·777'	28 "		·444'	16 "	
·75	27 "		·416'	15 "	
·722'	26 "		·388'	14 "	
·694'	25 "		·361'	13 "	
·666'	24 "		·333'	12 "	
·305'	11 inches.		·277'	10 "	
·25	9 "		·222'	8 "	
·194'	7 "		·166'	6 "	
·138'	5 "		·111'	4 "	
·083'	3 "		·055'	2 "	
·027'	1 "				

TABLE

Containing the price of metals, or other materials, by the ton, cwt., quarter, or lb.

Per ton.	Per cwt.	Per grtr.	Per lb.	Per ton.	Per cwt.	Per grtr.	Per lb.	Per ton.	Per cwt.	Per grtr.	Per lb.
£ s. d.	£ s. d.	£ s. d.	£ s. d.	£ s. d.	£ s. d.	£ s. d.	£ s. d.	£ s. d.	£ s. d.	£ s. d.	£ s. d.
2 6 8	2 4 0	7 1	14 10 0	14 6 3	7 1	32 10 0	12 6 0	8 1 1	32 13 4	12 8 0	8 2 3
2 10 0	2 6 0	7 1	14 15 0	14 9 3	8 1	33 0 0	13 0 0	8 3	33 10 0	13 6 0	8 4 1
2 15 0	2 9 0	8 1	15 0 0	15 0 3	9	34 0 0	14 0 0	8 6	34 10 0	14 6 0	8 7 1
3 0 0	3 0 0	9	15 5 0	15 3 3	9 1	35 0 0	15 0 0	8 9	35 10 0	15 6 0	8 10 1
3 5 0	3 3 0	9 1	15 10 0	15 6 3	10 1	36 0 0	16 0 0	9 0	36 10 0	16 6 0	9 1 1
3 10 0	3 6 0	10 1	15 15 0	15 9 3	11 1	37 0 0	17 0 0	9 3	37 6 8	17 4 0	9 4 4
3 15 0	3 9 0	11 1	16 0 0	16 0 4	0	37 10 0	17 6 0	9 4 1	38 0 0	18 0 0	9 6
4 0 0	4 0 1	0	16 5 0	16 3 4	0 1	38 10 0	18 6 0	9 7 1	39 0 0	19 0 0	9 9
4 5 0	4 3 1	0 1	16 6 8	16 4 4	1 1 1	39 10 0	19 6 0	9 10 1	39 13 4	19 8 0	9 11 4
4 10 0	4 6 1	1 1	16 10 0	16 6 4	1 1 1	40 0 0	20 0 0	10 0	40 10 0	20 6 0	10 1 1
4 13 4	4 8 1	2 1	16 15 0	16 9 4	2 1	41 0 0	21 0 0	10 3	41 10 0	21 6 0	10 4 1
4 15 0	4 9 1	2 1	17 0 0	17 0 4	3	42 0 0	22 0 0	10 6	44 6 8	22 4 0	11 1 4
5 0 0	5 0 1	3	17 5 0	17 3 4	3 1	46 13 4	26 8 0	11 8 5	49 0 0	26 0 0	12 3 5
5 5 0	5 3 1	3 1	17 10 0	17 6 4	4 1	51 6 8	27 11 4	12 10 5	53 13 4	27 8 0	13 5 6
5 10 0	5 6 1	4 1	17 15 0	17 9 4	5 1	56 0 0	28 16 0	14 0 6	58 6 8	28 18 0	14 7 7
5 15 0	5 9 1	5 1	18 0 0	18 0 4	6	60 13 4	30 15 2	15 9 6	65 6 8	30 16 4	15 17 7
6 0 0	6 0 1	6	18 5 0	18 3 4	6 1	67 13 4	32 16 11	17 6 7	70 0 0	32 10 0	17 7 7
6 5 0	6 3 1	6 1	18 10 0	18 6 4	7 1	72 6 8	34 18 1	18 1 7	74 13 4	34 18 8	18 8 8
6 10 0	6 6 1	7 1	18 13 4	18 8 4	8 2	77 0 0	36 17 0	19 3 8	79 6 8	36 19 4	19 10 8
6 15 0	6 9 1	8 1	18 15 0	18 9 4	8 1	81 13 4	38 18 1	20 5 8	84 0 0	38 20 1	20 9 9
7 0 0	7 0 1	9	19 0 0	19 0 4	9	86 6 8	40 18 1	21 7 9	88 13 4	40 21 8	21 12 9
7 5 0	7 3 1	9 1	19 5 0	19 3 4	9 1	91 0 0	42 18 1	22 9 9	93 6 8	42 23 3	22 14 10
7 10 0	7 6 1	10 1	19 10 0	19 6 4	10 1	95 13 4	44 18 1	23 11 10	98 0 0	44 25 1	23 16 11
7 15 0	7 9 1	11 1	19 15 0	19 9 4	11 1	100 6 8	46 18 1	24 13 11	102 13 4	46 27 1	24 18 12
8 0 0	8 0 2	0	20 0 0	20 0 5	0	106 6 8	48 18 1	25 15 12	107 6 8	48 29 1	25 20 13
8 5 0	8 3 2	0 1	20 10 0	20 6 5	1 1	112 0 0	50 18 1	26 17 13	112 0 0	50 31 1	26 22 14
8 10 0	8 6 2	1 1	21 0 0	21 0 5	3 2 1	118 0 0	52 18 1	27 19 14	118 0 0	52 33 1	27 24 15
8 15 0	8 9 2	2 1	21 10 0	21 6 5	4 1	124 0 0	54 18 1	28 21 15	124 0 0	54 35 1	28 26 16
9 0 0	9 0 2	3	22 0 0	22 0 5	6	130 0 0	56 18 1	29 23 16	130 0 0	56 37 1	29 28 17
9 5 0	9 3 2	3 1	22 10 0	22 6 5	7 1	136 0 0	58 18 1	30 25 17	136 0 0	58 39 1	30 30 18
9 6 8	9 4 2	4 1	23 0 0	23 0 5	9	142 0 0	60 18 1	31 27 18	142 0 0	60 41 1	31 32 19
9 10 0	9 6 2	4 1	23 6 8	23 4 5	10 2 1	148 0 0	62 18 1	32 29 19	148 0 0	62 43 1	32 34 20
9 15 0	9 9 2	5 1	23 10 0	23 6 5	10 1	154 0 0	64 18 1	33 31 20	154 0 0	64 45 1	33 36 21
10 0 0	10 0 2	6	24 0 0	24 0 6	0	160 0 0	66 18 1	34 33 21	160 0 0	66 47 1	34 38 22
10 5 0	10 3 2	6 1	24 10 0	24 6 6	1 1	166 0 0	68 18 1	35 35 22	166 0 0	68 49 1	35 40 23
10 10 0	10 6 2	7 1	25 0 0	25 0 6	3	172 0 0	70 18 1	36 37 23	172 0 0	70 51 1	36 42 24
10 15 0	10 9 2	8 1	25 10 0	25 6 6	4 1	178 0 0	72 18 1	37 39 24	178 0 0	72 53 1	37 44 25
11 0 0	11 0 2	9	25 13 4	25 8 6	5 2 1	184 0 0	74 18 1	38 41 25	184 0 0	74 55 1	38 46 26
11 5 0	11 3 2	9 1	26 0 0	26 0 6	6	190 0 0	76 18 1	39 43 26	190 0 0	76 57 1	39 48 27
11 10 0	11 6 2	10 1	26 10 0	26 6 6	7 1	196 0 0	78 18 1	40 45 27	196 0 0	78 59 1	40 50 28
11 13 4	11 8 2	11 1	27 0 0	27 0 6	9	202 0 0	80 18 1	41 47 28	202 0 0	80 61 1	41 52 29
11 15 0	11 9 2	11 1	27 10 0	27 6 6	10 1	208 0 0	82 18 1	42 49 29	208 0 0	82 63 1	42 54 30
12 0 0	12 0 3	0	28 0 0	28 0 7	0 3	214 0 0	84 18 1	43 51 30	214 0 0	84 65 1	43 56 31
12 5 0	12 3 3	0 1	28 10 0	28 6 7	1 1	220 0 0	86 18 1	44 53 31	220 0 0	86 67 1	44 58 32
12 10 0	12 6 3	1 1	29 0 0	29 0 7	3	226 0 0	88 18 1	45 55 32	226 0 0	88 69 1	45 60 33
12 15 0	12 9 3	2 1	29 10 0	29 6 7	4 1	232 0 0	90 18 1	46 57 33	232 0 0	90 71 1	46 62 34
13 0 0	13 0 3	3	30 0 0	30 0 7	6	238 0 0	92 18 1	47 59 34	238 0 0	92 73 1	47 64 35
13 5 0	13 3 3	3 1	30 6 8	30 4 7	7 1	244 0 0	94 18 1	48 61 35	244 0 0	94 75 1	48 66 36
13 10 0	13 6 3	4 1	30 10 0	30 6 7	8 1	250 0 0	96 18 1	49 63 36	250 0 0	96 77 1	49 68 37
13 15 0	13 9 3	5 1	31 0 0	31 0 7	9	256 0 0	98 18 1	50 65 37	256 0 0	98 79 1	50 70 38
14 0 0	14 0 3	6 1	31 10 0	31 6 7	10 1	262 0 0	100 18 1	51 67 38	262 0 0	100 81 1	51 72 39
14 5 0	14 3 3	6 1	32 0 0	32 0 8	0	268 0 0	102 18 1	52 69 39	268 0 0	102 83 1	52 74 40

A TABLE, BY SAMUEL MAYNARD,

Containing useful numbers often required in calculations, together with their Logarithms.

NOTE.—Some portion of the following Table has already appeared, by the present editor, in other publications; among which may be mentioned Keith's *Mensuration* (twenty-fourth edition, 1846). The numbers in that portion, however, have all been recomputed, and an amplification has been given to them which the Author believes to surpass any similar collection. He feels confidence in their strict accuracy, and hopes they will be found to be carried to an extent sufficient for every purpose of calculation.

By the combination of the six following logarithms, viz. of 2, 3, 15, 77, 94, and 487 (extracted from the table of logarithms to 61 decimal places in Abraham Sharp's *Geometry Improved*, p. 56, edit. of 1717), with $\log. \pi$, all the logarithms in the following table have been constructed; with the exception of the logarithm of the Imperial gallon, No. (86), which has been computed by Hutton's 20-figure logarithm table.

Log. 2 = 0.30102 99956 63881 19521 87388 94724 49303 | Log. 77 = 1.88649 07251 72481 87146 24162 29835 66044
 " 3 = 0.47712 12547 19662 43729 50279 03255 11531 | " 94 = 1.97312 78535 99698 65962 79582 94173 69367
 " 15 = 1.17609 12590 55681 24208 12890 08530 62228 | " 487 = 2.68752 89612 14634 33246 32050 64435 75372

NUMBERS.

LOGARITHMS.

By means of Euler's formula, $\frac{\pi}{4} = 4 \tan^{-1} \frac{1}{5} - \tan^{-1} \frac{1}{7} + \tan^{-1} \frac{1}{99}$ wherein π represents the circumference of a circle whose diameter is unity, Mr. William Rutherford, F.R.A.S., of the Royal Military Academy, Woolwich, has obtained the following value of π to 208 decimal places, (see *Philosophical Transactions of London*, Part II. for 1841, page 281): viz.

(1) $\pi = 3.14159 26535 89793 23846 26433 83279 50288$
 = 41971 69399 37510 58209 74944 59230 78164
 = 06286 20899 86280 84825 34211 70679 82148
 = 08651 32823 06647 09354 46095 50582 23172
 = 55594 08128 48473 78139 20386 33830 21574
 = 73996 00825 93125 91294 01832 80651 744 + &c.

the circumference of a circle whose diameter is 1 = area of a circle whose radius is 1; which, to 185 decimals, agrees with V.zea.

0.48714 98726 94133 85435 12682 88291
 (Abraham Sharp, in *Shirwin's Table of Logarithms*,
 edit. of 1726, p. 42.)

NUMBERS.	LOGARITHMS.
(2) $2\pi = 6.28318$ 53071 79586 47692 52867 66559, the circumference of a circle whose radius is 1.	0.79817 98683 58115 04956 50071 83015
(3) $4\pi = 12.56637$ 06143 59172 95385 05735 33118 $= 2\pi \times 2$, the surface of a sphere whose radius is 1.	1.09920 98640 22096 24477 87460 77740
(4) $36\pi = 113.09733$ 55292 32556 58465 51617 98062 $= 4\pi \times 9 =$ the surface and solidity of a sphere whose diameter is 6.	2.05345 23734 61421 11936 88018 84250
(5) $\frac{1}{2}\pi = 1.57079$ 63267 94896 61023 13216 91640 $=$ semicircular surface of a circle whose diameter is 1 $=$ quadrant arc of a circle whose radius is 1.	0.19611 98770 30152 65913 75293 93566
(6) $\frac{1}{3}\pi = 1.04719$ 75311 96397 74615 42144 61093 $=$ arc of a circle whose chord equals the radius.	0.02002 86179 74471 41705 62403 85036
(7) $\frac{1}{4}\pi = 0.78539$ 81633 97448 30961 56608 45820 $= \frac{1}{4}$ of $\frac{1}{2}\pi =$ quadrant arc of a circle whose diameter is 1 $=$ area of a circle whose diameter is 1.	9.89408 98813 66171 46392 37904 98842—10
(8) $\frac{1}{5}\pi = 0.62831$ 85307 87755 98298 87307 71072 30547 $= \frac{1}{5}\pi - \frac{1}{5}$ of $\frac{1}{2}\pi =$ arc of 60° to diameter 1 $=$ solidity of a sphere whose diameter is 1.	9.71899 86223 10490 22184 25014 90311—10
(9) $\frac{1}{6}\pi = 0.52359$ 87755 98298 87307 71072 30547 $= \frac{1}{6}\pi - \frac{1}{6}$ of $\frac{1}{2}\pi =$ arc of 45° to diameter 1.	9.59405 98857 02190 26871 00516 04117—10
(10) $\frac{1}{7}\pi = 0.44879$ 93877 99149 43653 85586 15273 $= \frac{1}{7}\pi - \frac{1}{7}$ of $\frac{1}{2}\pi =$ arc of 30° to diameter 1.	9.44796 86266 46509 03692 87625 95587—10
(11) $\frac{1}{8}\pi = 0.39269$ 90816 98724 15480 78304 22910 $= \frac{1}{8}\pi - \frac{1}{8}$ of $\frac{1}{2}\pi =$ arc of 22.5° to diameter 1.	9.14693 86309 82527 83141 50237 00862—10
(12) $\frac{1}{10}\pi = 0.31415$ 92653 50943 29576 92369 07683 $= \frac{1}{10}\pi =$ arc of 18° to diameter 1.	8.24187 73675 90327 78454 74735 87056—10
(13) $\frac{1}{12}\pi = 0.26179$ 93877 99149 43653 85586 15273 $= \frac{1}{12}\pi =$ arc of 15° to diameter 1.	7.94084 73719 26846 58933 37346 92333—10

(14) $\frac{1}{10800}\pi = 0.00029$ 08882 08665 72159 61539 48461 = $\frac{1}{36}$ of $360^\circ\pi$ = arc of $1'$ to radius 1.	6-46372 61172 07184 15203 87067 89077-10
(15) $\frac{1}{72000}\pi = 0.00000$ 48481 26811 09535 99358 99141 = $\frac{1}{60}$ of 18000π = arc of $1''$ to radius 1.	4-68537 48668 23540 51952 99399 91097-10
(16) $\frac{2}{3}\pi = 2.09439$ 51023 93195 49230 84289 22186 = $\frac{2}{3}$ of 2π = arc of 120° to radius 1.	0-32105 86136 38452 61226 99792 79760
(17) $\frac{4}{3}\pi = 4.18879$ 02047 86390 98461 68578 44373 = $\frac{4}{3}$ of 4π = arc of 240° to radius 1 = solidity of a sphere whose radius is 1.	0-62208 86093 02433 80748 37181 74485
(18) $\sqrt{2} = 1.41421$ 35623 73695 04880 16887 24209 7, the diagonal of a square whose side is 1.	0-15031 49978 31990 59760 68694 47962
(19) $\sqrt{\frac{1}{2}} = 0.70710$ 67811 86547 52440 08443 62104 8 = $\frac{1}{2}\sqrt{2}$, the side of a square whose diagonal is 1.	9-84948 50021 68009 40239 31305 52638-10
(20) $\pi\sqrt{2} = 4.44288$ 29381 58366 24701 58809 90060 7, the circum- ference of the circle circumscribing a square whose side is 1.	0-64766 48705 26124 45195 81377 35653
(21) $\pi\sqrt{\frac{1}{2}} = 2.22144$ 14690 79183 12350 79404 95030 3, the $\frac{3}{2}$ of $\frac{\pi}{\sqrt{2}}$, the circumference of a circle circumscribing a square whose side = $\frac{1}{2}$.	0-34663 48748 62143 25674 43988 40929
(22) $\pi^2 = 9.86960$ 44010 89358 61883 44909 99876.....	0-99429 97453 88267 70870 25365 76582
(23) $6\pi^2 = 59.21762$ 64065 36151 71300 69459 99257.....	1-77245 09957 71911 34121 13033 74561
(24) $\frac{1}{\pi} = 0.31830$ 98861 83790 67153 77675 26745, the reciprocal of π , the diameter of a circle whose circumference is 1.	9-50285 01273 05866 14564 87317 11709-10
(25) $\frac{2}{\pi} = 0.63661$ 97723 67581 34307 55350 53490 = $\frac{1}{\pi} \times 2$.	9-80358 01229 69847 34086 24706 06494-10
(26) $\frac{4}{\pi} = 1.27323$ 95447 35162 68615 10701 06980 = $\frac{2}{\pi} \times 2$.	0-10491 01186 33828 53607 62095 01158

NUMBERS.		LOGARITHMS.	
(27)	$\frac{6}{\pi} = 1.90985$	93171 02744 02922 66051 60470 = $\frac{4}{\pi} + \frac{1}{2}$ of $\frac{4}{\pi}$	0.28100 13776 88509 77815 74985 09689
(28)	$\frac{180}{\pi} = 57.29577$	95130 82320 87679 81548 14111 = $\frac{6}{\pi} \times 30$, the number of degrees and decimal parts in an arc equal to the radius; which, expressed in degrees, minutes, and seconds, and decimals of a second, is $57^{\circ} 17' 44'' .806247096$.	1.75812 26324 09172 21545 25264 12944
(29)	$\frac{360}{\pi} = 114.59155$	90261 64641 75359 63096 28221 = $\frac{180}{\pi} \times 2$, the number of degrees and decimal parts in an arc equal to the diameter.	2.05915 26280 73153 41066 62653 07668
(30)	$\frac{10800}{\pi} = 3437.74677$	07849 39252 60788 92888 46637 = $\frac{360}{\pi} \times 30$, the number of minutes and decimal parts in an arc equal to the radius; which, expressed in minutes, seconds, and decimals of a second, is $3437^{\circ} 44' .806247096$.	3.53627 38827 92815 84796 12932 10923
(31)	$\frac{648000}{\pi} = 206264.90624$	70963 55156 47335 73307 98214 = $\frac{10800}{\pi} \times 60$, the number of seconds and decimal parts in an arc equal to the radius.	5.31442 51331 76459 48047 00600 08903
(32)	60 = the base of the sexagesimal scale.		1.77815 12503 83643 63250 87687 97980
(33)	360 = the number of degrees in the circle, sexagesimal or ancient division.		2.55630 25007 67287 26501 75335 95959
(34)	21600 = the number of minutes in the circle, according to the ancient or sexagesimal division = 360×60 .		4.33445 37511 50930 89752 63003 83939
(35)	1296000 = the number of seconds in the circle, according to the ancient or sexagesimal division = 21600×60 .		6.11260 50015 34574 53003 50671 91918
(36)	100 = base of the French, or centesimal scale.		2.00000 00000 00000 00000 00000 00000

(37)	400 = the number of degrees in the circle, according to the French or centesimal division.	2·60205	99913	27962	39042	74777	89449
(38)	40000 = 400×100 = the number of minutes in the circle, according to the French or centesimal division.	4·60205	99913	27962	39042	74777	89449
(39)	4000000 = 40000×100 = the number of seconds in the circle, according to the French or centesimal division.	6·60205	99913	27962	39042	74777	89449
(40)	$\frac{400}{\pi} = 127\cdot32395$ 44735 16268 61510 70106 98024 = $\frac{4}{\pi} \times 100$, the number of French or centesimal degrees and decimal parts in an arc equal to the radius.	2·10491	01186	33828	53607	62095	01158
(41)	$\frac{40000}{\pi} = 12732\cdot39544$ 73516 26861 51070 10698 02359 = $\frac{400}{\pi} \times 100$, the number of French or centesimal minutes in an arc equal to the radius.	4·10491	01186	33828	53607	62095	01158
(42)	$\frac{40000000}{\pi} = 1273239\cdot54473$ 51626 86151 07010 69802 35688 = $\frac{400000}{\pi} \times 100$, the number of French or centesimal seconds in an arc equal to the radius.	6·10491	01186	33828	53607	62095	01158
(43)	$\frac{\pi}{200} = 0\cdot01570$ 79632 67948 96619 23132 16916 = arc of 1 centesimal degree to radius 1.	8·19611	98770	30152	65913	75293	93566—10
(44)	$\frac{\pi}{20000} = 0\cdot00015$ 70796 32679 48966 19231 32169 = $\frac{1}{100}$ of $\frac{\pi}{200}$ = arc of 1 centesimal minute to radius 1.	6·19611	98770	30152	65913	75293	93566—10
(45)	$\frac{\pi}{2000000} = 0\cdot000000$ 15707 96326 79489 66192 31322 = $\frac{1}{100}$ of $\frac{\pi}{20000}$ = arc of 1 centesimal second to radius 1.	4·19611	98770	30152	65913	75293	93566—10
(46)	$\frac{1}{2\pi} = 0\cdot15915$ 49430 91895 33576 88837 63373 = $\frac{1}{2}$ of $\frac{1}{\pi}$	9·20182	01316	41864	95043	49628	16985—10

NUMBERS.		LOGARITHMS.	
(47)	$\frac{1}{4\pi} = 0.07957$ the area of a circle whose circumference is 1.	890079	01859 77903 75522 12539 22260—10
(48)	$\frac{1}{6\pi} = 0.05305$	879459	88769 22223 51313 99649 13729—10
(49)	$\frac{1}{\pi}\sqrt{2} = 0.45015$	965336	51251 37856 74325 56011 59071—10
(50)	$\frac{1}{\pi}\sqrt{\frac{1}{2}} = 0.22507$	935233	51294 73875 54804 18622 64347—10
(51)	$\frac{1}{\pi^2} = 0.10132$	900570	02546 11732 29129 74634 23418—10
(52)	$\frac{1}{2\pi^2} = 0.05066$	870467	02589 47751 09608 37245 28694—10
(53)	$\frac{1}{6\pi^2} = 0.01688$	822754	90042 29088 65878 86966 25439—10
(54)	$\sqrt{\pi} = 1.77245$ square equal in surface to a circle whose radius is 1.	024837	49363 47066 92717 56341 44145
(55)	$2\sqrt{\pi} = 3.54490$ square equal in surface to a circle whose radius is 1.	054960	49320 11048 12238 93730 38870
(56)	$\frac{1}{2}\sqrt{\pi} = 0.88622$ side of a square equal in surface to a circle whose diameter is 1.	994754	49406 83085 73196 18952 49421—10
(57)	$\frac{1}{3}\sqrt{\pi} = 0.32155$	934548	49493 55123 34153 44174 59972—10
(58)	$\sqrt{\frac{\pi}{2}} = 1.25331$	009805	99385 15076 32956 87646 96783

(59)	$\sqrt{\frac{1}{\pi}} = 0.56418$ 93835 47756 28694 80794 .51560, the reciprocal of $\sqrt{\pi}$, the diameter of a sphere whose surface is 1.	9-75142	50636	52933	07282	43658	55855—10
(60)	$2\sqrt{\frac{1}{\pi}} = 1.12837$ 91670 95512 57389 61589 03120, the diameter of a circle equal in surface to a square whose side is 1.	0-05245	50593	16914	26803	81047	50579
(61)	$\frac{1}{2}\sqrt{\frac{1}{\pi}} = 0.28209$ 47917 73878 14347 40387 25780 = $\frac{1}{2}$ of $2\sqrt{\frac{1}{\pi}}$	9-45939	50679	89951	87761	06269	61130—10
(62)	$\frac{1}{3}\sqrt{\frac{1}{\pi}} = 0.09403$ 15972 57959 38115 80132 41927 = $\frac{1}{3}$ of $\frac{1}{2}\sqrt{\frac{1}{\pi}}$	8-97327	38132	69289	44031	55990	57875—10
(63)	$\frac{1}{4}\sqrt{\frac{1}{\pi}} = 0.07052$ 36979 43469 53586 85099 31445 = $\frac{1}{4}$ of $\frac{1}{2}\sqrt{\frac{1}{\pi}}$	8-84833	50766	60989	48718	31491	71681—10
(64)	$\sqrt{\frac{2}{\pi}} = 0.79788$ 45608 02865 35587 98921 19876 = $\sqrt{2 \times \frac{1}{\pi}}$	9-90194	00614	84923	67043	12353	03217—10
(65)	$\sqrt[3]{\frac{1}{36\pi}} = 0.03597$ 58620 5, the side of a cube equal in solid content to a sphere whose diameter is 6.	0-68448	41244	87140	37312	29339	61417—10
(66)	$\sqrt[3]{\frac{\pi}{6}} = 0.80599$ 59770 1 = $\frac{1}{2}$ of $\sqrt[3]{\frac{1}{36\pi}}$, the side of a cube equal in solid content to a sphere whose diameter is 1.	9-90633	28741	03496	74061	41671	63437—10
(67)	$\sqrt[3]{6\pi^2} = 3.89777$ 707.....	0-59081	69985	90637	11373	71011	24854
(68)	$\sqrt[3]{\frac{1}{\pi}} = 1.24070$ 09818.....	0-09386	71258	96503	25938	58328	36563
(69)	24 = the number of hours in a day.	1-38021	12417	11606	02293	62445	87429
(70)	1440 = the number of minutes in a day = 24×60 .	3-15836	24920	95249	65544	50113	85408
(71)	86400 = the number of seconds in a day = 1440×60 .	4-98651	37424	78893	28795	37781	83388
(72)	60 = the number of minutes in 1 hour.	1-77815	12503	83643	63250	87667	97980
(73)	3600 = the number of seconds in 1 hour = $60 \times 60 = 60^2$.	3-55630	25007	67287	26501	75335	95959

NUMBERS.		LOGARITHMS.	
(74)	47356400000 = the number of days in which a body moving uniformly at the rate of 1" sexagesimal in a Julian century or 36525 days describes one revolution or 12960000" sexagesimal seconds, this number being equal to $36525 \times 1296000 = 36525000 \times 6 \times 6 \times 6$.	10 67519	52261 40008 90936 58223 70160
(75)	146100000000 = the number of days in which a body moving uniformly at the rate of 1" centesimal in a Julian century or 36525 days describes one revolution or 4000000 centesimal seconds, this number being equal to 36525×4000000 .	11 16465	02159 34296 76975 82329 67691
(76)	3999930 09 = the number of centesimal seconds (according to Laplace, Mécanique Céleste, tome troisième, page 64) which the earth describes in a Julian century or 36525 days.		
(77)	365 25638 37684 = the number of days and decimal parts in a sidereal year, equal to $146100000000 \div 399993039$.		
(78)	129597738 2344 = the number of sexagesimal seconds, according to Bessel, which the earth describes in a Julian century or 36525 days.— <i>Connaissance des Temps, Additions, Année 1831</i> , p. 154.		
(79)	365 25637 44159 = the number of days and decimal parts in a sidereal year, equal to $47356400000 \div 129597738 2344$; or,		
(80)	129597734 2891 = the number of sexagesimal seconds which the earth, according to another determination, describes in a Julian century or 36525 days.		
(81)	365 25638 55353 = the number of days and decimal parts in a sidereal year, according to the preceding motion = 365d. 6h. 9m. 11 ^s .71035s. 365d. 6h. 9m. 10 ^s .74953s., according to Bessel. 365d. 6h. 9m. 11 ^s .55759s., according to Delambre and Laplace.		

(82)	39-14997	= length of Beasel's seconds pendulum in a vacuum, at the level of the sea, in latitude $54^{\circ} 42' 50''$ (Königsberg).			
(83)	39-13907	= length of seconds pendulum, deduced from the preceding to London (lat. $51^{\circ} 31' 8''$).			
(84)	252-458	grains troy = weight of a cubic inch of distilled water at the temperature of 62° Fahrenheit. (English Measures, <i>Act 5 G. 4, c. 74.</i>)			
(85)	7000	grains = 1 lb. avoirdupois.			
(86)	277-273843570	= $70000 \div 252-458$ = number of cubic inches in the imperial gallon.			
(87)	$\sqrt{277-273843570}$	= $16-65154177565$.			
(88)	$\sqrt{765398163-974483}$	= $18-789252841825$			
(89)	277-273843570	= -00360 65428 57143 11166			
(90)	231	= the number of inches in the old wine gallon.			
(91)	$\sqrt{231}$	= $15-19868$ 41535 706636			
(92)	$\sqrt{231}$	= $17-14987$ 85661 54			
(93)	282	= the number of cubic inches in the old beer gallon.			
(94)	$\sqrt{282}$	= $16-79285$ 56237 466			
(95)	$\sqrt{765398, \&c.}$	= $18-94870$ 84418 76			
(96)	Common logarithm of 10 = 1.				
(97)	$k = 2-30258$ 50929 94045 68401 79914 54684 364;				
(98)	$\frac{1}{k}$ = Napier's logarithm of 10 (compared with <i>Wolfgram and Abraham Sharp</i>).				
(99)	$\frac{1}{k}$ = common logarithm of <i>Wolfgram and Abraham Sharp</i> = common logarithm of Napier's logarithm of this number = 1.				

(compared with the same in the *Life of Napier* by David Stewart, Earl of Buchan, and Walter Minto, LL.D., Edition of 1787, page 112.)

2-40218 91373 63903 3
3-84509 80400 11256 8
2-44290 89026 50353 6
1-22145 44513 25176 8
1-27390 95106 42091 0
7-55709 10973 49646 4-10
2-38361 19798 92144 3
1-18180 59899 46072 2
1-23426 10492 62986 4
2-45024 91083 19361 1
1-22512 45541 59680 5
1-27757 96134 76594 8

NUMBERS.	AUTHORITIES.
(100) 0.513074 of the toise of Peru = 1 metre	{(Commission des poids et mesures le 11 Floréal, an iii. p. 432.)
(101) 1 metre = 39.370069 English inches.....	{(Delambre Base du Systeme Métrique. Tom. iii. p. 432.)
(102) 76.733744 (= 39370069 ÷ 513074) English inches = toise of Peru.....	{(Report by Francis Baily, Esq., in Mem. of the Roy. Astron. Society of London. Vol. ix. p. 133.)
(103) Equatorial radius of the earth = 3272077.14 toises = 251078730 English inches = 20923227.5 feet English whence 7925.465 English geographical miles = equatorial diameter.	(computed from the preceding.)
(104) Polar radius of the earth = 3261189.33 toises = 250239431 English inches = 20855226.9 English feet whence 7898.972 English geogra- phical miles = polar diameter.	{(Toise Measure, by Bessel, Astronomische Nach- richten, 1841, No. 438, p. 116.)
(105) 299.153 polar radii = 298.153 equatorial radii.	{(Toise measure, by Bessel, as above.)
(106) 1901306 cubic inches of water at the temperature of 62° Fahrenheit weighs 1 oz. troy, or 480 gra.	(1901306 = 480 + 252.458.)
(107) 1.732962 cubic inches of water weighs 1 oz avoirdupois, or 437.75 gra. at 62° of Fahrenheit.	(1.732962 = 437.5 + 252.458.)
(108) 1 cubic foot of air weighs 528.367 grains.	(deduced from No. 101.)
(109) 100 of a square metre = 1 are = 1076.391602686 square feet = 119.599066965 sq. yards.	(deduced from No. 101.)
(110) 1 stere = cubic metre = 35.314694 cubic feet = 1.30795164187 cubic yards.	(deduced from No. 101.)
(111) 1 litre = $\frac{1}{1000}$ of a cubic metre = 61.0237918 cubic inches } = 2200.482 imperial gallon = 1.76067936 imperial pints	{(cubic inches, deduced from No. 101; gallons = [61.023, &c. x 252.458] + 70000.)
(112) 1 gramme = 15.434 grains	{(from weighings made at the Mint, London, and the Bureau des Poids et Mesures, Paris.)
(113) 1 metrical quintal = 100000 grammes = 1543400 grains = 220 $\frac{1}{2}$ lbs. avoirdupois }	(deduced from No. 112.)
(114) 1 franc = 9.32 pence.....	{(by relation value of gold and copper, London and Paris Mints.)

THE FOLLOWING QUANTITIES VARY FROM TIME TO TIME.

1 Julian year the unit of time; and t = the number of Julian years from the commencement of 1800.

(115) Precession	$\left\{ \begin{aligned} &= * 50'' 2235 t - 0'' 0001221805 t^2 \pm 0'' 000000000215 t^3 \text{ (Bessel, Astron. Nachr. 92, Feb. 1826).} \\ &= 1296027'' 565891 \pm 0'' 000244361 t + 0'' 000000000645 t^2. \end{aligned} \right.$
(116) Relative velocity to the equinoxes	$\left\{ \begin{aligned} &= 473364000 \text{ divided by the preceding velocity.} \\ &= 365 24223 \text{ 11650 05 days } \pm t \times 0 00000 \text{ 00688 67 days.} \end{aligned} \right.$
(117) Equinoctial or tropical year.	$\left\{ \begin{aligned} &= 365d. 5h. 48m. 48'' 7727s. \pm t \times 0 00595s. \\ &= 61'' 5171 \pm 0'' 000407393 t \text{ (Bessel, Ast. Nach. Nos. 83 and 92).} \end{aligned} \right.$
(118) Tropical velocity of solar perigee	$\left\{ \begin{aligned} &= 61'' 5171 \pm 0'' 000407393 t \text{ (Bessel, Ast. Nach. Nos. 83 and 92).} \\ &= 1295966'' 049291 \pm 0'' 000163232 t. \end{aligned} \right.$
(119) Velocity of mean anomaly of the sun	$\left\{ \begin{aligned} &= 473364000 \pm \text{by the preceding velocity.} \\ &= 365 24223 11650 05 days } \pm t \times 0 00000 \text{ 00460 06 days.} \end{aligned} \right.$
(120) Anomalistic year	$\left\{ \begin{aligned} &= 365d. 6h. 18m. 46'' 7214s. \pm t \times 0 00397 49s. \\ &= 17325593'' 3925 \pm 0'' 217572 t + 0'' 00047943 t^2 \text{ (Damoiseau [Tables de la Lune, 2nd edit. fol. 1925] and Bessel).} \end{aligned} \right.$
(121) Sidereal lunar velocity	$\left\{ \begin{aligned} &= 473364000 \pm \text{by the preceding velocity.} \\ &= 27 32166 16177 24 days } \pm t \times 0 00000 \text{ 03431 01 days.} \end{aligned} \right.$
(122) Sidereal month	$\left\{ \begin{aligned} &= 27d. 7h. 43m. 11 5688s. \pm t \times 0 02964s. \\ &= 17325643'' 616 \pm 0'' 217816 t + 0'' 00047943 t^2 \text{ (Damoiseau).} \end{aligned} \right.$
(123) Tropical lunar velocity	$\left\{ \begin{aligned} &= 473364000 \pm \text{by the preceding velocity.} \\ &= 27 32166 16177 24 days } \pm t \times 0 00000 \text{ 03431 01 days.} \end{aligned} \right.$
(124) Tropical month	$\left\{ \begin{aligned} &= 27d. 7h. 43m. 4'' 7209s. \pm t \times 0 02968s. \\ &= 16029616'' 049609 \pm 0'' 317572 t. \end{aligned} \right.$
(125) Synodic lunar velocity	$\left\{ \begin{aligned} &= 473364000 \pm \text{by the preceding velocity.} \\ &= 29 53056 57885 91 days } \pm t \times 0 00000 \text{ 04008 21 days.} \end{aligned} \right.$
(126) Synodic month	$\left\{ \begin{aligned} &= 29d. 12h. 44m. 2 8713s. \pm t \times 0 03463s. \end{aligned} \right.$

* When two signs occur, the upper is for time *a/ter* the beginning of the year 1800, and the lower for time *before* 1800.

QUANTITIES VARYING FROM TIME TO TIME—continued.

(127) Velocity of moon's mean anomaly	= $17179157''.95 \pm 1''.008406 t + 0''.00273105 t^2$ (<i>Damoiseau</i>).
(128) Anomalistic month	$\left\{ \begin{array}{l} = 27.55455 \text{ 19389 } 09 \text{ days } \pm t \times 0.00000 \text{ 16174 } 36 \text{ days.} \\ = 27d. 13h. 18m. 33.2875s. \pm t \times 0.139746s. \end{array} \right.$
(129) Velocity of moon from node	= $17395273''.591 \pm 0''.086308 t + 0''.12393 t^2$ (<i>Damoiseau</i>).
(130) Draconic month	$\left\{ \begin{array}{l} = 473864000 + \text{by the preceding velocity.} \\ = 27.21221 \text{ 93148 } 44 \text{ days } \pm t \times 0.00000 \text{ 01350 } 16 \text{ days.} \\ = 27d. 5h. 5m. 35.7488s. \pm t \times 0.01166 \text{ } s. \end{array} \right.$
(131) Sidereal velocity of lunar perigee	= $146435''.4425 \pm 0''.790834 t$ (<i>Damoiseau and Besse!</i>).
(132) Sidereal period of lunar perigee	$\left\{ \begin{array}{l} = 473864000 + \text{by the preceding velocity.} \\ = 3232.57806 \text{ 93427 } 41 \text{ days } \pm t \times 0.01745 \text{ 74577 } 48 \text{ days.} \\ = 3232d. 13h. 52m. 25.1912s. \pm t \times 25m. 8.3494s. \end{array} \right.$
(133) Tropical velocity of lunar perigee	= $146485''.666 \pm 0''.79059 t - 0''.00225162 t^2$ (<i>Damoiseau</i>).
(134) Tropical period of the lunar perigee	$\left\{ \begin{array}{l} = 473864000 + \text{by the preceding velocity.} \\ = 3231.46976 \text{ 03245 } 36 \text{ days } \pm t \times 0.01744 \text{ 03936 } 41 \text{ days.} \\ = 3231d. 11h. 16m. 27.29204s. \pm t \times 25m. 6.35s. \end{array} \right.$
(135) Sidereal velocity of the moon's node	= $-69680''.1985 \pm 0''.131264 t$ (<i>Damoiseau and Besse!</i>).
(136) Sidereal period of the node	$\left\{ \begin{array}{l} = 473864000 + \text{by the preceding velocity.} \\ = 6793.37904 \text{ 01147 } 16 \text{ days } \pm t \times 0.01279 \text{ 74105 } \text{ days.} \\ = 6793d. 9h. 5m. 49.0659s. \pm t \times 18m. 25.65027 s. \end{array} \right.$
(137) Tropical velocity of the moon's node	= $-69629''.975 \pm 0''.131508 t + 0''.0003555 t^2$ (<i>Damoiseau</i>).
(138) Tropical period of the node	$\left\{ \begin{array}{l} = 6798.27904 \text{ 57701 } 59 \text{ days } \pm t \times 0.01283 \text{ 97073 } \text{ days.} \\ = 6798d. 6h. 41m. 49.5545s. \pm t \times 18m. 29.2907s. \end{array} \right.$
(139) Inequality of long period (240 years) in the sun's longitude, discovered by G. B. Airy, Esq., Astronomer Royal (Phil. Trans. of Roy. Soc. of London, 1832, Part II. p. 67)	= $(29049 - 0''.0002076 t) \sin (258^\circ 43' 37'' + t \times 1^\circ 30' 15''.118)$. (<i>Airy, Besse, Lindensau.</i>)

APPENDIX.

TABLE I.

CONTAINING THE

CIRCUMFERENCES AND AREAS OF CIRCLES,

SIDES OF EQUAL SQUARES

SQUARES AND CUBES OF THE DIAMETERS,

FROM $\frac{1}{8}$ th TO 100 INCHES, FEET, YARDS, CHAINS, MILES, &c.,

ADVANCING BY AN $\frac{1}{8}$ th, AND ALSO THE SIDE OF EQUAL

SQUARE, ADVANCING AT AN EQUAL RATIO.

2] CIRCUMFERENCES AND AREAS OF CIRCLES, SIDES OF

TABLE I.

Dia. or Root.	Circum.	Area.	Side of = square.	Square.	Cube.
$\frac{1}{8}$	0.3927	0.0123	0.1108	0.0156	0.00195
$\frac{1}{4}$	0.7854	0.0491	0.2216	0.0625	0.01563
$\frac{3}{8}$	1.1781	0.1104	0.3323	0.1406	0.05273
$\frac{1}{2}$	1.5708	0.1963	0.4431	0.25	0.125
$\frac{5}{8}$	1.9635	0.3068	0.5539	0.3906	0.24414
$\frac{3}{4}$	2.3562	0.4418	0.6647	0.5625	0.42188
$\frac{7}{8}$	2.7489	0.6013	0.7754	0.7656	0.66992
1 in.	3.1416	0.7854	0.8862	1.	1.
$1\frac{1}{8}$	3.5343	0.9940	0.9970	1.2656	1.42383
$1\frac{1}{4}$	3.9270	1.2272	1.1078	1.5625	1.95313
$1\frac{3}{8}$	4.3197	1.4849	1.2186	1.8906	2.59961
$1\frac{1}{2}$	4.7124	1.7671	1.3293	2.25	3.375
$1\frac{5}{8}$	5.1051	2.0739	1.4401	2.6406	4.29102
$1\frac{3}{4}$	5.4978	2.4053	1.5509	3.0625	5.35938
$1\frac{7}{8}$	5.8905	2.7612	1.6617	3.5156	6.59180
2 in.	6.2832	3.1416	1.7725	4.	8.
$2\frac{1}{8}$	6.6759	3.5466	1.8832	4.5156	9.5957
$2\frac{1}{4}$	7.0686	3.9761	1.9940	5.0625	11.3906
$2\frac{3}{8}$	7.4613	4.4301	2.1048	5.6406	13.3965
$2\frac{1}{2}$	7.8540	4.9087	2.2156	6.25	15.625
$2\frac{5}{8}$	8.2467	5.4119	2.3263	6.8906	18.0879
$2\frac{3}{4}$	8.6394	5.9396	2.4371	7.5625	20.7969
$2\frac{7}{8}$	9.0321	6.4918	2.5479	8.2656	23.7637
3 in.	9.4248	7.0686	2.6587	9.	27.
$3\frac{1}{8}$	9.8175	7.6699	2.7695	9.7656	30.5176
$3\frac{1}{4}$	10.2102	8.2958	2.8802	10.5625	34.3281
$3\frac{3}{8}$	10.6029	8.9462	2.9910	11.3906	38.4434
$3\frac{1}{2}$	10.9956	9.6211	3.1018	12.25	42.875
$3\frac{5}{8}$	11.3883	10.3206	3.2126	13.1406	47.6348
$3\frac{3}{4}$	11.7810	11.0447	3.3234	14.0625	52.7344
$3\frac{7}{8}$	12.1737	11.7932	3.4341	15.0156	58.1855
4 in.	12.5664	12.5664	3.5449	16.	64.
$4\frac{1}{8}$	12.9591	13.3640	3.6557	17.0156	70.1895
$4\frac{1}{4}$	13.3518	14.1863	3.7665	18.0625	76.7656
$4\frac{3}{8}$	13.7445	15.0390	3.8772	19.1406	83.7402
$4\frac{1}{2}$	14.1372	15.9043	3.9880	20.25	91.125
$4\frac{5}{8}$	14.5299	16.8002	4.0988	21.3906	98.9316
$4\frac{3}{4}$	14.9226	17.7205	4.2096	22.5625	107.1719
$4\frac{7}{8}$	15.3153	18.6655	4.3204	23.7656	115.8574
5 in.	15.7080	19.6350	4.4311	25.	125.
$5\frac{1}{8}$	16.1007	20.6290	4.5419	26.2656	134.6113
$5\frac{1}{4}$	16.4934	21.6475	4.6527	27.5625	144.7031
$5\frac{3}{8}$	16.8861	22.6906	4.7635	28.8906	155.2871
$5\frac{1}{2}$	17.2788	23.7583	4.8742	30.25	166.375
$5\frac{5}{8}$	17.6715	24.8505	4.9850	31.6406	177.9785
$5\frac{3}{4}$	18.0642	25.9672	5.0958	33.0625	190.1094
$5\frac{7}{8}$	18.4569	27.1085	5.2066	34.5156	202.7793

EQUAL SQUARES, SQUARES AND CUBES OF THE DIAMETERS. [3

TABLE I.

Dia. or Root.	Circum.	Area.	Side of = square.	Square.	Cube.
6 in.	18.8496	28.2743	5.3174	36.	216.
$\frac{1}{2}$	19.2423	29.4647	5.4281	37.5156	229.7832
$\frac{1}{4}$	19.6350	30.6796	5.5389	39.0625	244.1406
$\frac{3}{4}$	20.0277	31.9191	5.6497	40.6406	259.0840
$\frac{1}{2}$	20.4204	33.1831	5.7605	42.25	274.625
$\frac{3}{4}$	20.8131	34.4716	5.8713	43.8906	290.7754
$\frac{1}{2}$	21.2058	35.7847	5.9820	45.5625	307.5469
$\frac{3}{4}$	21.5985	37.1223	6.0928	47.2656	324.9512
7 in.	21.9912	38.4846	6.2036	49.	343.
$\frac{1}{2}$	22.3839	39.8712	6.3144	50.7656	361.7051
$\frac{1}{4}$	22.7766	41.2825	6.4251	52.5625	381.0781
$\frac{3}{4}$	23.1693	42.7183	6.5359	54.3906	401.1309
$\frac{1}{2}$	23.5620	44.1786	6.6467	56.25	421.875
$\frac{3}{4}$	23.9547	45.6635	6.7575	58.1406	443.3223
$\frac{1}{2}$	24.3474	47.1730	6.8683	60.0625	465.4844
$\frac{3}{4}$	24.7401	48.7070	6.9790	62.0156	488.3730
8 in.	25.1328	50.2655	7.0898	64.	512.
$\frac{1}{2}$	25.5255	51.8486	7.2006	66.0156	536.3770
$\frac{1}{4}$	25.9182	53.4562	7.3114	68.0625	561.6156
$\frac{3}{4}$	26.3109	55.0883	7.4222	70.1406	587.4277
$\frac{1}{2}$	26.7036	56.7450	7.5329	72.25	614.125
$\frac{3}{4}$	27.0963	58.4263	7.6437	74.3906	641.6191
$\frac{1}{2}$	27.4890	60.1320	7.7545	76.5625	669.9219
$\frac{3}{4}$	27.8817	61.8624	7.8653	78.7656	699.0449
9 in.	28.2743	63.6173	7.9760	81.	729.
$\frac{1}{2}$	28.6670	65.3967	8.0868	83.2656	759.7988
$\frac{1}{4}$	29.0597	67.2006	8.1976	85.5625	791.4531
$\frac{3}{4}$	29.4524	69.0291	8.3084	87.8906	823.9746
$\frac{1}{2}$	29.8451	70.8822	8.4192	90.25	857.375
$\frac{3}{4}$	30.2378	72.7598	8.5299	92.6406	891.6660
$\frac{1}{2}$	30.6305	74.6619	8.6407	95.0625	926.8594
$\frac{3}{4}$	31.0232	76.5886	8.7515	97.5156	962.9668
10 in.	31.4159	78.5398	8.8623	100.	1000.
$\frac{1}{2}$	31.8086	80.5156	8.9730	102.5156	1037.9707
$\frac{1}{4}$	32.2013	82.5159	9.0838	105.0625	1076.8906
$\frac{3}{4}$	32.5940	84.5407	9.1946	107.6406	1116.7715
$\frac{1}{2}$	32.9867	86.5901	9.3054	110.25	1157.625
$\frac{3}{4}$	33.3794	88.664	9.4162	112.8906	1199.4629
$\frac{1}{2}$	33.7721	90.7626	9.5269	115.5625	1242.2969
$\frac{3}{4}$	34.1648	92.8856	9.6377	118.2656	1286.1387
11 in.	34.5575	95.0332	9.7485	121.	1331.
$\frac{1}{2}$	34.9502	97.2053	9.8593	123.7656	1376.8926
$\frac{1}{4}$	35.3429	99.4020	9.9701	126.5625	1423.8281
$\frac{3}{4}$	35.7356	101.6232	10.0808	129.3906	1471.8184
$\frac{1}{2}$	36.1283	103.8689	10.1916	132.25	1520.875
$\frac{3}{4}$	36.5210	106.1392	10.3024	135.1406	1571.0098
$\frac{1}{2}$	36.9137	108.4340	10.4132	138.0625	1622.2344
$\frac{3}{4}$	37.3064	110.7534	10.5239	141.0156	1674.5605

TABLE I.

Dia. or Root.	Circum.	Area.	Side of = square.	Square.	Cube.
12 in.	37.6991	113.0973	10.6347	144.	1728.
$\frac{1}{2}$	38.0918	115.4658	10.7455	147.0156	1782.5645
$\frac{1}{4}$	38.4845	117.8588	10.8563	150.0625	1838.2656
$\frac{3}{4}$	38.8772	120.2764	10.9671	153.1406	1895.1152
$\frac{1}{2}$	39.2699	122.7185	11.0778	156.25	1953.1250
$\frac{3}{4}$	39.6626	125.1851	11.1886	159.3906	2012.3066
$\frac{1}{2}$	40.0553	127.6763	11.2994	162.5625	2072.6719
$\frac{3}{4}$	40.4480	130.1920	11.4102	165.7656	2134.2324
13 in.	40.8407	132.7323	11.5210	169.	2197.
$\frac{1}{2}$	41.2334	135.2971	11.6317	172.2656	2260.9863
$\frac{1}{4}$	41.6261	137.8865	11.7425	175.5625	2326.2031
$\frac{3}{4}$	42.0188	140.5004	11.8533	178.8906	2392.6621
$\frac{1}{2}$	42.4115	143.1388	11.9641	182.25	2460.3750
$\frac{3}{4}$	42.8042	145.8018	12.0748	185.6406	2529.3535
$\frac{1}{2}$	43.1969	148.4893	12.1856	189.0625	2599.6094
$\frac{3}{4}$	43.5896	151.2014	12.2964	192.5156	2671.1543
14 in.	43.9823	153.9380	12.4072	196.	2744.
$\frac{1}{2}$	44.3750	156.6992	12.5180	199.5156	2818.1582
$\frac{1}{4}$	44.7677	159.4849	12.6287	203.0625	2893.6406
$\frac{3}{4}$	45.1504	162.2952	12.7395	206.6406	2970.4590
$\frac{1}{2}$	45.5531	165.1300	12.8503	210.25	3048.6250
$\frac{3}{4}$	45.9458	167.9893	12.9611	213.8906	3128.1504
$\frac{1}{2}$	46.3385	170.8732	13.0718	217.5625	3209.0469
$\frac{3}{4}$	46.7312	173.7816	13.1826	221.2656	3291.3262
15 in.	47.1239	176.7146	13.2934	225.	3375.
$\frac{1}{2}$	47.5166	179.6721	13.4042	228.7656	3460.0801
$\frac{1}{4}$	47.9093	182.6542	13.5150	232.5625	3546.5781
$\frac{3}{4}$	48.3020	185.6608	13.6257	236.3906	3634.5059
$\frac{1}{2}$	48.6947	188.6919	13.7365	240.25	3723.8750
$\frac{3}{4}$	49.0874	191.7476	13.8473	244.1406	3814.6973
$\frac{1}{2}$	49.4801	194.8278	13.9581	248.0625	3906.9844
$\frac{3}{4}$	49.8728	197.9326	14.0689	252.0156	4000.7480
16 in.	50.2655	201.0619	14.1796	256.	4096.
$\frac{1}{2}$	50.6582	204.2158	14.2904	260.0156	4192.7520
$\frac{1}{4}$	51.0509	207.3942	14.4012	264.0625	4291.0156
$\frac{3}{4}$	51.4436	210.5972	14.5120	268.1406	4390.8027
$\frac{1}{2}$	51.8363	213.8246	14.6227	272.25	4492.1250
$\frac{3}{4}$	52.2290	217.0767	14.7335	276.3906	4594.9941
$\frac{1}{2}$	52.6217	220.3533	14.8443	280.5625	4699.4219
$\frac{3}{4}$	53.0144	223.6544	14.9551	284.7656	4805.4199
17 in.	53.4071	226.9801	15.0659	289.	4913.
$\frac{1}{2}$	53.7998	230.3303	15.1766	293.2656	5022.1738
$\frac{1}{4}$	54.1925	233.7050	15.2874	297.5625	5132.9531
$\frac{3}{4}$	54.5852	237.1044	15.3982	301.8906	5245.3496
$\frac{1}{2}$	54.9779	240.5282	15.5090	306.25	5359.3750
$\frac{3}{4}$	55.3706	243.9766	15.6197	310.6406	5475.0410
$\frac{1}{2}$	55.7633	247.4495	15.7305	315.0625	5592.3594
$\frac{3}{4}$	56.1560	250.9470	15.8413	319.5156	5711.3418

TABLE I.

Dia. or Root.	Circum.	Area.	Side of = square.	Square.	Cube.
18 in.	56.5487	254.4690	15.9521	324.	5832.
$\frac{1}{2}$	56.9414	258.0159	16.0629	328.5156	5954.3457
$\frac{1}{4}$	57.3341	261.5867	16.1736	333.0625	6078.3906
$\frac{3}{4}$	57.7268	265.1824	16.2844	337.6406	6204.1465
$\frac{1}{2}$	58.1195	268.8025	16.3952	342.25	6331.6250
$\frac{3}{4}$	58.5122	272.4473	16.5060	346.8906	6460.8379
$\frac{1}{2}$	58.9049	276.1165	16.6168	351.5625	6591.7969
$\frac{3}{4}$	59.2976	279.8104	16.7275	356.2656	6724.5137
19 in.	59.6903	283.5287	16.8383	361.	6859.
$\frac{1}{2}$	60.0830	287.2717	16.9491	365.7656	6995.2676
$\frac{1}{4}$	60.4757	291.0599	17.0599	370.5625	7133.3281
$\frac{3}{4}$	60.8684	294.8311	17.1706	375.3906	7273.1934
$\frac{1}{2}$	61.2611	298.6477	17.2814	380.25	7414.8750
$\frac{3}{4}$	61.6538	302.4887	17.3922	385.1406	7558.3848
$\frac{1}{2}$	62.0465	306.3544	17.5030	390.0625	7703.7344
$\frac{3}{4}$	62.4392	310.2446	17.6138	395.0156	7850.9355
20 in.	62.8319	314.1593	17.7245	400.	8000.
$\frac{1}{2}$	63.2245	318.0985	17.8353	405.0156	8150.9395
$\frac{1}{4}$	63.6173	322.0623	17.9461	410.0625	8303.7656
$\frac{3}{4}$	64.0100	326.0507	18.0569	415.1406	8458.4902
$\frac{1}{2}$	64.4026	330.0636	18.1677	420.25	8615.1250
$\frac{3}{4}$	64.7953	334.1010	18.2784	425.3906	8773.6816
$\frac{1}{2}$	65.1880	338.1630	18.3892	430.5625	8934.1719
$\frac{3}{4}$	65.5807	342.2495	18.5000	435.7656	9096.6074
21 in.	65.9734	346.3606	18.6108	441.	9261.
$\frac{1}{2}$	66.3661	350.4962	18.7215	446.2656	9427.3613
$\frac{1}{4}$	66.7588	354.6564	18.8323	451.5625	9595.7031
$\frac{3}{4}$	67.1515	358.8411	18.9431	456.8906	9766.0371
$\frac{1}{2}$	67.5442	363.0503	19.0539	462.25	9938.3750
$\frac{3}{4}$	67.9369	367.2841	19.1647	467.6406	10112.7285
$\frac{1}{2}$	68.3296	371.5424	19.2754	473.0625	10289.1094
$\frac{3}{4}$	68.7223	375.8253	19.3862	478.5156	10467.5293
22 in.	69.1150	380.1327	19.4970	484.	10648.
$\frac{1}{2}$	69.5077	384.4647	19.6078	489.5156	10830.5332
$\frac{1}{4}$	69.9004	388.8212	19.7185	495.0625	11015.1406
$\frac{3}{4}$	70.2931	393.2022	19.8293	500.6406	11201.8340
$\frac{1}{2}$	70.6858	397.6078	19.9401	506.25	11390.6250
$\frac{3}{4}$	71.0785	402.0380	20.0509	511.8906	11581.5254
$\frac{1}{2}$	71.4712	406.4926	20.1617	517.5625	11774.5469
$\frac{3}{4}$	71.8639	410.9719	20.2724	523.2656	11969.7012
23 in.	72.2566	415.4756	20.3832	529.	12167.
$\frac{1}{2}$	72.6493	420.0039	20.4940	534.7656	12366.4551
$\frac{1}{4}$	73.0420	424.5568	20.6048	540.5625	12568.0781
$\frac{3}{4}$	73.4347	429.1342	20.7156	546.3906	12771.8809
$\frac{1}{2}$	73.8274	433.7361	20.8263	552.25	12977.8750
$\frac{3}{4}$	74.2201	438.3626	20.9371	558.1406	13186.0723
$\frac{1}{2}$	74.6128	443.0137	21.0479	564.0625	13396.4844
$\frac{3}{4}$	75.0055	447.6892	21.1587	570.0156	13609.1230

TABLE I.

Dia. or Root.	Circum.	Area.	Side of = square.	Square.	Cube.
24 in.	75.3982	452.3893	21.2694	576	13824
$\frac{1}{2}$	75.7909	457.1140	21.3802	582.0156	14041.1270
$\frac{1}{4}$	76.1836	461.8632	21.4910	588.0625	14260.5156
$\frac{3}{4}$	76.5763	466.6370	21.6018	594.1406	14482.1777
$\frac{1}{2}$	76.9690	471.4352	21.7126	600.25	14706.125
$\frac{3}{4}$	77.3617	476.2581	21.8233	606.3906	14932.3691
$\frac{1}{2}$	77.7544	481.1055	21.9341	612.5625	15160.9219
$\frac{3}{4}$	78.1471	485.9774	22.0449	618.7656	15391.7949
25 in.	78.5398	490.8739	22.1557	625	15625
$\frac{1}{2}$	78.9325	495.7949	22.2665	631.2656	15860.5488
$\frac{1}{4}$	79.3252	500.7404	22.3772	637.5625	16096.4331
$\frac{3}{4}$	79.7179	505.7105	22.4880	643.8906	16338.7246
$\frac{1}{2}$	80.1106	510.7052	22.5988	650.25	16581.375
$\frac{3}{4}$	80.5033	515.7243	22.7096	656.6406	16826.4160
$\frac{1}{2}$	80.8960	520.7681	22.8203	663.0625	17073.8594
$\frac{3}{4}$	81.2887	525.8363	22.9311	669.5156	17323.7168
26 in.	81.6814	530.9292	23.0419	676	17576
$\frac{1}{2}$	82.0741	536.0465	23.1527	682.5156	17830.7207
$\frac{1}{4}$	82.4668	541.1884	23.2635	689.0625	18087.8906
$\frac{3}{4}$	82.8595	546.3549	23.3742	695.6406	18347.5215
$\frac{1}{2}$	83.2522	551.5459	23.4850	702.25	18609.625
$\frac{3}{4}$	83.6449	556.7614	23.5958	708.8906	18874.2129
$\frac{1}{2}$	84.0376	562.0015	23.7066	715.5625	19141.2969
$\frac{3}{4}$	84.4303	567.2661	23.8173	722.2656	19410.8887
27 in.	84.8230	572.5553	23.9281	729	19683
$\frac{1}{2}$	85.2157	577.8690	24.0389	735.7656	19957.6426
$\frac{1}{4}$	85.6084	583.2072	24.1497	742.5625	20234.8281
$\frac{3}{4}$	86.0011	588.5700	24.2605	749.3906	20514.5684
$\frac{1}{2}$	86.3938	593.9574	24.3712	756.25	20796.875
$\frac{3}{4}$	86.7865	599.3692	24.4820	763.1406	21081.7598
$\frac{1}{2}$	87.1792	604.8057	24.5928	770.0625	21369.2344
$\frac{3}{4}$	87.5719	610.2666	24.7036	777.0156	21659.3105
28 in.	87.9646	615.7522	24.8144	784	21952
$\frac{1}{2}$	88.3573	621.2622	24.9251	791.0156	22247.3145
$\frac{1}{4}$	88.7500	626.7968	25.0359	798.0625	22545.2656
$\frac{3}{4}$	89.1427	632.3560	25.1467	805.1406	22845.8652
$\frac{1}{2}$	89.5354	637.9397	25.2575	812.25	23149.125
$\frac{3}{4}$	89.9281	643.5479	25.3682	819.3906	23455.0566
$\frac{1}{2}$	90.3208	649.1807	25.4790	826.5625	23763.6719
$\frac{3}{4}$	90.7135	654.8380	25.5898	833.7656	24074.9824
29 in.	91.1062	660.5199	25.7006	841	24389
$\frac{1}{2}$	91.4989	666.2263	25.8114	848.2656	24705.7363
$\frac{1}{4}$	91.8916	671.9572	25.9221	855.5625	25025.2031
$\frac{3}{4}$	92.2843	677.7127	26.0329	862.8906	25347.4121
$\frac{1}{2}$	92.6770	683.4928	26.1437	870.25	25672.375
$\frac{3}{4}$	93.0697	689.2973	26.2545	877.6406	26000.1035
$\frac{1}{2}$	93.4624	695.1265	26.3653	885.0625	26330.6094
$\frac{3}{4}$	93.8551	700.9801	26.4760	892.5156	26663.9043

EQUAL SQUARES, SQUARES AND CUBES OF THE DIAMETERS. [7

TABLE I.

Dia. or Root.	Circum.	Area.	Side of = square.	Square.	Cube.
30 in.	94.2478	706.8583	26.5868	900.	27000.
$\frac{1}{2}$	94.6405	712.7611	26.6976	907.5156	27338.9082
$\frac{1}{4}$	95.0332	718.6884	26.8084	915.0625	27680.6406
$\frac{3}{4}$	95.4259	724.6403	26.9191	922.6406	28025.2090
1	95.8186	730.6166	27.0299	930.25	28372.625
$\frac{1}{2}$	96.2113	736.6176	27.1407	937.8906	28722.9004
$\frac{1}{4}$	96.6040	742.6431	27.2515	945.5625	29076.0469
$\frac{3}{4}$	96.9967	748.6931	27.3623	953.2656	29432.0762
31 in.	97.3894	754.7676	27.4730	961.	29791.
$\frac{1}{2}$	97.7821	760.8667	27.5838	968.7656	30152.8301
$\frac{1}{4}$	98.1748	766.9904	27.6946	976.5625	30517.5781
$\frac{3}{4}$	98.5675	773.1386	27.8054	984.3906	30885.2559
1	98.9602	779.3113	27.9161	992.25	31255.875
$\frac{1}{2}$	99.3529	785.5086	28.0269	1000.1406	31629.4473
$\frac{1}{4}$	99.7456	791.7304	28.1377	1008.0625	32005.9844
$\frac{3}{4}$	100.1383	797.9768	28.2485	1016.0156	32385.4980
32 in.	100.5310	804.2477	28.3593	1024.	32768.
$\frac{1}{2}$	100.9237	810.5432	28.4700	1032.0156	33153.5020
$\frac{1}{4}$	101.3164	816.8632	28.5808	1040.0625	33542.0156
$\frac{3}{4}$	101.7091	823.2077	28.6916	1048.1406	33933.5527
1	102.1018	829.5768	28.8024	1056.25	34328.125
$\frac{1}{2}$	102.4945	835.9704	28.9132	1064.3906	34725.7441
$\frac{1}{4}$	102.8872	842.3886	29.0239	1072.5625	35126.4219
$\frac{3}{4}$	103.2799	848.8313	29.1347	1080.7656	35530.1699
33 in.	103.6726	855.2986	29.2455	1089.	35937.
$\frac{1}{2}$	104.0653	861.7904	29.3563	1097.2656	36346.9238
$\frac{1}{4}$	104.4580	868.3068	29.4670	1105.5625	36759.9531
$\frac{3}{4}$	104.8507	874.8477	29.5778	1113.8906	37176.0996
1	105.2434	881.4131	29.6886	1122.25	37595.375
$\frac{1}{2}$	105.6361	888.0031	29.7994	1130.6406	38017.7910
$\frac{1}{4}$	106.0288	894.6176	29.9102	1139.0625	38443.3594
$\frac{3}{4}$	106.4215	901.2576	30.0209	1147.5156	38872.0918
34 in.	106.8142	907.9203	30.1317	1156.	39304.
$\frac{1}{2}$	107.2068	914.6084	30.2425	1164.5156	39739.0957
$\frac{1}{4}$	107.5995	921.3211	30.3533	1173.0625	40177.3906
$\frac{3}{4}$	107.9922	928.0584	30.4641	1181.6406	40618.8965
1	108.3849	934.8202	30.5748	1190.25	41063.625
$\frac{1}{2}$	108.7776	941.6065	30.6856	1198.8906	41511.5879
$\frac{1}{4}$	109.1703	948.4174	30.7964	1207.5625	41962.7969
$\frac{3}{4}$	109.5630	955.2528	30.9072	1216.2656	42417.2636
35 in.	109.9557	962.1128	31.0179	1225.	42875.
$\frac{1}{2}$	110.3484	968.9973	31.1287	1233.7656	43336.0176
$\frac{1}{4}$	110.7411	975.9063	31.2395	1242.5625	43800.3281
$\frac{3}{4}$	111.1338	982.8399	31.3503	1251.3906	44267.9433
1	111.5265	989.7980	31.4611	1260.25	44738.875
$\frac{1}{2}$	111.9192	996.7807	31.5718	1269.1406	45213.1348
$\frac{1}{4}$	112.3119	1003.7879	31.6826	1278.0625	45690.7344
$\frac{3}{4}$	112.7046	1010.8197	31.7934	1287.0156	46171.6855

TABLE I.

Dia. or Root.	Circum.	Area.	Side of = square.	Square.	Cube.
36 in.	113.0973	1017.8760	31.9042	1296.	46656.
$\frac{1}{2}$	113.4900	1024.9569	32.0149	1305.0156	47143.6895
$\frac{1}{4}$	113.8827	1032.0623	32.1257	1314.0625	47634.7656
$\frac{3}{4}$	114.2754	1039.1922	32.2365	1323.1406	48129.2402
$\frac{1}{2}$	114.6681	1046.3467	32.3473	1332.25	48627.125
$\frac{3}{4}$	115.0608	1053.5257	32.4581	1341.3906	49128.4316
$\frac{1}{2}$	115.4535	1060.7293	32.5688	1350.5625	49653.1719
$\frac{3}{4}$	115.8462	1067.9574	32.6796	1359.7656	50141.3574
37 in.	116.2389	1075.2101	32.7904	1369.	50653.
$\frac{1}{2}$	116.6316	1082.4873	32.9012	1378.2656	51168.1113
$\frac{1}{4}$	117.0243	1089.7890	33.0120	1387.5625	51686.7031
$\frac{3}{4}$	117.4170	1097.1153	33.0227	1396.8906	52208.7871
$\frac{1}{2}$	117.8097	1104.4662	33.2335	1406.25	52734.375
$\frac{3}{4}$	118.2024	1111.8415	33.3443	1415.6406	53263.4785
$\frac{1}{2}$	118.5951	1119.2415	33.4551	1425.0625	53796.1094
$\frac{3}{4}$	118.9878	1126.6659	33.5658	1434.5156	54332.2793
38 in.	119.3805	1134.1149	33.6766	1444.	54872.
$\frac{1}{2}$	119.7732	1141.5885	33.7874	1453.5156	55415.2832
$\frac{1}{4}$	120.1659	1149.0866	33.8982	1463.0625	55962.1406
$\frac{3}{4}$	120.5586	1156.6092	34.0090	1472.6406	56512.5839
$\frac{1}{2}$	120.9513	1164.1564	34.1197	1482.25	57066.625
$\frac{3}{4}$	121.3440	1171.7282	34.2305	1491.8906	57624.2754
$\frac{1}{2}$	121.7367	1179.3244	34.3413	1501.5625	58185.5469
$\frac{3}{4}$	122.1294	1186.9452	34.4521	1511.2656	58750.4512
39 in.	122.5221	1194.5906	34.5629	1521.	59319.
$\frac{1}{2}$	122.9148	1202.2605	34.6736	1530.7656	59891.2051
$\frac{1}{4}$	123.3075	1209.9550	34.7844	1540.5625	60467.0781
$\frac{3}{4}$	123.7002	1217.6739	34.8952	1550.3906	61046.6309
$\frac{1}{2}$	124.0929	1225.4175	35.0060	1560.25	61629.875
$\frac{3}{4}$	124.4856	1233.1855	35.1167	1570.1406	62216.8223
$\frac{1}{2}$	124.8783	1240.9782	35.2275	1580.0625	62807.4844
$\frac{3}{4}$	125.2710	1248.7954	35.3383	1590.0156	63401.8730
40 in.	125.6637	1256.6370	35.4491	1600.	64000.
$\frac{1}{2}$	126.0564	1264.5033	35.5599	1610.0156	64601.8770
$\frac{1}{4}$	126.4491	1272.3941	35.6706	1620.0625	65207.5156
$\frac{3}{4}$	126.8418	1280.3095	35.7814	1630.1406	65816.9277
$\frac{1}{2}$	127.2345	1288.2493	35.8922	1640.25	66430.125
$\frac{3}{4}$	127.6272	1296.2138	36.0030	1650.3906	67047.1191
$\frac{1}{2}$	128.0199	1304.2027	36.1137	1660.5625	67667.9219
$\frac{3}{4}$	128.4126	1312.2163	36.2245	1670.7656	68292.5449
41 in.	128.8053	1320.2543	36.3353	1681.	68921.
$\frac{1}{2}$	129.1980	1328.3169	36.4461	1691.2656	69553.2988
$\frac{1}{4}$	129.5907	1336.4041	36.5569	1701.5625	70189.4531
$\frac{3}{4}$	129.9834	1344.5158	36.6676	1711.8906	70829.4746
$\frac{1}{2}$	130.3761	1352.6520	36.7784	1722.25	71473.375
$\frac{3}{4}$	130.7688	1360.8128	36.8892	1732.6406	72121.1660
$\frac{1}{2}$	131.1615	1368.9981	37.0000	1743.0625	72772.8594
$\frac{3}{4}$	131.5542	1377.2080	37.1108	1753.5156	73428.4668

TABLE I.

Dia. or Root.	Circum.	Area.	Side of = square.	Square.	Cube.
42 in.	131.9469	1385.4424	37.2215	1764.	74088.
1/4	132.3396	1393.7013	37.3323	1774.5156	74751.4707
1/2	132.7323	1401.9848	37.4431	1785.0625	75418.8906
3/4	133.1250	1410.2928	37.5539	1795.6406	76090.2715
1	133.5177	1418.6254	37.6646	1806.25	76765.625
1 1/4	133.9104	1426.9826	37.7754	1816.8906	77444.9629
1 1/2	134.3031	1435.3642	37.8862	1827.5625	78128.2969
1 3/4	134.6958	1443.7704	37.9970	1838.2656	78815.6387
43 in.	135.0885	1452.2012	38.1078	1849.	79507.
1/4	135.4812	1460.6565	38.2185	1859.7656	80202.3926
1/2	135.8739	1469.1364	38.3293	1870.5625	80901.8281
3/4	136.2666	1477.6407	38.4401	1881.3906	81605.3183
1	136.6593	1486.1697	38.5509	1892.25	82312.875
1 1/4	137.0520	1494.7232	38.6616	1903.1406	83024.5097
1 1/2	137.4447	1503.3012	38.7724	1914.0625	83740.2344
1 3/4	137.8374	1511.9037	38.8832	1925.0156	84460.0605
44 in.	138.2301	1520.5308	38.9940	1936.	85184.
1/4	138.6228	1529.1825	39.1048	1947.0156	85912.0644
1/2	139.0155	1537.8587	39.2155	1958.0625	86644.2656
3/4	139.4082	1546.5594	39.3263	1969.1406	87380.6152
1	139.8009	1555.2847	39.4371	1980.25	88121.125
1 1/4	140.1936	1564.0345	39.5479	1991.3906	88865.8066
1 1/2	140.5863	1572.8089	39.6587	2002.5625	89614.6719
1 3/4	140.9790	1581.6078	39.7694	2013.7656	90367.7324
45 in.	141.3717	1590.4313	39.8802	2025.	91125.
1/4	141.7644	1599.2793	39.9910	2036.2656	91886.4863
1/2	142.1571	1608.1518	40.1018	2047.5625	92652.2031
3/4	142.5498	1617.0489	40.2125	2058.8906	93422.1621
1	142.9425	1625.9705	40.3233	2070.25	94196.375
1 1/4	143.3352	1634.9167	40.4341	2081.6406	94974.8535
1 1/2	143.7279	1643.8874	40.5449	2093.0625	95757.6094
1 3/4	144.1206	1652.8827	40.6557	2104.5156	96544.6543
46 in.	144.5133	1661.9025	40.7664	2116.	97336.
1/4	144.9060	1670.9469	40.8772	2127.5156	98131.6582
1/2	145.2987	1680.0158	40.9880	2139.0625	98931.6406
3/4	145.6914	1689.1092	41.0988	2150.6406	99735.9589
1	146.0841	1698.2272	41.2096	2162.25	100544.625
1 1/4	146.4768	1707.3697	41.3203	2173.8906	101357.6504
1 1/2	146.8695	1716.5368	41.4311	2185.5625	102175.0469
1 3/4	147.2622	1725.7284	41.5419	2197.2656	102996.8261
47 in.	147.6549	1734.9445	41.6527	2209.	103823.
1/4	148.0476	1744.1852	41.7634	2220.7656	104653.5800
1/2	148.4403	1753.4505	41.8742	2232.5625	105488.5781
3/4	148.8329	1762.7403	41.9850	2244.3906	106328.0058
1	149.2257	1772.0546	42.0958	2256.25	107171.875
1 1/4	149.6183	1781.3935	42.2066	2268.1406	108020.1973
1 1/2	150.0110	1790.7569	42.3173	2280.0625	108872.9844
1 3/4	150.4037	1800.1449	42.4281	2292.0156	109730.2480

TABLE I.

Dia. or Root.	Circum.	Area.	Side of = square.	Square.	Cube.
48 in.	150.7964	1809.5574	42.5389	2304.	110692.
1/2	151.1891	1818.9944	42.6497	2316.0156	111458.2520
1/4	151.5818	1828.4560	42.7604	2328.0625	112329.0156
3/4	151.9745	1837.9421	42.8712	2340.1406	113204.3027
1	152.3672	1847.4528	42.9820	2352.25	114084.125
1 1/4	152.7599	1856.9881	43.0928	2364.3906	114968.4941
1 1/2	153.1526	1866.5478	43.2036	2376.5625	115857.4219
1 3/4	153.5453	1876.1321	43.3143	2388.7656	116750.9199
49 in.	153.9380	1885.7410	43.4251	2401.	117649.
1/2	154.3307	1895.3744	43.5359	2413.2656	118551.6738
1/4	154.7234	1905.0323	43.6467	2425.5625	119458.9531
3/4	155.1161	1914.7148	43.7575	2437.8906	120370.8496
1	155.5088	1924.4218	43.8682	2450.25	121287.375
1 1/4	155.9015	1934.1534	43.9790	2462.6406	122208.5410
1 1/2	156.2942	1943.9095	44.0898	2475.0625	123134.3594
1 3/4	156.6869	1953.6902	44.2006	2487.5156	124064.8418
50 in.	157.0796	1963.4954	44.3113	2500.	125000.
1/2	157.4723	1973.3252	44.4221	2512.5156	125939.8457
1/4	157.8650	1983.1794	44.5329	2525.0625	126884.3906
3/4	158.2577	1993.0583	44.6437	2537.6406	127833.645
1	158.6504	2002.9617	44.7545	2550.25	128787.625
1 1/4	159.0431	2012.8896	44.8652	2562.8906	129746.3379
1 1/2	159.4358	2022.8421	44.9760	2575.5625	130701.7969
1 3/4	159.8285	2032.8191	45.0868	2588.2656	131678.0136
51 in.	160.2212	2042.8206	45.1976	2601.	132651.
1/2	160.6139	2052.8467	45.3084	2613.7656	133628.7676
1/4	161.0066	2062.8974	45.4191	2626.5625	134611.3281
3/4	161.3993	2072.9725	45.5299	2639.3906	135598.6933
1	161.7920	2083.0723	45.6407	2652.25	136590.875
1 1/4	162.1847	2093.1966	45.7515	2665.1406	137587.8847
1 1/2	162.5774	2103.3454	45.8622	2678.0625	138589.7344
1 3/4	162.9701	2113.5187	45.9730	2691.0156	139596.4355
52 in.	163.3628	2123.7166	46.0838	2704.	140608.
1/2	163.7555	2133.9391	46.1946	2717.0156	141624.435
1/4	164.1482	2144.1861	46.3054	2730.0625	142645.7656
3/4	164.5409	2154.4576	46.4161	2743.1406	143671.9902
1	164.9336	2164.7537	46.5269	2756.25	144703.125
1 1/4	165.3263	2175.0743	46.6377	2769.3906	145739.1816
1 1/2	165.7190	2185.4195	46.7485	2782.5625	146780.1719
1 3/4	166.1117	2195.7892	46.8592	2795.7656	147826.1074
53 in.	166.5044	2206.1834	46.9700	2809.	148877.
1/2	166.8971	2216.6022	47.0808	2822.2656	149932.8613
1/4	167.2898	2227.0456	47.1916	2835.5625	150993.7031
3/4	167.6825	2237.5135	47.3024	2848.8906	152059.5371
1	168.0752	2248.0059	47.4131	2862.25	153130.375
1 1/4	168.4679	2258.5229	47.5239	2875.6406	154206.2285
1 1/2	168.8606	2269.0644	47.6347	2889.0625	155287.1094
1 3/4	169.2533	2279.6304	47.7455	2902.5156	156373.0292

EQUAL SQUARES, SQUARES AND CUBES OF THE DIAMETERS. [1]

TABLE I.

Dia. or Root.	Circum.	Area.	Side of = square.	Square.	Cube.
54 in.	169-6460	2290-2210	47-8563	2916-	157464-
$\frac{1}{2}$	170-0367	2300-8362	47-9670	2929-5156	158560-0332
$\frac{1}{4}$	170-4314	2311-4759	48-0778	2943-0625	159661-1406
$\frac{3}{4}$	170-8241	2322-1401	48-1886	2956-6406	160767-3339
$\frac{1}{2}$	171-2168	2332-8289	48-2994	2970-25	161878-625
$\frac{1}{4}$	171-6095	2343-5422	48-4101	2983-8906	162995-0254
$\frac{3}{4}$	172-0022	2354-2801	48-5209	2997-5625	164116-5469
$\frac{1}{2}$	172-3949	2365-0425	48-6317	3011-2656	165243-2011
55 in.	172-7856	2375-8294	48-7425	3025-	166375-
$\frac{1}{2}$	173-1803	2386-6409	48-8533	3038-7656	167511-9550
$\frac{1}{4}$	173-5730	2397-4770	48-9640	3052-5625	168654-0781
$\frac{3}{4}$	173-9657	2408-3376	49-0748	3066-3906	169801-3809
$\frac{1}{2}$	174-3584	2419-2227	49-1856	3080-25	170953-875
$\frac{1}{4}$	174-7511	2430-1324	49-2964	3094-1406	172111-5722
$\frac{3}{4}$	175-1438	2441-0666	49-4072	3108-0625	173274-4844
$\frac{1}{2}$	175-5365	2452-0253	49-5179	3122-0156	174442-6230
56 in.	175-9292	2463-0086	49-6287	3136-	175616-
$\frac{1}{2}$	176-3219	2474-0165	49-7395	3150-0156	176794-6269
$\frac{1}{4}$	176-7146	2485-0489	49-8503	3164-0625	177978-5156
$\frac{3}{4}$	177-1073	2496-1058	49-9610	3178-1406	179167-6777
$\frac{1}{2}$	177-5000	2507-1873	50-0718	3192-25	180362-125
$\frac{1}{4}$	177-8927	2518-2933	50-1826	3206-3906	181561-8691
$\frac{3}{4}$	178-2854	2529-4239	50-2934	3220-5625	182766-9219
$\frac{1}{2}$	178-6781	2540-5790	50-4042	3234-7656	183977-2949
57 in.	179-0708	2551-7586	50-5149	3249-	185193-
$\frac{1}{2}$	179-4635	2562-9628	50-6257	3263-2656	186414-0488
$\frac{1}{4}$	179-8562	2574-1916	50-7365	3277-5625	187640-4531
$\frac{3}{4}$	180-2489	2585-4449	50-8473	3291-8906	188872-2246
$\frac{1}{2}$	180-6416	2596-7227	50-9580	3306-25	190109-375
$\frac{1}{4}$	181-0343	2608-0250	51-0688	3320-6406	191351-9160
$\frac{3}{4}$	181-4270	2619-3520	51-1796	3335-0625	192599-8594
$\frac{1}{2}$	181-8197	2630-7034	51-2904	3349-5156	193853-2168
58 in.	182-2124	2642-0794	51-4012	3364-	195112-
$\frac{1}{2}$	182-6051	2653-4800	51-5119	3378-5156	196376-2207
$\frac{1}{4}$	182-9978	2664-9051	51-6227	3393-0625	197645-8906
$\frac{3}{4}$	183-3905	2676-3547	51-7335	3407-6406	198921-0214
$\frac{1}{2}$	183-7832	2687-8289	51-8443	3422-25	200201-625
$\frac{1}{4}$	184-1759	2699-3276	51-9551	3436-8906	201487-7129
$\frac{3}{4}$	184-5686	2710-8508	52-0658	3451-5625	202779-2969
$\frac{1}{2}$	184-9613	2722-3987	52-1766	3466-2656	204076-3886
59 in.	185-3540	2733-9710	52-2874	3481-	205379-
$\frac{1}{2}$	185-7467	2745-5679	52-3982	3495-7656	206687-1426
$\frac{1}{4}$	186-1394	2757-1893	52-5089	3510-5625	208000-8281
$\frac{3}{4}$	186-5321	2768-8353	52-6197	3525-3906	209320-0683
$\frac{1}{2}$	186-9248	2780-5058	52-7305	3540-25	210644-875
$\frac{1}{4}$	187-3175	2792-2009	52-8413	3555-1406	211975-2598
$\frac{3}{4}$	187-7102	2803-9205	52-9521	3570-0625	213311-2344
$\frac{1}{2}$	188-1029	2815-6646	53-0628	3585-0156	214642-8105

TABLE I.

Dia. or Root.	Circum.	Area.	Side of square.	Square.	Cube.
60 in.	188.4956	2827.4334	53.1736	3600.	216000.
1/2	188.8883	2839.2266	53.2844	3615.0156	217352.8145
1/4	189.2810	2851.0444	53.3952	3630.0625	218711.2656
3/4	189.6737	2862.8868	53.5060	3645.1406	220075.3652
1	190.0664	2874.7536	53.6167	3660.25	221445.125
1 1/4	190.4591	2886.6450	53.7275	3675.3906	222820.5566
1 1/2	190.8518	2898.5610	53.8383	3690.5625	224201.6719
1 3/4	191.2445	2910.5015	53.9491	3705.7656	225588.4824
61 in.	191.6372	2922.4666	54.0598	3721.	226981.
1/2	192.0299	2934.4562	54.1706	3736.2656	228379.2363
1/4	192.4226	2946.4703	54.2814	3751.5625	229783.2031
3/4	192.8152	2958.5090	54.3922	3766.8906	231192.9121
1	193.2079	2970.5722	54.5030	3782.25	232608.375
1 1/4	193.6006	2982.6600	54.6137	3797.6406	234029.6035
1 1/2	193.9933	2994.7723	54.7245	3813.0625	235456.6094
1 3/4	194.3860	3006.9091	54.8353	3828.5156	236889.4043
62 in.	194.7787	3019.0705	54.9461	3844.	238328.
1/2	195.1714	3031.2565	55.0568	3859.5156	239772.4082
1/4	195.5641	3043.4670	55.1676	3875.0625	241222.6406
3/4	195.9568	3055.7020	55.2784	3890.6406	242678.7089
1	196.3495	3067.9616	55.3892	3906.25	244140.625
1 1/4	196.7422	3080.2457	55.5000	3921.8906	245608.4004
1 1/2	197.1349	3092.5544	55.6107	3937.5625	247082.0469
1 3/4	197.5276	3104.8876	55.7215	3953.2656	248561.5761
63 in.	197.9203	3117.2453	55.8323	3969.	250047.
1/2	198.3130	3129.6276	55.9431	3984.7656	251538.3300
1/4	198.7057	3142.0344	56.0539	4000.5625	253035.5781
3/4	199.0984	3154.4658	56.1646	4016.3906	254538.7558
1	199.4911	3166.9217	56.2754	4032.25	256047.875
1 1/4	199.8838	3179.4022	56.3862	4048.1406	257562.9472
1 1/2	200.2765	3191.9072	56.4970	4064.0625	259083.9844
1 3/4	200.6692	3204.4368	56.6077	4080.0156	260610.9980
64 in.	201.0619	3216.9909	56.7185	4096.	262144.
1/2	201.4546	3229.5695	56.8293	4112.0156	263683.0019
1/4	201.8473	3242.1727	56.9401	4128.0625	265228.0156
3/4	202.2400	3254.8004	57.0509	4144.1406	266779.0527
1	202.6327	3267.4527	57.1616	4160.25	268336.125
1 1/4	203.0254	3280.1295	57.2724	4176.3906	269899.2441
1 1/2	203.4181	3292.8309	57.3832	4192.5625	271468.4219
1 3/4	203.8108	3305.5568	57.4940	4208.7656	273043.6699
65 in.	204.2035	3318.3072	57.6048	4225.	274625.
1/2	204.5962	3331.0822	57.7155	4241.2656	276212.4238
1/4	204.9889	3343.8818	57.8263	4257.5625	277805.9531
3/4	205.3816	3356.7058	57.9371	4273.8906	279405.6996
1	205.7743	3369.5545	58.0479	4290.25	281011.375
1 1/4	206.1670	3382.4276	58.1586	4306.6406	282623.2910
1 1/2	206.5597	3395.3253	58.2694	4323.0625	284241.3594
1 3/4	206.9524	3408.2476	58.3802	4339.5156	285865.6918

EQUAL SQUARES, SQUARES AND CUBES OF THE DIAMETERS. [13

TABLE I.

Dia. or Root.	Circum.	Area.	Side of = square.	Square.	Cube.
66 in.	207.3451	3421.1944	58.4910	4356	287496
$\frac{1}{2}$ in.	207.7378	3434.1657	58.6018	4372.5156	289132.5957
$\frac{1}{4}$ in.	208.1305	3447.1616	58.7125	4389.0625	290775.3906
$\frac{3}{4}$ in.	208.5232	3460.1821	58.8233	4405.6406	292424.3964
1 in.	208.9159	3473.2270	58.9341	4422.25	294079.625
$\frac{1}{2}$ in.	209.3086	3486.2965	59.0449	4438.8906	295741.0879
$\frac{1}{4}$ in.	209.7013	3499.3906	59.1556	4455.5625	297408.7969
$\frac{3}{4}$ in.	210.0940	3512.5092	59.2664	4472.2656	299082.7637
67 in.	210.4867	3525.6524	59.3772	4489	300763
$\frac{1}{2}$ in.	210.8794	3538.8200	59.4880	4505.7656	302449.5176
$\frac{1}{4}$ in.	211.2721	3552.0123	59.5988	4522.5625	304142.3281
$\frac{3}{4}$ in.	211.6648	3565.2291	59.7095	4539.3906	305841.4433
1 in.	212.0575	3578.4704	59.8203	4556.25	307546.875
$\frac{1}{2}$ in.	212.4502	3591.7362	59.9311	4573.1406	309258.6348
$\frac{1}{4}$ in.	212.8429	3605.0267	60.0419	4590.0625	310976.7344
$\frac{3}{4}$ in.	213.2356	3618.3416	60.1527	4607.0156	312701.1855
68 in.	213.6283	3631.6811	60.2634	4624	314432
$\frac{1}{2}$ in.	214.0210	3645.0451	60.3742	4641.0156	316169.1894
$\frac{1}{4}$ in.	214.4137	3658.4337	60.4850	4658.0625	317912.7656
$\frac{3}{4}$ in.	214.8064	3671.8469	60.5958	4675.1406	319662.7402
1 in.	215.1991	3685.2845	60.7065	4692.25	321419.125
$\frac{1}{2}$ in.	215.5918	3698.7467	60.8173	4709.3906	323181.9316
$\frac{1}{4}$ in.	215.9845	3712.2335	60.9281	4726.5625	324951.1719
$\frac{3}{4}$ in.	216.3772	3725.7448	61.0389	4743.7656	326726.8574
69 in.	216.7699	3739.2807	61.1497	4761	328509
$\frac{1}{2}$ in.	217.1626	3752.8410	61.2604	4778.2656	330297.6113
$\frac{1}{4}$ in.	217.5553	3766.4260	61.3712	4795.5625	332092.7031
$\frac{3}{4}$ in.	217.9480	3780.0355	61.4820	4812.8906	333894.2871
1 in.	218.3407	3793.6695	61.5928	4830.25	335702.375
$\frac{1}{2}$ in.	218.7334	3807.3280	61.7035	4847.6406	337516.9785
$\frac{1}{4}$ in.	219.1261	3821.0112	61.8143	4865.0625	339338.1094
$\frac{3}{4}$ in.	219.5188	3834.7188	61.9251	4882.5156	341165.7793
70 in.	219.9115	3848.4510	62.0359	4900	343000
$\frac{1}{2}$ in.	220.3042	3862.2077	62.1467	4917.5156	344840.7832
$\frac{1}{4}$ in.	220.6969	3875.9890	62.2574	4935.0625	346688.1406
$\frac{3}{4}$ in.	221.0896	3889.7949	62.3682	4952.6406	348542.0839
1 in.	221.4823	3903.6252	62.4790	4970.25	350402.625
$\frac{1}{2}$ in.	221.8750	3917.4801	62.5898	4987.8906	352269.7754
$\frac{1}{4}$ in.	222.2677	3931.3596	62.7006	5005.5625	354143.5469
$\frac{3}{4}$ in.	222.6604	3945.2636	62.8113	5023.2656	356023.9511
71 in.	223.0531	3959.1921	62.9221	5041	357911
$\frac{1}{2}$ in.	223.4458	3973.1452	63.0329	5058.7656	359804.7051
$\frac{1}{4}$ in.	223.8385	3987.1229	63.1437	5076.5625	361705.0781
$\frac{3}{4}$ in.	224.2312	4001.1250	63.2544	5094.3906	363612.1308
1 in.	224.6239	4015.1518	63.3652	5112.25	365525.875
$\frac{1}{2}$ in.	225.0166	4029.2030	63.4760	5130.1406	367446.3222
$\frac{1}{4}$ in.	225.4093	4043.2788	63.5868	5148.0625	369373.4844
$\frac{3}{4}$ in.	225.8020	4057.3792	63.6976	5166.0156	371307.3730

14] CIRCUMFERENCES AND AREAS OF CIRCLES, SIDES OF

TABLE I.

Dia. or Root.	Circum.	Area.	Side of = square.	Square.	Cube.
72 in.	226-1947	4071-5041	63-8083	5184-	373248-
$\frac{1}{2}$	226-5874	4085-6535	63-9191	5202-0156	375195-3770
$\frac{1}{4}$	226-9801	4099-8275	64-0299	5220-0625	377149-5156
$\frac{3}{4}$	227-3728	4114-0260	64-1407	5238-1406	379110-4277
$\frac{1}{2}$	227-7655	4128-2491	64-2515	5256-25	381078-125
$\frac{1}{4}$	228-1582	4142-4967	64-3622	5274-3906	383052-6191
$\frac{3}{4}$	228-5509	4156-7689	64-4730	5292-5625	385033-9219
$\frac{1}{2}$	228-9436	4171-0656	64-5838	5310-7656	387022-0449
73 in.	229-3363	4185-3868	64-6946	5329-	389017-
$\frac{1}{2}$	229-7290	4199-7326	64-8053	5347-2656	391018-7988
$\frac{1}{4}$	230-1217	4214-1029	64-9161	5365-5625	393027-4531
$\frac{3}{4}$	230-5144	4228-4978	65-0269	5383-8906	395042-9746
$\frac{1}{2}$	230-9071	4242-9172	65-1377	5402-25	397065-375
$\frac{1}{4}$	231-2998	4257-3612	65-2485	5420-6406	399094-6660
$\frac{3}{4}$	231-6925	4271-8297	65-3592	5439-0625	401130-8594
$\frac{1}{2}$	232-0852	4286-8227	65-4700	5457-5156	403173-9668
74 in.	232-4779	4300-8403	65-5808	5476-	405224-
$\frac{1}{2}$	232-8706	4315-3825	65-6916	5494-5156	407280-9707
$\frac{1}{4}$	233-2633	4329-9492	65-8023	5513-0625	409344-8906
$\frac{3}{4}$	233-6560	4344-5404	65-9131	5531-6406	411415-7714
$\frac{1}{2}$	234-0487	4359-1562	66-0239	5550-25	413493-625
$\frac{1}{4}$	234-4414	4373-7965	66-1347	5568-8906	415578-4629
$\frac{3}{4}$	234-8341	4388-4618	66-2455	5587-5625	417670-2969
$\frac{1}{2}$	235-2267	4403-1507	66-3562	5606-2656	419769-1386
75 in.	235-6194	4417-8647	66-4670	5625-	421875-
$\frac{1}{2}$	236-0121	4432-6032	66-5778	5643-7656	423987-8926
$\frac{1}{4}$	236-4048	4447-3662	66-6886	5662-5625	426107-8281
$\frac{3}{4}$	236-7975	4462-1538	66-7994	5681-3906	428234-8183
$\frac{1}{2}$	237-1902	4476-9639	66-9104	5700-25	430368-875
$\frac{1}{4}$	237-5829	4491-8025	67-0209	5719-1406	432510-0098
$\frac{3}{4}$	237-9756	4506-6637	67-1317	5738-0625	434658-2344
$\frac{1}{2}$	238-3683	4521-5495	67-2425	5757-0156	436813-5605
76 in.	238-7610	4536-4598	67-3532	5776-	438976-
$\frac{1}{2}$	239-1537	4551-3946	67-4640	5795-0156	441145-5644
$\frac{1}{4}$	239-5464	4566-3540	67-5748	5814-0625	443322-2656
$\frac{3}{4}$	239-9391	4581-3379	67-6856	5833-1406	445506-1152
$\frac{1}{2}$	240-3318	4596-3464	67-7964	5852-25	447697-125
$\frac{1}{4}$	240-7245	4611-3794	67-9071	5871-3906	449895-3066
$\frac{3}{4}$	241-1172	4626-4370	68-0179	5890-5625	452100-6719
$\frac{1}{2}$	241-5099	4641-5191	68-1287	5909-7656	454313-2324
77 in.	241-9026	4656-6257	68-2395	5929-	456533-
$\frac{1}{2}$	242-2953	4671-7569	68-3503	5948-2656	458759-9863
$\frac{1}{4}$	242-6880	4686-9126	68-4610	5967-5625	460994-2031
$\frac{3}{4}$	243-0807	4702-0929	68-5718	5986-8906	463235-6621
$\frac{1}{2}$	243-4734	4717-2977	68-6826	6006-25	465484-375
$\frac{1}{4}$	243-8661	4732-5271	68-7937	6025-6406	467740-3535
$\frac{3}{4}$	244-2588	4747-7810	68-9041	6045-0625	470003-6094
$\frac{1}{2}$	244-6515	4763-0594	69-0149	6064-5156	472274-1543

EQUAL SQUARES, SQUARES AND CUBES OF THE DIAMETERS. [15

TABLE I.

Dia. or Root.	Circum.	Area.	Side of = square.	Square.	Cube.
78 in.	245-0442	4778-3624	69-1257	6084-	474552-
$\frac{1}{2}$	245-4369	4793-6900	69-2365	6103-5156	476637-1582
$\frac{1}{4}$	245-8296	4809-0420	69-3473	6123-0625	479129-6406
$\frac{3}{4}$	246-2223	4824-4187	69-4580	6142-6406	481429-4589
$\frac{1}{2}$	246-6150	4839-8198	69-5688	6162-25	483736-625
$\frac{3}{4}$	247-0077	4855-2455	69-6796	6181-8906	486051-1504
$\frac{1}{2}$	247-4004	4870-6958	69-7904	6201-5625	488373-0469
$\frac{3}{4}$	247-7931	4886-1706	69-9011	6221-2656	490702-3261
79 in.	248-1858	4901-6699	70-0119	6241-	493039-
$\frac{1}{2}$	248-5785	4917-1938	70-1227	6260-7656	495383-0801
$\frac{1}{4}$	248-9712	4932-7423	70-2335	6280-5625	497734-5781
$\frac{3}{4}$	249-3639	4948-3152	70-3443	6300-3906	500093-5058
$\frac{1}{2}$	249-7566	4963-9127	70-4550	6320-25	502459-875
$\frac{3}{4}$	250-1493	4979-5348	70-5658	6340-1406	504833-6972
$\frac{1}{2}$	250-5420	4995-1814	70-6766	6360-0625	507214-9844
$\frac{3}{4}$	250-9347	5010-8526	70-7874	6380-0156	509603-7480
80 in.	251-3274	5026-5482	70-8982	6400-	512000-
$\frac{1}{2}$	251-7201	5042-2685	71-0089	6420-0156	514403-7519
$\frac{1}{4}$	252-1128	5058-0133	71-1197	6440-0625	516815-0156
$\frac{3}{4}$	252-5055	5073-7826	71-2305	6460-1406	519233-8027
$\frac{1}{2}$	252-8982	5089-5764	71-3413	6480-25	521660-125
$\frac{3}{4}$	253-2909	5105-3949	71-4520	6500-3906	524093-9941
$\frac{1}{2}$	253-6836	5121-2378	71-5628	6520-5625	526535-4219
$\frac{3}{4}$	254-0763	5137-1053	71-6736	6540-7656	528984-4199
81 in.	254-4690	5152-9974	71-7844	6561-	531441-
$\frac{1}{2}$	254-8617	5168-9139	71-8952	6581-2656	533905-1738
$\frac{1}{4}$	255-2544	5184-8551	72-0059	6601-5625	536376-9531
$\frac{3}{4}$	255-6471	5200-8207	72-1167	6621-8906	538856-3496
$\frac{1}{2}$	256-0398	5216-8110	72-2275	6642-25	541343-875
$\frac{3}{4}$	256-4325	5232-8257	72-3383	6662-6406	543838-0410
$\frac{1}{2}$	256-8252	5248-8650	72-4491	6683-0625	546340-3594
$\frac{3}{4}$	257-2179	5264-9289	72-5598	6703-5156	548850-3418
82 in.	257-6106	5281-0173	72-6706	6724-	551368-
$\frac{1}{2}$	258-0033	5297-1302	72-7814	6744-5156	553893-3457
$\frac{1}{4}$	258-3960	5313-2677	72-8922	6765-0625	556426-3906
$\frac{3}{4}$	258-7887	5329-4297	73-0029	6785-6406	558967-1464
$\frac{1}{2}$	259-1814	5345-6162	73-1137	6806-25	561515-625
$\frac{3}{4}$	259-5741	5361-8274	73-2245	6826-8906	564071-8379
$\frac{1}{2}$	259-9668	5378-0630	73-3353	6847-5625	566635-7969
$\frac{3}{4}$	260-3595	5394-3232	73-4461	6868-2656	569207-5137
83 in.	260-7522	5410-6079	73-5568	6889-	571787-
$\frac{1}{2}$	261-1449	5426-9172	73-6676	6909-7656	574374-2676
$\frac{1}{4}$	261-5376	5443-2511	73-7784	6930-5625	576969-3281
$\frac{3}{4}$	261-9303	5459-6094	73-8892	6951-3906	579572-1933
$\frac{1}{2}$	262-3230	5475-9923	73-9999	6972-25	582182-875
$\frac{3}{4}$	262-7157	5492-3998	74-1107	6993-1406	584801-3848
$\frac{1}{2}$	263-1084	5508-8318	74-2215	7014-0625	587427-7344
$\frac{3}{4}$	263-5011	5525-2884	74-3323	7035-0156	590061-9355

TABLE I.

Dia. or Root.	Circum.	Area.	Side of = square.	Square.	Cube.
84 in.	263.8938	5541.7694	74.4431	7056	592704
$\frac{1}{2}$	264.2865	5558.2751	74.5538	7077.0156	595353.9394
$\frac{1}{4}$	264.6792	5574.8053	74.6646	7098.0625	598011.7656
$\frac{3}{4}$	265.0719	5591.3600	74.7754	7119.1406	600677.4902
$\frac{1}{2}$	265.4646	5607.9392	74.8862	7140.25	603351.125
$\frac{1}{4}$	265.8573	5624.5430	74.9970	7161.3906	606032.6816
$\frac{3}{4}$	266.2500	5641.1714	75.1077	7182.5625	608722.1719
$\frac{1}{2}$	266.6427	5657.8243	75.2185	7203.7656	611419.6074
85 in.	267.0354	5674.5017	75.3293	7225	614125
$\frac{1}{2}$	267.4281	5691.2037	75.4401	7246.2656	616838.3613
$\frac{1}{4}$	267.8208	5707.9302	75.5508	7267.5625	619559.7031
$\frac{3}{4}$	268.2135	5724.6813	75.6616	7288.8906	622289.0371
$\frac{1}{2}$	268.6062	5741.4569	75.7724	7310.25	625026.375
$\frac{1}{4}$	268.9989	5758.2571	75.8832	7331.6406	627771.7285
$\frac{3}{4}$	269.3916	5775.0818	75.9940	7353.0625	630525.1094
$\frac{1}{2}$	269.7843	5791.9310	76.1047	7374.5156	633286.5293
86 in.	270.1770	5808.8048	76.2155	7396	636056
$\frac{1}{2}$	270.5697	5825.7031	76.3263	7417.5156	638833.5332
$\frac{1}{4}$	270.9624	5842.6260	76.4371	7439.0625	641619.1406
$\frac{3}{4}$	271.3551	5859.5734	76.5479	7460.6406	644412.8339
$\frac{1}{2}$	271.7478	5876.5454	76.6586	7482.25	647214.625
$\frac{1}{4}$	272.1405	5893.5419	76.7694	7503.8906	650024.5254
$\frac{3}{4}$	272.5332	5910.5630	76.8802	7525.5625	652842.5469
$\frac{1}{2}$	272.9259	5927.6086	76.9910	7547.2656	655668.7011
87 in.	273.3186	5944.6787	77.1017	7569	658503
$\frac{1}{2}$	273.7113	5961.7734	77.2125	7590.7656	661345.4551
$\frac{1}{4}$	274.1040	5978.8926	77.3233	7612.5625	664196.0781
$\frac{3}{4}$	274.4967	5996.0364	77.4341	7634.3906	667054.8808
$\frac{1}{2}$	274.8894	6013.2047	77.5449	7656.25	669921.875
$\frac{1}{4}$	275.2821	6030.3975	77.6556	7678.1406	672797.0722
$\frac{3}{4}$	275.6748	6047.6149	77.7664	7700.0625	675680.4844
$\frac{1}{2}$	276.0675	6064.8569	77.8772	7722.0156	678572.1230
88 in.	276.4602	6082.1234	77.9880	7744	681472
$\frac{1}{2}$	276.8529	6099.4144	78.0987	7766.0156	684380.1269
$\frac{1}{4}$	277.2456	6116.7300	78.2095	7788.0625	687296.5156
$\frac{3}{4}$	277.6382	6134.0701	78.3203	7810.1406	690221.1777
$\frac{1}{2}$	278.0309	6151.4348	78.4310	7832.25	693154.125
$\frac{1}{4}$	278.4236	6168.8240	78.5419	7854.3906	696095.3691
$\frac{3}{4}$	278.8163	6186.2377	78.6526	7876.5625	699044.9219
$\frac{1}{2}$	279.2090	6203.6760	78.7634	7898.7656	702002.7949
89 in.	279.6017	6221.1389	78.8742	7921	704969
$\frac{1}{2}$	279.9944	6238.6262	78.9850	7943.2656	707943.5488
$\frac{1}{4}$	280.3871	6256.1382	79.0958	7965.5625	710926.4531
$\frac{3}{4}$	280.7798	6273.6746	79.2065	7987.8906	713917.7246
$\frac{1}{2}$	281.1725	6291.2356	79.3173	8010.25	716917.375
$\frac{1}{4}$	281.5652	6308.8212	79.4281	8032.6406	719925.4160
$\frac{3}{4}$	281.9579	6326.4313	79.5389	8055.0625	722941.8594
$\frac{1}{2}$	282.3506	6344.0659	79.6496	8077.5156	725966.7168

EQUAL SQUARES, SQUARES AND CUBES OF THE DIAMETERS. [17

TABLE I.

Dia. or Root.	Circum.	Area.	Side of = square.	Square.	Cube.
90 in.	282.7433	6361.7251	79.7604	8100.	729000.
$\frac{1}{2}$	283.1360	6379.4089	79.8712	8122.5156	732041.7207
$\frac{1}{4}$	283.5287	6397.1171	79.9820	8145.0625	735091.8906
$\frac{3}{4}$	283.9214	6414.8499	80.0928	8167.6406	738150.5214
$\frac{1}{2}$	284.3141	6432.6073	80.2035	8190.25	741217.625
$\frac{1}{4}$	284.7068	6450.3892	80.3143	8212.8906	744293.2129
$\frac{3}{4}$	285.0995	6468.1957	80.4251	8235.5625	747377.2969
$\frac{1}{2}$	285.4922	6486.0267	80.5359	8258.2656	750469.8886
91 in.	285.8849	6503.8822	80.6467	8281.	753571.
$\frac{1}{2}$	286.2776	6521.7623	80.7574	8303.7656	756680.6426
$\frac{1}{4}$	286.6703	6539.6669	80.8682	8326.5625	759798.8281
$\frac{3}{4}$	287.0630	6557.5961	80.9790	8349.3906	762925.5683
$\frac{1}{2}$	287.4557	6575.5498	81.0898	8372.25	766060.875
$\frac{1}{4}$	287.8484	6593.5280	81.2005	8395.1406	769204.7598
$\frac{3}{4}$	288.2411	6611.5308	81.3113	8418.0625	772357.2344
$\frac{1}{2}$	288.6338	6629.5582	81.4221	8441.0156	775518.3105
92 in.	289.0265	6647.6101	81.5329	8464.	778688.
$\frac{1}{2}$	289.4192	6665.6865	81.6437	8487.0156	781866.8144
$\frac{1}{4}$	289.8119	6683.7875	81.7544	8510.0625	785053.2656
$\frac{3}{4}$	290.2046	6701.9130	81.8652	8533.1406	788248.8652
$\frac{1}{2}$	290.5973	6720.0630	81.9760	8556.25	791453.125
$\frac{1}{4}$	290.9900	6738.2376	82.0868	8579.3906	794666.0566
$\frac{3}{4}$	291.3827	6756.4368	82.1975	8602.5625	797887.6719
$\frac{1}{2}$	291.7754	6774.6605	82.3083	8625.7656	801117.9824
93 in.	292.1681	6792.9087	82.4191	8649.	804357.
$\frac{1}{2}$	292.5608	6811.1815	82.5299	8672.2656	807604.7363
$\frac{1}{4}$	292.9535	6829.4788	82.6407	8695.5625	810861.2031
$\frac{3}{4}$	293.3462	6847.8007	82.7514	8718.8906	814126.4121
$\frac{1}{2}$	293.7389	6866.1471	82.8622	8742.25	817400.375
$\frac{1}{4}$	294.1316	6884.5180	82.9730	8765.6406	820683.1035
$\frac{3}{4}$	294.5243	6902.9135	83.0838	8789.0625	823974.6094
$\frac{1}{2}$	294.9170	6921.3336	83.1946	8812.5156	827274.9043
94 in.	295.3097	6939.7782	83.3053	8836.	830584.
$\frac{1}{2}$	295.7024	6958.2473	83.4161	8859.5156	833901.9082
$\frac{1}{4}$	296.0951	6976.7410	83.5269	8883.0625	837228.6406
$\frac{3}{4}$	296.4878	6995.2592	83.6377	8906.6406	840564.2089
$\frac{1}{2}$	296.8805	7013.8019	83.7484	8930.25	843908.625
$\frac{1}{4}$	297.2732	7032.3693	83.8592	8953.8906	847261.9004
$\frac{3}{4}$	297.6659	7050.9611	83.9700	8977.5625	850624.0469
$\frac{1}{2}$	298.0586	7069.5775	84.0808	9001.2656	853995.0761
95 in.	298.4513	7088.2184	84.1916	9025.	857375.
$\frac{1}{2}$	298.8440	7106.8839	84.3023	9048.7656	860763.8301
$\frac{1}{4}$	299.2367	7125.5799	84.4131	9072.5625	864161.5781
$\frac{3}{4}$	299.6294	7144.2885	84.5239	9096.3906	867568.2558
$\frac{1}{2}$	300.0221	7163.0276	84.6347	9120.25	870983.875
$\frac{1}{4}$	300.4148	7181.7913	84.7454	9144.1406	874408.4472
$\frac{3}{4}$	300.8075	7200.5794	84.8562	9168.0625	877841.9844
$\frac{1}{2}$	301.2002	7219.3922	84.9670	9192.0156	881284.4980

TABLE I.

Dia. or Root.	Circum.	Area.	Side of = square.	Square.	Cube.
96 in.	301.5929	7238.2295	85.0778	9216.	884736.
$\frac{1}{2}$	301.9856	7257.0913	85.1886	9240.0156	888196.5019
$\frac{1}{4}$	302.3783	7275.9777	85.2993	9264.0625	891666.0156
$\frac{3}{8}$	302.7710	7294.8886	85.4101	9288.1406	895144.5527
$\frac{1}{2}$	303.1637	7313.8240	85.5209	9312.25	898632.125
$\frac{5}{8}$	303.5564	7332.7840	85.6317	9336.3906	902128.7441
$\frac{3}{4}$	303.9491	7351.7686	85.7425	9360.5625	905634.4219
$\frac{7}{8}$	304.3418	7370.7777	85.8532	9384.7656	909149.1699
97 in.	304.7345	7389.8113	85.9640	9409.	912673.
$\frac{1}{2}$	305.1272	7408.8695	86.0748	9433.2656	916205.9238
$\frac{1}{4}$	305.5199	7427.9522	86.1856	9457.5625	919747.9531
$\frac{3}{8}$	305.9126	7447.0595	86.2963	9481.8906	923299.0996
$\frac{1}{2}$	306.3053	7466.1913	86.4071	9506.25	926859.375
$\frac{5}{8}$	306.6980	7485.3476	86.5179	9530.6406	930428.7910
$\frac{3}{4}$	307.0907	7504.5285	86.6287	9555.0625	934007.3594
$\frac{7}{8}$	307.4834	7523.7340	86.7395	9579.5156	937595.0918
98 in.	307.8761	7542.9640	86.8502	9604.	941192.
$\frac{1}{2}$	308.2688	7562.2185	86.9610	9628.5156	944798.0957
$\frac{1}{4}$	308.6615	7581.4976	87.0718	9653.0625	948413.3906
$\frac{3}{8}$	309.0542	7600.8012	87.1826	9677.6406	952037.8965
$\frac{1}{2}$	309.4469	7620.1293	87.2934	9702.25	955671.625
$\frac{5}{8}$	309.8396	7639.4820	87.4041	9726.8906	959314.5879
$\frac{3}{4}$	310.2323	7658.8593	87.5149	9751.5625	962966.7969
$\frac{7}{8}$	310.6250	7678.2611	87.6257	9776.2656	966628.2637
99 in.	311.0177	7697.6874	87.7365	9801.	970299.
$\frac{1}{2}$	311.4104	7717.1383	87.8472	9825.7656	973979.0176
$\frac{1}{4}$	311.8031	7736.6137	87.9580	9850.5625	977668.3281
$\frac{3}{8}$	312.1958	7756.1137	88.0688	9875.3906	981366.9433
$\frac{1}{2}$	312.5885	7775.6382	88.1796	9900.25	985074.875
$\frac{5}{8}$	312.9812	7795.1872	88.2904	9925.1406	988792.1348
$\frac{3}{4}$	313.3739	7814.7608	88.4011	9950.0625	992518.7344
$\frac{7}{8}$	313.7666	7834.3590	88.5119	9975.0156	996254.6855
100 in.	314.1593	7853.9816	88.6227	10000.	1000000.
$\frac{1}{2}$	314.5447	7893.3006	88.8442	10050.0625	1007518.7656
$\frac{1}{4}$	315.7301	7932.7178	89.0658	10100.25	1015075.125
$\frac{3}{8}$	316.5155	7972.2331	89.2874	10150.5625	1022669.1719
101 in.	317.3009	8011.8467	89.5089	10201.	1030301.
$\frac{1}{2}$	318.0863	8051.5584	89.7305	10251.5625	1037970.7031
$\frac{1}{4}$	318.8717	8091.3682	89.9520	10302.25	1045678.375
$\frac{3}{8}$	319.6571	8131.2763	90.1736	10353.0625	1053424.1093
102 in.	320.4425	8171.2825	90.3951	10404.	1061208.
$\frac{1}{2}$	321.2278	8211.3869	90.6167	10455.0625	1069030.1406
$\frac{1}{4}$	322.0132	8251.5895	90.8383	10506.25	1076890.625
$\frac{3}{8}$	322.7986	8291.8902	91.0598	10557.5625	1084789.5468

EQUAL SQUARES, SQUARES AND CUBES OF THE DIAMETERS. [19

TABLE I.

Dia. or Root.	Circum.	Area.	Side of =square.	Square.	Cube.
103 in.	323.5840	8332.2891	91.2814	10609.	1092727.
$\frac{1}{4}$	324.3694	8372.7862	91.5029	10660.5625	1100703.0781
$\frac{1}{2}$	325.1548	8413.3815	91.7245	10712.25	1108717.875
$\frac{3}{4}$	325.9402	8454.0749	91.9460	10764.0625	1116771.4843
104 in.	326.7256	8494.8665	92.1676	10816.	1124864.
$\frac{1}{4}$	327.5110	8535.7563	92.3892	10868.0625	1132995.5156
$\frac{1}{2}$	328.2964	8576.7443	92.6107	10920.25	1141166.125
$\frac{3}{4}$	329.0818	8617.8304	92.8323	10972.5625	1149375.9218
105 in.	329.8672	8659.0148	93.0538	11028.	1157625.
$\frac{1}{4}$	330.6526	8700.2972	93.2754	11077.5625	1165913.4531
$\frac{1}{2}$	331.4380	8741.6779	93.4969	11130.25	1174241.375
$\frac{3}{4}$	332.2234	8783.1567	93.7185	11183.0625	1182608.8593
106 in.	333.0088	8824.7338	93.9401	11236.	1191016.
$\frac{1}{4}$	333.7942	8866.4090	94.1616	11289.0625	1199462.8906
$\frac{1}{2}$	334.5796	8908.1823	94.3832	11342.25	1207949.625
$\frac{3}{4}$	335.3650	8950.0539	94.6047	11395.5625	1216476.2968
107 in.	336.1504	8992.0236	94.8263	11449.	1225043.
$\frac{1}{4}$	336.9358	9034.0915	95.0478	11502.5625	1233649.8281
$\frac{1}{2}$	337.7212	9076.2575	95.2694	11556.25	1242296.875
$\frac{3}{4}$	338.5066	9118.5218	95.4910	11610.0625	1250984.2343
108 in.	339.2920	9160.8842	95.7125	11664.	1259712.
$\frac{1}{4}$	340.0774	9203.3448	95.9341	11718.0625	1268480.2656
$\frac{1}{2}$	340.8628	9245.9035	96.1556	11772.25	1277289.125
$\frac{3}{4}$	341.6482	9288.5605	96.3772	11826.5625	1286138.6718
109 in.	342.4336	9331.3156	96.5987	11881.	1295029.
$\frac{1}{4}$	343.2190	9374.1689	96.8203	11935.5625	1303960.2031
$\frac{1}{2}$	344.0044	9417.1203	97.0418	11990.25	1312932.375
$\frac{3}{4}$	344.7898	9460.1700	97.2634	12045.0625	1321945.6093
110 in.	345.5752	9503.3178	97.4850	12100.	1331000.

EXAMPLES TO THE PRECEDING TABLE I.*

Ex. 1. The diameter of a circle being $83\frac{3}{4}$ inches, required its circumference?

Here, in column 1, page 15, we find the diameter $83\frac{3}{4}$ in *inches, feet, yards, chains, miles, &c.*; the corresponding circumference will be found in column 2; hence, the diameter being $83\frac{3}{4}$ inches, the circumference is 263·5011 inches = *Ans.* The same result may be found by Prob. IX., page 50; thus $3\cdot14159265 \times 83\frac{3}{4}$ in. = 263·50108352 inches = *Ans. as before.*

Ex. 2. What is the area of a square whose side is 35·25 chains = $35\frac{1}{2}$ chains?

Here, in column 1, page 7, we find the side $35\frac{1}{2}$ in *inches, feet, yards, chains, miles, &c.*; the corresponding square will be found in column 5; hence, the side being $35\frac{1}{2}$ chains, the area of the square will be 1242·5625 square chains = 124·25625 acres = 124 acres 1 rood 1 perch = *Ans.* The same result may also be found by Prob. I. page 44; thus $35\cdot25 \times 35\cdot25 = 1242\cdot5625$ sq. chains = *Ans. as before.*

Ex. 3. A cellar is to be dug whose length, breadth, and depth are each 10 feet 3 inches (= $10\frac{1}{4}$ inches), how many solid feet does it contain, and what will it cost digging at 1s. per solid yard?

Here, in column 1, page 3, we find the side $10\frac{1}{4}$ in *inches, feet, yards, chains, miles, &c.*; the corresponding cube will be found in column 6; hence, the side being $10\frac{1}{4}$ feet, the solid content or cube will be 1076·8906 cubic feet = 39·8848 cubic yards = 1st *Ans.*, and the cost of digging will be 39·8848 shillings = 1l. 19s. $10\frac{1}{2}$ d. = 2nd *Ans.* Similarly, by Prob. I. page 44, the area of any side or base of the cube will be $10\cdot25 \times 10\cdot25 = 105\cdot0625$ sq. feet; and by Prob. I. page 59, the solid content may be found = $1076\cdot890625$ cubic feet, and therefore the rest as before.

Ex. 4. The length of a line with which a gardener formed a circular fish-pond was exactly $27\frac{3}{4}$ yards, what quantity of ground did the fish-pond take up.

Here, in column 1, page 11, we find the diameter $55\frac{1}{2}$ (= $27\frac{3}{4} \times 2$) in *inches, feet, yards, chains, miles, &c.*; the corresponding area of the circle will be found in column 3; hence, the diameter being $55\frac{1}{2}$ yards, the area will be 2419·2227 square yards, or half an acre nearly = *Ans.* The same result may be found as in Ex. 1, page 55, using ·78539816 in place of ·7854; thus $55\frac{1}{2}$ squared \times ·78539816 = $55\cdot5^2 \times$ ·78539816 = 2419·22268234 square yards = *Ans. as before.*

Ex. 5. If the diameter of a circle be 91 miles 5 furlongs (= 91½ miles), what is the side of a square equal in area to that circle?

Here, in column 1, page 17, we find the diameter $91\frac{1}{2}$ in *inches, feet, yards, chains, miles, &c.*; the corresponding side of the equal square will be found in column 4; hence, the diameter being $91\frac{1}{2}$ miles, the sides of the equal square will be 81·2005 miles = 81 miles 1 furlong 24 poles 2 feet 2 inches = *Ans.* The same result may be found as in Ex. 1, page 54, using ·8862269 in place of ·8862; thus $91\frac{1}{2} \times$ ·8862269 = $81\cdot2005397125$ square miles = *Ans. as before.*

* The references to the Problems made use of in the arithmetical computations for testing the tables, are taken from Templeton's "Millwright and Engineer's Pocket Companion." Edition 1852.

TABLE II.

CONTAINING THE
CIRCUMFERENCES AND AREAS OF CIRCLES;

ALSO,

THE SIDE OF A SQUARE OF EQUAL AREA,

AND THE

CONTENTS OF CYLINDERS IN CUBIC YARDS AND IMPERIAL GALLONS,
AT 1 FOOT IN DEPTH, FROM 1 TO 50 FEET DIAMETER,
ADVANCING BY AN INCH.

TABLE II.

Dia. in feet and inches.	Circum. in feet and inches.	Area in square feet.	Side of = square in ft. and in.	Cubic yards at one foot in depth.	Gallons at one foot in depth.
<i>ft. in.</i>	<i>ft. in.</i>		<i>ft. in.</i>		
0 . 1	0 . 3 $\frac{1}{2}$	·0055	0 . 0 $\frac{7}{8}$	·0002	·0340
2	0 . 6 $\frac{1}{2}$	·0218	0 . 1 $\frac{1}{2}$	·0008	·1360
3	0 . 9 $\frac{1}{2}$	·0491	0 . 2 $\frac{1}{2}$	·0018	·3059
4	1 . 0 $\frac{1}{2}$	·0873	0 . 3 $\frac{1}{2}$	·0032	·5439
5	1 . 3 $\frac{1}{2}$	·1364	0 . 4 $\frac{1}{2}$	·0051	·8498
6	1 . 6 $\frac{1}{2}$	·1963	0 . 5 $\frac{1}{2}$	·0073	1·2237
7	1 . 10	·2673	0 . 6 $\frac{1}{2}$	·0099	1·6656
8	2 . 1 $\frac{1}{2}$	·3491	0 . 7 $\frac{1}{2}$	·0129	2·1754
9	2 . 4 $\frac{1}{2}$	·4418	0 . 8 $\frac{1}{2}$	·0164	2·7533
10	2 . 7 $\frac{1}{2}$	·5454	0 . 8 $\frac{3}{4}$	·0202	3·3991
11	2 . 10 $\frac{1}{2}$	·6600	0 . 9 $\frac{1}{2}$	·0244	4·1129
1 <i>ft.</i>	3 . 1 $\frac{1}{2}$	·7854	0 . 10 $\frac{1}{2}$	·0291	4·8947
1	3 . 4 $\frac{1}{2}$	·9218	0 . 11 $\frac{1}{2}$	·0341	5·7445
2	3 . 8	1·0690	1 . 0 $\frac{1}{2}$	·0396	6·6622
3	3 . 11 $\frac{1}{2}$	1·2272	1 . 1 $\frac{1}{2}$	·0455	7·6479
4	4 . 2 $\frac{1}{2}$	1·3963	1 . 2 $\frac{1}{2}$	·0517	8·7017
5	4 . 5 $\frac{1}{2}$	1·5763	1 . 3 $\frac{1}{2}$	·0584	9·8234
6	4 . 8 $\frac{1}{2}$	1·7671	1 . 4	·0654	11·0130
7	4 . 11 $\frac{1}{2}$	1·9689	1 . 4 $\frac{1}{2}$	·0729	12·2707
8	5 . 2 $\frac{1}{2}$	2·1817	1 . 5 $\frac{1}{2}$	·0808	13·5963
9	5 . 6	2·4053	1 . 6 $\frac{1}{2}$	·0891	14·9900
10	5 . 9 $\frac{1}{2}$	2·6398	1 . 7 $\frac{1}{2}$	·0978	16·4516
11	6 . 0 $\frac{1}{2}$	2·8852	1 . 8 $\frac{1}{2}$	·1069	17·9812
2 <i>ft.</i>	6 . 3 $\frac{1}{2}$	3·1416	1 . 9 $\frac{1}{2}$	·1164	19·5787
1	6 . 6 $\frac{1}{2}$	3·4088	1 . 10 $\frac{1}{2}$	·1263	21·2423
2	6 . 9 $\frac{1}{2}$	3·6870	1 . 11	·1366	22·9778
3	7 . 0 $\frac{1}{2}$	3·9761	1 . 11 $\frac{1}{2}$	·1473	24·7793
4	7 . 4	4·2761	2 . 0 $\frac{1}{2}$	·1584	26·6488
5	7 . 7 $\frac{1}{2}$	4·5869	2 . 1 $\frac{1}{2}$	·1699	28·5863
6	7 . 10 $\frac{1}{2}$	4·9087	2 . 2 $\frac{1}{2}$	·1818	30·5918
7	8 . 1 $\frac{1}{2}$	5·2414	2 . 3 $\frac{1}{2}$	·1941	32·6652
8	8 . 4 $\frac{1}{2}$	5·5851	2 . 4 $\frac{1}{2}$	·2069	34·8066
9	8 . 7 $\frac{1}{2}$	5·9396	2 . 5 $\frac{1}{2}$	·2200	37·0161
10	8 . 10 $\frac{1}{2}$	6·3050	2 . 6 $\frac{1}{2}$	·2335	39·2934
11	9 . 2	6·6813	2 . 7	·2475	41·6388
3 <i>ft.</i>	9 . 5 $\frac{1}{2}$	7·0686	2 . 7 $\frac{1}{2}$	·2618	44·0522
1	9 . 8 $\frac{1}{2}$	7·4667	2 . 8 $\frac{1}{2}$	·2765	46·5335
2	9 . 11 $\frac{1}{2}$	7·8758	2 . 9 $\frac{1}{2}$	·2917	49·0828
3	10 . 2 $\frac{1}{2}$	8·2958	2 . 10 $\frac{1}{2}$	·3073	51·7001
4	10 . 5 $\frac{1}{2}$	8·7266	2 . 11 $\frac{1}{2}$	·3232	54·3854
5	10 . 8 $\frac{1}{2}$	9·1684	3 . 0 $\frac{1}{2}$	·3396	57·1386
6	11 . 0	9·6211	3 . 1 $\frac{1}{2}$	·3563	59·9599
7	11 . 3 $\frac{1}{2}$	10·0847	3 . 2 $\frac{1}{2}$	·3735	62·8491
8	11 . 6 $\frac{1}{2}$	10·5592	3 . 3 $\frac{1}{2}$	·3911	65·8063
9	11 . 9 $\frac{1}{2}$	11·0447	3 . 3 $\frac{3}{4}$	·4091	68·8315
10	12 . 0 $\frac{1}{2}$	11·5410	3 . 4 $\frac{1}{2}$	·4274	71·9247
11	12 . 3 $\frac{1}{2}$	12·0482	3 . 5 $\frac{1}{2}$	·4462	75·0858

TABLE II.

Dia. in feet and inches.		Circum. in feet and inches.		Area in square feet.	Side of = square in ft. and in.	Cubic yards at one foot in depth.	Gallons at one foot in depth.
<i>ft.</i>	<i>in.</i>	<i>ft.</i>	<i>in.</i>		<i>ft.</i>	<i>in.</i>	
4	0	12	6 $\frac{3}{4}$	12-5664	3	6 $\frac{3}{4}$	78-3150
	1	12	10	13-0954	3	7 $\frac{7}{8}$	81-6121
	2	13	1 $\frac{1}{8}$	13-6354	3	8 $\frac{1}{4}$	84-9772
	3	13	4 $\frac{1}{2}$	14-1863	3	9 $\frac{1}{2}$	88-4102
	4	13	7 $\frac{7}{8}$	14-7480	3	10 $\frac{1}{8}$	91-9113
	5	13	10 $\frac{3}{4}$	15-3207	3	11 $\frac{1}{8}$	95-4803
	6	14	1 $\frac{1}{8}$	15-9043	3	11 $\frac{1}{2}$	99-1174
	7	14	4 $\frac{1}{2}$	16-4988	4	0 $\frac{1}{2}$	102-8224
	8	14	7 $\frac{7}{8}$	17-1042	4	1 $\frac{1}{8}$	106-5954
	9	14	11 $\frac{1}{8}$	17-7205	4	2 $\frac{1}{8}$	110-4363
	10	15	2 $\frac{1}{4}$	18-3478	4	3 $\frac{1}{8}$	114-3453
	11	15	5 $\frac{1}{8}$	18-9859	4	4 $\frac{1}{4}$	118-3222
5	<i>ft.</i>	15	8 $\frac{1}{2}$	19-6350	4	5 $\frac{1}{8}$	122-3671
	1	15	11 $\frac{1}{8}$	20-2949	4	6 $\frac{1}{8}$	126-4800
	2	16	2 $\frac{1}{4}$	20-9658	4	7 $\frac{1}{8}$	130-6609
	3	16	5 $\frac{1}{8}$	21-6475	4	7 $\frac{7}{8}$	134-9097
	4	16	9	22-3402	4	8 $\frac{1}{2}$	139-2266
	5	17	0 $\frac{1}{4}$	23-0438	4	9 $\frac{1}{8}$	143-6114
	6	17	3 $\frac{1}{8}$	23-7583	4	10 $\frac{1}{8}$	148-0642
	7	17	6 $\frac{3}{4}$	24-4837	4	11 $\frac{1}{8}$	152-5850
	8	17	9 $\frac{7}{8}$	25-2200	5	0 $\frac{1}{4}$	157-1738
	9	18	0 $\frac{3}{4}$	25-9672	5	1 $\frac{1}{8}$	161-8305
	10	18	3 $\frac{1}{4}$	26-7254	5	2 $\frac{1}{8}$	166-5552
	11	18	7	27-4944	5	2 $\frac{1}{2}$	171-3480
6	<i>ft.</i>	18	10 $\frac{1}{4}$	28-2743	5	3 $\frac{1}{8}$	176-2086
	1	19	1 $\frac{1}{8}$	29-0652	5	4 $\frac{1}{8}$	181-1373
	2	19	4 $\frac{1}{2}$	29-8669	5	5 $\frac{1}{8}$	186-1340
	3	19	7 $\frac{7}{8}$	30-6796	5	6 $\frac{1}{8}$	191-1986
	4	19	10 $\frac{3}{4}$	31-5032	5	7 $\frac{1}{8}$	196-3312
	5	20	1 $\frac{1}{4}$	32-3377	5	8 $\frac{1}{8}$	201-5318
	6	20	5	33-1831	5	9 $\frac{1}{8}$	206-8004
	7	20	8 $\frac{1}{4}$	34-0394	5	10 $\frac{1}{8}$	212-1370
	8	20	11 $\frac{1}{8}$	34-9066	5	10 $\frac{3}{4}$	217-5415
	9	21	2 $\frac{1}{4}$	35-7847	5	11 $\frac{1}{8}$	223-0141
	10	21	5 $\frac{1}{8}$	36-6737	6	0 $\frac{1}{8}$	228-5546
	11	21	8 $\frac{1}{4}$	37-5737	6	1 $\frac{1}{8}$	234-1631
7	<i>ft.</i>	21	11 $\frac{1}{4}$	38-4846	6	2 $\frac{1}{8}$	239-8395
	1	22	3	39-4063	6	3 $\frac{1}{8}$	245-5840
	2	22	6 $\frac{1}{4}$	40-3389	6	4 $\frac{1}{8}$	251-3964
	3	22	9 $\frac{1}{4}$	41-2825	6	5 $\frac{1}{8}$	257-2769
	4	23	0 $\frac{1}{4}$	42-2370	6	6 $\frac{1}{8}$	263-2253
	5	23	3 $\frac{1}{8}$	43-2024	6	6 $\frac{3}{4}$	269-2416
	6	23	6 $\frac{3}{8}$	44-1786	6	7 $\frac{1}{8}$	275-3260
	7	23	9 $\frac{7}{8}$	45-1658	6	8 $\frac{1}{8}$	281-4784
	8	24	1	46-1640	6	9 $\frac{1}{8}$	287-6987
	9	24	4 $\frac{1}{4}$	47-1730	6	10 $\frac{1}{8}$	293-9870
	10	24	7 $\frac{1}{4}$	48-1929	6	11 $\frac{1}{8}$	300-3433
	11	24	10 $\frac{1}{4}$	49-2237	7	0 $\frac{1}{4}$	306-7676

TABLE II.

Dia. in feet and inches.		Circum. in feet and inches.		Area in square feet.	Side of = square in ft. and in.	Cubic yards at one foot in depth.	Gallons at one foot in depth.	
ft.	in.	ft.	in.		ft.	in.		
8	0	25	1½	50-2655	7	1½	1-8617	313-2598
	1	25	4½	51-3181	7	2	1-9007	319-8201
	2	25	7½	52-3817	7	2½	1-9401	326-4483
	3	25	11	53-4562	7	3½	1-9799	333-1445
	4	26	2½	54-5415	7	4½	2-0201	339-9087
	5	26	5½	55-6378	7	5½	2-0607	346-7408
	6	26	8½	56-7450	7	6½	2-1017	353-6410
	7	26	11½	57-8631	7	7½	2-1431	360-6091
	8	27	2½	58-9921	7	8½	2-1849	367-6452
	9	27	5½	60-1320	7	9	2-2271	374-7493
	10	27	9	61-2829	7	10	2-2697	381-9214
	11	28	0½	62-4446	7	10½	2-3128	389-1614
9 ft.		28	3½	63-6173	7	11½	2-3562	396-4695
	1	28	6½	64-8008	8	0½	2-4000	403-8455
	2	28	9½	65-9953	8	1½	2-4443	411-2895
	3	29	0½	67-2006	8	2½	2-4889	418-8015
	4	29	3½	68-4169	8	3½	2-5340	426-3814
	5	29	7	69-6441	8	4½	2-5794	434-0294
	6	29	10½	70-8822	8	5	2-6253	441-7453
	7	30	1½	72-1312	8	5½	2-6715	449-5292
	8	30	4½	73-3911	8	6½	2-7182	457-3811
	9	30	7½	74-6619	8	7½	2-7653	465-3010
	10	30	10½	75-9436	8	8½	2-8127	473-2888
	11	31	1½	77-2363	8	9½	2-8606	481-3446
10 ft.		31	5	78-5398	8	10½	2-9089	489-4685
	1	31	8½	79-8543	8	11½	2-9576	497-6603
	2	31	11½	81-1796	9	0½	3-0067	505-9200
	3	32	2½	82-5159	9	1	3-0561	514-2478
	4	32	5½	83-8631	9	1½	3-1060	522-6435
	5	32	8½	85-2212	9	2½	3-1563	531-1073
	6	32	11½	86-5901	9	3½	3-2070	539-6390
	7	33	3	87-9700	9	4½	3-2581	548-2387
	8	33	6½	89-3609	9	5½	3-3097	556-9063
	9	33	9½	90-7626	9	6½	3-3616	565-6420
	10	34	0½	92-1752	9	7½	3-4139	574-4456
	11	34	3½	93-5987	9	8½	3-4666	583-3172
11 ft.		34	6½	95-0332	9	9	3-5197	592-2568
	1	34	9½	96-4785	9	9½	3-5733	601-2644
	2	35	1	97-9348	9	10½	3-6272	610-3400
	3	35	4½	99-4020	9	11½	3-6816	619-4835
	4	35	7½	100-8800	10	0½	3-7363	628-6951
	5	35	10½	102-3690	10	1½	3-7914	637-9746
	6	36	1½	103-8689	10	2½	3-8470	647-3220
	7	36	4½	105-3797	10	3½	3-9030	656-7375
	8	36	7½	106-9014	10	4½	3-9593	666-2210
	9	36	11	108-4340	10	5	4-0161	675-7724
	10	37	2½	109-9776	10	5½	4-0732	685-3918
	11	37	5½	111-5320	10	6½	4-1308	695-0792

TABLE II.

Dia. in feet and inches.		Circum. in feet and inches.		Area in square feet.	Side of = square in ft. and in.	Cubic yards at one foot in depth.	Gallons at one foot in depth.	
ft.	in.	ft.	in.		ft.	in.		
12.	0	37.	8 $\frac{1}{2}$	113.0973	10.	7 $\frac{1}{2}$	4.1888	704.8346
	1	37.	11 $\frac{1}{2}$	114.6736	10.	8 $\frac{1}{2}$	4.2472	714.6579
	2	38.	2 $\frac{1}{2}$	116.2607	10.	9 $\frac{1}{2}$	4.3060	724.5493
	3	38.	5 $\frac{1}{2}$	117.8588	10.	10 $\frac{1}{2}$	4.3651	734.5086
	4	38.	9	119.4678	10.	11 $\frac{1}{2}$	4.4247	744.5359
	5	39.	0 $\frac{1}{2}$	121.0877	11.	0	4.4847	754.6312
	6	39.	3 $\frac{1}{2}$	122.7185	11.	0 $\frac{1}{2}$	4.5451	764.7945
	7	39.	6 $\frac{1}{2}$	124.3602	11.	1 $\frac{1}{2}$	4.6059	775.0257
	8	39.	9 $\frac{1}{2}$	126.0128	11.	2 $\frac{1}{2}$	4.6671	785.3250
	9	40.	0 $\frac{1}{2}$	127.6763	11.	3 $\frac{1}{2}$	4.7288	795.6922
	10	40.	3 $\frac{1}{2}$	129.3507	11.	4 $\frac{1}{2}$	4.7908	806.1274
	11	40.	7	131.0360	11.	5 $\frac{1}{2}$	4.8532	816.6305
13 ft.	0	40.	10 $\frac{1}{2}$	132.7323	11.	6 $\frac{1}{2}$	4.9160	827.2017
	1	41.	1 $\frac{1}{2}$	134.4394	11.	7 $\frac{1}{2}$	4.9792	837.8408
	2	41.	4 $\frac{1}{2}$	136.1575	11.	8 $\frac{1}{2}$	5.0429	848.5480
	3	41.	7 $\frac{1}{2}$	137.8865	11.	9 $\frac{1}{2}$	5.1069	859.3231
	4	41.	10 $\frac{1}{2}$	139.6263	11.	10 $\frac{1}{2}$	5.1713	870.1662
	5	42.	1 $\frac{1}{2}$	141.3771	11.	11 $\frac{1}{2}$	5.2362	881.0772
	6	42.	5	143.1388	11.	11 $\frac{1}{2}$	5.3014	892.0563
	7	42.	8 $\frac{1}{2}$	144.9114	12.	0	5.3671	903.1033
	8	42.	11 $\frac{1}{2}$	146.6949	12.	1 $\frac{1}{2}$	5.4331	914.2183
	9	43.	2 $\frac{1}{2}$	148.4893	12.	2 $\frac{1}{2}$	5.4996	925.4013
	10	43.	5 $\frac{1}{2}$	150.2947	12.	3 $\frac{1}{2}$	5.5665	936.6523
	11	43.	8 $\frac{1}{2}$	152.1109	12.	4 $\frac{1}{2}$	5.6337	947.9713
14 ft.	0	43.	11 $\frac{1}{2}$	153.9380	12.	5 $\frac{1}{2}$	5.7014	959.3582
	1	44.	2 $\frac{1}{2}$	155.7761	12.	6 $\frac{1}{2}$	5.7695	970.8131
	2	44.	6 $\frac{1}{2}$	157.6250	12.	6 $\frac{1}{2}$	5.8380	982.3360
	3	44.	9 $\frac{1}{2}$	159.4849	12.	7 $\frac{1}{2}$	5.9068	993.9269
	4	45.	0 $\frac{1}{2}$	161.3557	12.	8 $\frac{1}{2}$	5.9761	1005.5858
	5	45.	3 $\frac{1}{2}$	163.2374	12.	9 $\frac{1}{2}$	6.0458	1017.3126
	6	45.	6 $\frac{1}{2}$	165.1300	12.	10 $\frac{1}{2}$	6.1159	1029.1074
	7	45.	9 $\frac{1}{2}$	167.0335	12.	11 $\frac{1}{2}$	6.1864	1040.9703
	8	46.	0 $\frac{1}{2}$	168.9479	13.	0	6.2573	1052.9011
	9	46.	4	170.8732	13.	0 $\frac{1}{2}$	6.3286	1064.8998
	10	46.	7 $\frac{1}{2}$	172.8094	13.	1 $\frac{1}{2}$	6.4003	1076.9666
	11	46.	10 $\frac{1}{2}$	174.7565	13.	2 $\frac{1}{2}$	6.4725	1089.1013
15 ft.	0	47.	1 $\frac{1}{2}$	176.7146	13.	3 $\frac{1}{2}$	6.5450	1101.3040
	1	47.	4 $\frac{1}{2}$	178.6835	13.	4 $\frac{1}{2}$	6.6179	1113.5747
	2	47.	7 $\frac{1}{2}$	180.6634	13.	5 $\frac{1}{2}$	6.6912	1125.9134
	3	47.	10 $\frac{1}{2}$	182.6542	13.	6 $\frac{1}{2}$	6.7650	1138.3201
	4	48.	2 $\frac{1}{2}$	184.6558	13.	7 $\frac{1}{2}$	6.8391	1150.7947
	5	48.	5 $\frac{1}{2}$	186.6694	13.	8 $\frac{1}{2}$	6.9136	1163.3374
	6	48.	8 $\frac{1}{2}$	188.6919	13.	9 $\frac{1}{2}$	6.9886	1175.9480
	7	48.	11 $\frac{1}{2}$	190.7263	13.	10 $\frac{1}{2}$	7.0639	1188.6266
	8	49.	2 $\frac{1}{2}$	192.7716	13.	11 $\frac{1}{2}$	7.1397	1201.3732
	9	49.	5 $\frac{1}{2}$	194.8278	13.	11 $\frac{1}{2}$	7.2158	1214.1877
	10	49.	8 $\frac{1}{2}$	196.8950	14.	0	7.2924	1227.0702
	11	50.	0	198.9730	14.	1 $\frac{1}{2}$	7.3694	1240.0208

TABLE II.

Dia. in feet and inches.		Circum. in feet and inches.		Area in square feet.	Side of = square in ft. and in.		Cubic yards at one foot in depth.	Gallons at one foot in depth.
ft.	in.	ft.	in.		ft.	in.		
16	0	50	3 $\frac{1}{2}$	201-0619	14	2 $\frac{1}{2}$	7-4467	1253-0393
	1	50	6 $\frac{1}{2}$	203-1618	14	3	7-5245	1266-1258
	2	50	9 $\frac{1}{2}$	205-2725	14	3 $\frac{1}{2}$	7-6027	1279-2802
	3	51	0 $\frac{1}{2}$	207-3942	14	4 $\frac{1}{2}$	7-6813	1292-5027
	4	51	3 $\frac{1}{2}$	209-5268	14	5 $\frac{1}{2}$	7-7603	1305-7931
	5	51	6 $\frac{1}{2}$	211-6703	14	6 $\frac{1}{2}$	7-8396	1319-1515
	6	51	10	213-8246	14	7 $\frac{1}{2}$	7-9194	1332-5779
	7	52	1 $\frac{1}{2}$	215-9899	14	8 $\frac{1}{2}$	7-9996	1346-0723
	8	52	4 $\frac{1}{2}$	218-1662	14	9 $\frac{1}{2}$	8-0802	1359-6346
	9	52	7 $\frac{1}{2}$	220-3533	14	10 $\frac{1}{2}$	8-1612	1373-2650
	10	52	10 $\frac{1}{2}$	222-5513	14	11 $\frac{1}{2}$	8-2426	1386-9633
	11	53	1 $\frac{1}{2}$	224-7602	14	11 $\frac{1}{2}$	8-3445	1400-7296
17	ft.	53	4 $\frac{1}{2}$	226-9801	15	0 $\frac{1}{2}$	8-4067	1414-5639
	1	53	8	229-2108	15	1 $\frac{1}{2}$	8-4893	1428-4661
	2	53	11 $\frac{1}{2}$	231-4525	15	2 $\frac{1}{2}$	8-5723	1442-4364
	3	54	2 $\frac{1}{2}$	233-7050	15	3 $\frac{1}{2}$	8-6557	1456-4746
	4	54	5 $\frac{1}{2}$	235-9685	15	4 $\frac{1}{2}$	8-7396	1470-5808
	5	54	8 $\frac{1}{2}$	238-2429	15	5 $\frac{1}{2}$	8-8238	1484-7550
	6	54	11 $\frac{1}{2}$	240-5282	15	6 $\frac{1}{2}$	8-9085	1498-9972
	7	55	2 $\frac{1}{2}$	242-8244	15	7	8-9935	1513-3073
	8	55	6	245-1315	15	7 $\frac{1}{2}$	9-0789	1527-6855
	9	55	9 $\frac{1}{2}$	247-4495	15	8 $\frac{1}{2}$	9-1648	1542-1316
	10	56	0 $\frac{1}{2}$	249-7784	15	9 $\frac{1}{2}$	9-2511	1556-6457
	11	56	3 $\frac{1}{2}$	252-1183	15	10 $\frac{1}{2}$	9-3377	1571-2278
18	ft.	56	6 $\frac{1}{2}$	254-4690	15	11 $\frac{1}{2}$	9-4248	1585-8778
	1	56	9 $\frac{1}{2}$	256-8307	16	0 $\frac{1}{2}$	9-5122	1600-5959
	2	57	0 $\frac{1}{2}$	259-2032	16	1 $\frac{1}{2}$	9-6001	1615-3819
	3	57	4 $\frac{1}{2}$	261-5867	16	2 $\frac{1}{2}$	9-6884	1630-2359
	4	57	7 $\frac{1}{2}$	263-9810	16	3 $\frac{1}{2}$	9-7771	1645-1579
	5	57	10 $\frac{1}{2}$	266-3863	16	4 $\frac{1}{2}$	9-8662	1660-1479
	6	58	1 $\frac{1}{2}$	268-8025	16	5 $\frac{1}{2}$	9-9556	1675-2058
	7	58	4 $\frac{1}{2}$	271-2296	16	6 $\frac{1}{2}$	10-0455	1690-3318
	8	58	7 $\frac{1}{2}$	273-6676	16	7 $\frac{1}{2}$	10-1358	1705-5257
	9	58	10 $\frac{1}{2}$	276-1165	16	8 $\frac{1}{2}$	10-2265	1720-7876
	10	59	2	278-5764	16	9 $\frac{1}{2}$	10-3176	1736-1175
	11	59	5 $\frac{1}{2}$	281-0471	16	10 $\frac{1}{2}$	10-4092	1751-5153
19	ft.	59	8 $\frac{1}{2}$	283-5287	16	11 $\frac{1}{2}$	10-5011	1766-9812
	1	59	11 $\frac{1}{2}$	286-0213	16	12 $\frac{1}{2}$	10-5934	1782-5150
	2	60	2 $\frac{1}{2}$	288-5247	17	0 $\frac{1}{2}$	10-6861	1798-1168
	3	60	5 $\frac{1}{2}$	291-0391	17	1 $\frac{1}{2}$	10-7792	1813-7866
	4	60	8 $\frac{1}{2}$	293-5644	17	2 $\frac{1}{2}$	10-8728	1829-5244
	5	61	0	296-1106	17	3 $\frac{1}{2}$	10-9667	1845-3301
	6	61	3 $\frac{1}{2}$	298-6477	17	4 $\frac{1}{2}$	11-0610	1861-2038
	7	61	6 $\frac{1}{2}$	301-2057	17	5 $\frac{1}{2}$	11-1558	1877-1456
	8	61	9 $\frac{1}{2}$	303-7746	17	6 $\frac{1}{2}$	11-2509	1893-1553
	9	62	0 $\frac{1}{2}$	306-3544	17	7 $\frac{1}{2}$	11-3465	1909-2329
	10	62	3 $\frac{1}{2}$	308-9451	17	8 $\frac{1}{2}$	11-4424	1925-3786
	11	62	6 $\frac{1}{2}$	311-5467	17	9 $\frac{1}{2}$	11-5388	1941-5922

TABLE II.

Dia. in feet and inches.		Circum. in feet and inches.		Area in square feet.	Side of = square in ft. and in.		Cubic yards at one foot in depth.	Gallons at one foot in depth.
ft.	in.	ft.	in.		ft.	in.		
20	0	62	10	314.1593	17	8 ³ / ₈	11.6355	1957.8739
	1	63	1 ¹ / ₈	316.7827	17	9 ³ / ₈	11.7327	1974.2235
	2	63	4 ¹ / ₈	319.4171	17	10 ³ / ₈	11.8303	1990.6411
	3	63	7 ¹ / ₈	322.0623	17	11 ³ / ₈	11.9282	2007.1266
	4	63	10 ¹ / ₈	324.7185	18	0	12.0266	2023.6802
	5	64	1 ¹ / ₈	327.3856	18	1 ¹ / ₈	12.1254	2040.3017
	6	64	4 ¹ / ₈	330.0636	18	2	12.2246	2056.9912
	7	64	8	332.7525	18	2 ³ / ₈	12.3242	2073.7487
	8	64	11 ¹ / ₈	335.4523	18	3 ³ / ₈	12.4242	2090.5742
	9	65	2 ¹ / ₈	338.1630	18	4 ¹ / ₈	12.5246	2107.4677
	10	65	5 ¹ / ₈	340.8846	18	5 ¹ / ₈	12.6254	2124.4291
	11	65	8 ¹ / ₈	343.6172	18	6 ¹ / ₈	12.7266	2141.4585
21 ft.		65	11 ¹ / ₈	346.3606	18	7 ¹ / ₈	12.8282	2158.5559
	1	66	2 ¹ / ₈	349.1149	18	8 ¹ / ₈	12.9302	2175.7213
	2	66	6	351.8802	18	9 ¹ / ₈	13.0326	2192.9547
	3	66	9 ¹ / ₈	354.6564	18	10	13.1354	2210.2560
	4	67	0	357.4434	18	10 ³ / ₈	13.2386	2227.6254
	5	67	3 ¹ / ₈	360.2414	18	11 ¹ / ₈	13.3423	2245.0627
	6	67	6 ¹ / ₈	363.0503	19	0	13.4463	2262.5680
	7	67	9 ¹ / ₈	365.8701	19	1 ¹ / ₈	13.5507	2280.1413
	8	68	0	368.7008	19	2 ¹ / ₈	13.6556	2297.7825
	9	68	4	371.5424	19	3 ¹ / ₈	13.7608	2315.4918
	10	68	7 ¹ / ₈	374.3949	19	4 ¹ / ₈	13.8665	2333.2690
	11	68	10 ¹ / ₈	377.2584	19	5 ¹ / ₈	13.9725	2351.1142
22 ft.		69	1 ¹ / ₈	380.1327	19	6	14.0790	2369.0274
	1	69	4 ¹ / ₈	383.0180	19	6 ³ / ₈	14.1859	2387.0085
	2	69	7 ¹ / ₈	385.9141	19	7 ¹ / ₈	14.2931	2405.0577
	3	69	10 ¹ / ₈	388.8212	19	8 ¹ / ₈	14.4008	2423.1748
	4	70	1 ¹ / ₈	391.7392	19	9 ¹ / ₈	14.5089	2441.3599
	5	70	5 ¹ / ₈	394.6680	19	10 ¹ / ₈	14.6173	2459.6130
	6	70	8 ¹ / ₈	397.6078	19	11 ¹ / ₈	14.7262	2477.9341
	7	70	11 ¹ / ₈	400.5585	20	0 ¹ / ₈	14.8355	2496.3232
	8	71	2 ¹ / ₈	403.5201	20	1	14.9452	2514.7802
	9	71	5 ¹ / ₈	406.4926	20	2	15.0553	2533.3052
	10	71	8 ¹ / ₈	409.4761	20	2 ³ / ₈	15.1658	2551.8982
	11	72	0	412.4704	20	3 ¹ / ₈	15.2767	2570.5592
23 ft.		72	3 ¹ / ₈	415.4756	20	4 ¹ / ₈	15.3880	2589.2882
	1	72	6 ¹ / ₈	418.4918	20	5 ¹ / ₈	15.4997	2608.0851
	2	72	9 ¹ / ₈	421.5188	20	6 ¹ / ₈	15.6118	2626.9501
	3	73	0	424.5568	20	7 ¹ / ₈	15.7243	2645.8830
	4	73	3 ¹ / ₈	427.6057	20	8 ¹ / ₈	15.8372	2664.8839
	5	73	6 ¹ / ₈	430.6654	20	9	15.9506	2683.9527
	6	73	9 ¹ / ₈	433.7361	20	10 ¹ / ₈	16.0643	2703.0896
	7	74	1 ¹ / ₈	436.8177	20	10 ³ / ₈	16.1784	2722.2944
	8	74	4 ¹ / ₈	439.9102	20	11 ¹ / ₈	16.2930	2741.5673
	9	74	7 ¹ / ₈	443.0137	21	0	16.4079	2760.9081
	10	74	10 ¹ / ₈	446.1280	21	1 ¹ / ₈	16.5233	2780.3168
	11	75	1 ¹ / ₈	449.2532	21	2 ¹ / ₈	16.6390	2799.7936

TABLE II.

Dia. in feet and inches.		Circum. in feet and inches.		Area in square feet.	Side of = square in ft. and in.		Cubic yards at one foot in depth.	Gallons at one foot in depth.
ft.	in.	ft.	in.		ft.	in.		
24	0	75	43	452-3893	21	34	16-7552	2819-3384
	1	75	74	455-5364	21	44	16-8717	2838-9511
	2	75	11	458-6943	21	5	16-9887	2858-6318
	3	76	24	461-8632	21	54	17-1060	2878-3805
	4	76	54	465-0430	21	64	17-2238	2898-1972
	5	76	84	468-2337	21	74	17-3420	2918-0318
	6	76	114	471-4352	21	84	17-4606	2938-0345
	7	77	24	474-6477	21	94	17-5795	2958-0551
	8	77	54	477-8711	21	104	17-6989	2978-1437
	9	77	9	481-1055	21	114	17-8187	2998-3003
	10	78	04	484-3507	22	04	17-9389	3018-5248
25	11	78	34	487-6068	22	1	18-0595	3038-8174
	12	78	64	490-8739	22	12	18-1805	3059-1779
	1	78	94	494-1518	22	22	18-3019	3079-6064
	2	79	04	497-4407	22	34	18-4237	3100-1029
	3	79	34	500-7404	22	44	18-5459	3120-6674
	4	79	7	504-0511	22	54	18-6686	3141-2998
	5	79	104	507-3727	22	64	18-7916	3162-0003
	6	80	14	510-7052	22	74	18-9150	3182-7687
	7	80	44	514-0486	22	84	19-0388	3203-6051
	8	80	74	517-4029	22	94	19-1631	3224-5095
	9	80	104	520-7681	22	94	19-2877	3245-4818
26	10	81	14	524-1442	22	104	19-4127	3266-5222
	11	81	5	527-5312	22	114	19-5382	3287-6305
	12	81	84	530-9292	23	04	19-6640	3308-8068
	1	81	114	534-3380	23	14	19-7903	3330-0511
	2	82	24	537-7578	23	24	19-9170	3351-3634
	3	82	54	541-1884	23	34	20-0440	3372-7436
	4	82	84	544-6300	23	44	20-1715	3394-1919
	5	82	114	548-0825	23	44	20-2994	3415-7081
	6	83	3	551-5459	23	54	20-4276	3437-2923
	7	83	64	555-0202	23	64	20-5563	3458-9445
	8	83	94	558-5054	23	74	20-6854	3480-6646
27	9	84	04	562-0015	23	84	20-8149	3502-4528
	10	84	34	565-5085	23	94	20-9448	3524-3089
	11	84	64	569-0264	23	104	21-0751	3546-2330
	12	84	94	572-5553	23	114	21-2058	3568-2251
	1	85	1	576-0950	24	0	21-3369	3590-2852
	2	85	44	579-6457	24	04	21-4684	3612-4132
	3	85	74	583-2072	24	14	21-6003	3634-6093
	4	85	104	586-7797	24	24	21-7326	3656-8733
	5	86	14	590-3631	24	34	21-8653	3679-2053
	6	86	44	593-9574	24	44	21-9984	3701-6053
	7	86	74	597-5626	24	54	22-1319	3724-0732
28	8	86	11	601-1787	24	64	22-2659	3746-6092
	9	87	24	604-8057	24	74	22-4002	3769-2181
	10	87	54	608-4436	24	84	22-5349	3791-8850
	11	87	84	612-0921	24	84	22-6701	3814-6249

TABLE II.

Dia. in feet and inches.		Circum. in feet and inches.		Area in square feet.	Side of = square in ft. and in.		Cubic yards at one foot in depth.	Gallons at one foot in depth.
ft.	in.	ft.	in.		ft.	in.		
28	0	87	11 ⁸ / ₈	615.7522	24	9 ³ / ₈	22.8056	3837.4328
	1	88	2 ³ / ₈	619.4228	24	10 ³ / ₈	22.9416	3860.3086
	2	88	5 ³ / ₈	623.1044	24	11 ³ / ₈	23.0779	3883.2525
	3	88	9	626.7968	25	0 ³ / ₈	23.2147	3906.2643
	4	89	0 ³ / ₈	630.5002	25	1 ³ / ₈	23.3519	3929.3441
	5	89	3 ³ / ₈	634.2145	25	2 ³ / ₈	23.4894	3952.4918
	6	89	6 ³ / ₈	637.9397	25	3 ³ / ₈	23.6274	3975.7076
	7	89	9 ³ / ₈	641.6758	25	4 ³ / ₈	23.7658	3998.9914
	8	90	0 ³ / ₈	645.4228	25	4 ³ / ₈	23.9045	4022.3431
	9	90	3 ³ / ₈	649.1807	25	5 ³ / ₈	24.0437	4045.7628
	10	90	7	652.9495	25	6 ³ / ₈	24.1833	4069.2505
29	ft.	90	10 ³ / ₈	656.7292	25	7 ³ / ₈	24.3233	4092.8061
	1	91	1 ³ / ₈	660.5199	25	8 ³ / ₈	24.4637	4116.4298
	2	91	4 ³ / ₈	664.3214	25	9 ³ / ₈	24.6045	4140.1214
	3	91	7 ³ / ₈	668.1339	25	10 ³ / ₈	24.7457	4163.8810
	4	91	10 ³ / ₈	671.9572	25	11 ³ / ₈	24.8873	4187.7086
	5	92	1 ³ / ₈	675.7915	26	0	25.0293	4211.6042
	6	92	5	679.6367	26	0 ³ / ₈	25.1717	4235.5678
	7	92	8 ³ / ₈	683.4928	26	1 ³ / ₈	25.3145	4259.5993
	8	92	11 ³ / ₈	687.3597	26	2 ³ / ₈	25.4578	4283.6988
	9	93	2 ³ / ₈	691.2377	26	3 ³ / ₈	25.6014	4307.8663
	10	93	5 ³ / ₈	695.1265	26	4 ³ / ₈	25.7454	4332.1018
30	ft.	93	8 ³ / ₈	699.0262	26	5 ³ / ₈	25.8899	4356.4053
	1	93	11 ³ / ₈	702.9368	26	6 ³ / ₈	26.0347	4380.7768
	2	94	3	706.8583	26	7	26.1799	4405.2162
	3	94	6 ³ / ₈	710.7908	26	7 ³ / ₈	26.3256	4429.7236
	4	94	9 ³ / ₈	714.7341	26	8 ³ / ₈	26.4716	4454.2990
	5	95	0 ³ / ₈	718.6884	26	9 ³ / ₈	26.6181	4478.9424
	6	95	3 ³ / ₈	722.6536	26	10 ³ / ₈	26.7649	4503.6537
	7	95	6 ³ / ₈	726.6297	26	11 ³ / ₈	26.9122	4528.4331
	8	95	9 ³ / ₈	730.6166	27	0 ³ / ₈	27.0599	4553.2804
	9	96	1	734.6145	27	1 ³ / ₈	27.2079	4578.1957
	10	96	4 ³ / ₈	738.6233	27	2 ³ / ₈	27.3564	4603.1790
31	ft.	96	7 ³ / ₈	742.6431	27	3 ³ / ₈	27.5053	4628.2303
	1	96	10 ³ / ₈	746.6737	27	3 ³ / ₈	27.6546	4653.3495
	2	97	1	750.7152	27	4 ³ / ₈	27.8043	4678.5367
	3	97	4 ³ / ₈	754.7676	27	5 ³ / ₈	27.9544	4703.7919
	4	97	7 ³ / ₈	758.8310	27	6 ³ / ₈	28.1049	4729.1151
	5	97	11 ³ / ₈	762.9052	27	7 ³ / ₈	28.2557	4754.5063
	6	98	2 ³ / ₈	766.9904	27	8 ³ / ₈	28.4071	4779.9655
	7	98	5 ³ / ₈	771.0865	27	9 ³ / ₈	28.5588	4805.4926
	8	98	8 ³ / ₈	775.1934	27	10 ³ / ₈	28.7109	4831.0877
	9	98	11 ³ / ₈	779.3113	27	11 ³ / ₈	28.8634	4856.7508
	10	99	2 ³ / ₈	783.4401	27	11 ³ / ₈	29.0163	4882.4819
32	ft.	99	5 ³ / ₈	787.5798	28	0 ³ / ₈	29.1696	4908.2810
	1	99	9	791.7304	28	1 ³ / ₈	29.3233	4934.1480
	2	100	0 ³ / ₈	795.8920	28	2 ³ / ₈	29.4775	4960.0831
	3	100	3 ³ / ₈	800.0644	28	3 ³ / ₈	29.6320	4986.0861

TABLE II.

Dia. in feet and inches.		Circum. in feet and inches.		Area in square feet.	Side of square in ft. and in.	Cubic yards at one foot in depth.	Gallons at one foot in depth.	
ft.	in.	ft.	in.		ft.	in.		
32	0	100	68	804-2477	28	4½	29-7870	5012-1571
	1	100	99	808-4420	28	5½	29-9423	5038-2961
	2	101	08	812-6471	28	6½	30-0980	5064-5030
	3	101	38	816-8632	28	7	30-2542	5090-7780
	4	101	7	821-0901	28	7½	30-4107	5117-1209
	5	101	104	825-3280	28	8½	30-5678	5143-5318
	6	102	11	829-5768	28	9½	30-7251	5170-0107
	7	102	48	833-8365	28	10½	30-8828	5196-5575
	8	102	74	838-1071	28	11½	31-0410	5223-1724
	9	102	106	842-3886	29	0½	31-1996	5249-8552
	10	103	13	846-6810	29	1½	31-3586	5276-6060
	11	103	47	850-9844	29	2	31-5179	5303-4248
33	ft.	103	84	855-2986	29	3	31-6777	5330-3116
	1	103	112	859-6237	29	3½	31-8379	5357-2663
	2	104	28	863-9598	29	4½	31-9985	5384-2891
	3	104	54	868-3068	29	5½	32-1595	5411-3798
	4	104	88	872-6646	29	6½	32-3209	5438-5385
	5	104	118	877-0334	29	7½	32-4827	5465-7652
	6	105	24	881-4131	29	8½	32-6449	5493-0598
	7	105	6	885-8037	29	9½	32-8075	5520-4225
	8	105	94	890-2052	29	10	32-9706	5547-8531
	9	106	08	894-6176	29	10½	33-1340	5575-3517
	10	106	34	899-0409	29	11½	33-2978	5602-9183
	11	106	68	903-4751	30	0½	33-4620	5630-5529
34	ft.	106	94	907-9203	30	1½	33-6267	5658-2555
	1	107	04	912-3763	30	2½	33-7917	5686-0260
	2	107	4	916-8433	30	3½	33-9572	5713-8645
	3	107	74	921-3211	30	4½	34-1230	5741-7710
	4	107	108	925-8099	30	5½	34-2893	5769-7455
	5	108	14	930-3096	30	6	34-4559	5797-7880
	6	108	48	934-8202	30	6½	34-6230	5825-8984
	7	108	78	939-3417	30	7½	34-7904	5854-0768
	8	108	104	943-8741	30	8½	34-9583	5882-3232
	9	109	2	948-4174	30	9½	35-1266	5910-6376
	10	109	54	952-9716	30	10½	35-2952	5939-0200
	11	109	88	957-5367	30	11½	35-4643	5967-4704
35	ft.	109	114	962-1128	31	0½	35-6338	5995-9887
	1	110	28	966-6997	31	1½	35-8037	6024-5750
	2	110	54	971-2975	31	2	35-9740	6053-2293
	3	110	84	975-9063	31	3	36-1447	6081-9516
	4	111	0	980-5260	31	3½	36-3158	6110-7419
	5	111	34	985-1566	31	4½	36-4873	6139-6001
	6	111	68	989-7980	31	5½	36-6592	6168-5263
	7	111	94	994-4504	31	6½	36-8315	6197-5205
	8	112	08	999-1137	31	7½	37-0042	6226-5827
	9	112	38	1003-7879	31	8½	37-1773	6255-7129
	10	112	68	1008-4731	31	9½	37-3509	6284-9111
	11	112	10	1013-1691	31	10	37-5248	6314-1772

TABLE II.

Dia. in feet and inches.		Circum. in feet and inches.		Area in square feet.	Side of = square in ft. and in.	Cubic yards at one foot in depth.	Gallons at one foot in depth.	
ft.	in.	ft.	in.		ft.	in.		
36	0	113	1 $\frac{1}{2}$	1017.8760	31	10 $\frac{1}{2}$	37.6991	6343.5113
1		113	4 $\frac{1}{2}$	1022.5939	31	11 $\frac{1}{2}$	37.6738	6372.9134
2		113	7 $\frac{1}{2}$	1027.3226	32	0 $\frac{1}{2}$	38.0490	6402.3835
3		113	10 $\frac{1}{2}$	1032.0623	32	1 $\frac{1}{2}$	38.2245	6431.9215
4		114	1 $\frac{1}{2}$	1036.8128	32	2 $\frac{1}{2}$	38.4005	6461.5276
5		114	4 $\frac{1}{2}$	1041.5743	32	3 $\frac{1}{2}$	38.5768	6491.2016
6		114	8	1046.3467	32	4 $\frac{1}{2}$	38.7536	6520.9436
7		114	11 $\frac{1}{2}$	1051.1300	32	5	38.9307	6550.7536
8		115	2 $\frac{1}{2}$	1055.9242	32	6	39.1083	6580.6316
9		115	5 $\frac{1}{2}$	1060.7293	32	6 $\frac{1}{2}$	39.2863	6610.5775
10		115	8 $\frac{1}{2}$	1065.5453	32	7 $\frac{1}{2}$	39.4646	6640.5915
11		115	11 $\frac{1}{2}$	1070.3723	32	8 $\frac{1}{2}$	39.6434	6670.6734
37 ft.		116	2 $\frac{1}{2}$	1075.2101	32	9 $\frac{1}{2}$	39.8226	6700.8233
1		116	6	1080.0588	32	10 $\frac{1}{2}$	40.0222	6731.0412
2		116	9 $\frac{1}{2}$	1084.9185	32	11 $\frac{1}{2}$	40.1822	6761.3270
3		117	0 $\frac{1}{2}$	1089.7890	33	0 $\frac{1}{2}$	40.3626	6791.6809
4		117	3 $\frac{1}{2}$	1094.6705	33	1	40.5434	6822.1027
5		117	6 $\frac{1}{2}$	1099.5629	33	1 $\frac{1}{2}$	40.7246	6852.5925
6		117	9 $\frac{1}{2}$	1104.4662	33	2 $\frac{1}{2}$	40.9062	6883.1503
7		118	0 $\frac{1}{2}$	1109.3804	33	3 $\frac{1}{2}$	41.0882	6913.7761
8		118	4	1114.3055	33	4 $\frac{1}{2}$	41.2706	6944.4698
9		118	7 $\frac{1}{2}$	1119.2415	33	5 $\frac{1}{2}$	41.4534	6975.2315
10		118	10 $\frac{1}{2}$	1124.1884	33	6 $\frac{1}{2}$	41.6366	7006.0613
11		119	1 $\frac{1}{2}$	1129.1462	33	7 $\frac{1}{2}$	41.8202	7036.9590
38 ft.		119	4 $\frac{1}{2}$	1134.1149	33	8 $\frac{1}{2}$	42.0043	7067.9246
1		119	7 $\frac{1}{2}$	1139.0946	33	9	42.1887	7098.9583
2		119	10 $\frac{1}{2}$	1144.0851	33	9 $\frac{1}{2}$	42.3735	7130.0599
3		120	2	1149.0866	33	10 $\frac{1}{2}$	42.5588	7161.2296
4		120	5 $\frac{1}{2}$	1154.0990	33	11 $\frac{1}{2}$	42.7444	7192.4672
5		120	8 $\frac{1}{2}$	1159.1222	34	0 $\frac{1}{2}$	42.9305	7223.7728
6		120	11 $\frac{1}{2}$	1164.1564	34	1 $\frac{1}{2}$	43.1169	7255.1463
7		121	2 $\frac{1}{2}$	1169.2015	34	2 $\frac{1}{2}$	43.3038	7286.5879
8		121	5 $\frac{1}{2}$	1174.2575	34	3 $\frac{1}{2}$	43.4910	7318.0974
9		121	8 $\frac{1}{2}$	1179.3244	34	4 $\frac{1}{2}$	43.6787	7349.6749
10		122	0	1184.4022	34	5	43.8668	7381.3204
11		122	3 $\frac{1}{2}$	1189.4910	34	5 $\frac{1}{2}$	44.0552	7413.0539
39 ft.		122	6 $\frac{1}{2}$	1194.5906	34	6 $\frac{1}{2}$	44.2441	7444.8154
1		122	9 $\frac{1}{2}$	1199.7011	34	7 $\frac{1}{2}$	44.4334	7476.6648
2		123	0 $\frac{1}{2}$	1204.8226	34	8 $\frac{1}{2}$	44.6231	7508.5822
3		123	3 $\frac{1}{2}$	1209.9550	34	9 $\frac{1}{2}$	44.8131	7540.5676
4		123	6 $\frac{1}{2}$	1215.0982	34	10 $\frac{1}{2}$	45.0036	7572.6210
5		123	10	1220.2524	34	11 $\frac{1}{2}$	45.1945	7604.7424
6		124	1 $\frac{1}{2}$	1225.4175	35	0 $\frac{1}{2}$	45.3858	7636.9317
7		124	4 $\frac{1}{2}$	1230.5935	35	1	45.5775	7669.1891
8		124	7 $\frac{1}{2}$	1235.7804	35	1 $\frac{1}{2}$	45.7696	7701.5144
9		124	10 $\frac{1}{2}$	1240.9782	35	2 $\frac{1}{2}$	45.9622	7733.9077
10		125	1 $\frac{1}{2}$	1246.1869	35	3 $\frac{1}{2}$	46.1551	7766.3689
11		125	4 $\frac{1}{2}$	1251.4065	35	4 $\frac{1}{2}$	46.3484	7798.8982

TABLE II.

Dia. in feet and inches.		Circum. in feet and inches.		Area in square feet.	Side of square in ft. and in.		Cubic yards at one foot in depth.	Gallons at one foot in depth.
ft.	in.	ft.	in.		ft.	in.		
40	0	125	8	1256.6370	35	5 1/2	46.5421	7831.4954
	1	125	11 1/2	1261.8785	35	6 1/4	46.7362	7864.1607
	2	126	2	1267.1309	35	7 1/8	46.9308	7896.8939
	3	126	5 1/2	1272.3941	35	8 1/4	47.1257	7929.6951
	4	126	8 1/2	1277.6683	35	8 3/4	47.3210	7962.5642
	5	126	11 1/2	1282.9534	35	9 1/4	47.5168	7995.5014
	6	127	2	1288.2493	35	10 1/8	47.7129	8028.5065
	7	127	6	1293.5562	35	11 1/8	47.9095	8061.5796
	8	127	9 1/2	1298.8740	36	0	48.1064	8094.7207
	9	128	0	1304.2027	36	1 1/8	48.3038	8127.9298
	10	128	3 1/2	1309.5424	36	2 1/8	48.5016	8161.2068
	11	128	6 1/2	1314.8929	36	3 1/8	48.6997	8194.5519
41	ft.	128	9 1/2	1320.2543	36	4	48.8983	8227.9649
	1	129	0	1325.6267	36	4 1/4	49.0973	8261.4459
	2	129	4	1331.0099	36	5 1/8	49.2967	8294.9949
	3	129	7 1/2	1336.4041	36	6 1/8	49.4964	8328.6119
	4	129	10 1/2	1341.8091	36	7 1/8	49.6966	8362.2968
	5	130	1	1347.2251	36	8 1/8	49.8972	8396.0497
	6	130	4 1/2	1352.6520	36	9 1/8	50.0982	8429.8706
	7	130	7 1/2	1358.0898	36	10 1/8	50.2996	8463.7595
	8	130	10 1/2	1363.5385	36	11 1/8	50.5014	8497.7164
	9	131	2	1368.9981	37	0	50.7036	8531.7413
	10	131	5 1/2	1374.4686	37	0 1/4	50.9062	8565.8341
	11	131	8 1/2	1379.9500	37	1 1/8	51.1093	8599.9949
42	ft.	131	11 1/2	1385.4424	37	2 1/8	51.3127	8634.2257
	1	132	2	1390.9456	37	3 1/8	51.5165	8668.5205
	2	132	5 1/2	1396.4598	37	4 1/8	51.7207	8702.8853
	3	132	8 1/2	1401.9848	37	5 1/8	51.9254	8737.3180
	4	132	11 1/2	1407.5208	37	6 1/8	52.1304	8771.8187
	5	133	3 1/2	1413.0676	37	7 1/8	52.3358	8806.3875
	6	133	6 1/2	1418.6254	37	8 1/8	52.5417	8841.0242
	7	133	9 1/2	1424.1941	37	9 1/8	52.7479	8875.7288
	8	134	0	1429.7737	37	9 3/4	52.9546	8910.5015
	9	134	3 1/2	1435.3642	37	10 1/8	53.1616	8945.3421
	10	134	6 1/2	1440.9656	37	11 1/8	53.3691	8980.2507
	11	134	9 1/2	1446.5780	38	0	53.5770	9015.2273
43	ft.	135	1	1452.2012	38	1 1/8	53.7852	9050.2719
	1	135	4 1/2	1457.8353	38	2 1/8	53.9939	9085.3845
	2	135	7 1/2	1463.4804	38	3 1/8	54.2030	9120.5550
	3	135	10 1/2	1469.1364	38	4 1/8	54.4125	9155.8136
	4	136	1	1474.8032	38	4 1/4	54.6223	9191.1301
	5	136	4 1/2	1480.4810	38	5 1/8	54.8326	9226.5146
	6	136	7 1/2	1486.1697	38	6 1/8	55.0433	9261.9670
	7	136	11 1/2	1491.8693	38	7 1/8	55.2544	9297.4875
	8	137	2 1/2	1497.5798	38	8 1/8	55.4659	9333.0759
	9	137	5 1/2	1503.3012	38	9 1/8	55.6778	9368.7323
	10	137	8 1/2	1509.0335	38	10 1/8	55.8901	9404.4567
	11	137	11 1/2	1514.7767	38	11 1/8	56.1028	9440.2491

TABLE II.

Dia. in feet and inches.		Circum. in feet and inches.		Area in square feet.	Side of = square in ft. and in.	Cubic yards at one foot in depth.	Gallons at one foot in depth.	
ft.	in.	ft.	in.		ft.	in.		
44.	0	138	23	1520-5308	38	11½	56-3160	9476-1095
	1	138	58	1526-2959	39	0	56-5295	9512-0378
	2	138	9	1532-0718	39	13	56-7434	9548-0342
	3	139	0	1537-8587	39	28	56-9577	9584-0985
	4	139	38	1543-6565	39	34	57-1725	9620-2308
	5	139	62	1549-4651	39	48	57-3876	9656-4310
	6	139	96	1555-2847	39	54	57-6031	9692-6993
	7	140	0	1561-1152	39	68	57-8191	9729-0355
	8	140	38	1566-9566	39	72	58-0354	9765-4397
	9	140	7	1572-8089	39	78	58-2522	9801-9119
	10	140	104	1578-6721	39	88	58-4693	9838-4521
45 ft.	11	141	18	1584-5462	39	98	58-6869	9875-0603
	1	141	42	1590-4313	39	108	58-9049	9911-7364
	2	141	78	1596-3272	39	118	59-1232	9948-4805
	3	141	108	1602-2341	40	0	59-3420	9985-2927
	4	142	16	1608-1518	40	14	59-5612	10022-1727
	5	142	5	1614-0805	40	28	59-7808	10059-1208
	6	142	84	1620-0201	40	3	60-0007	10096-1369
	7	142	114	1625-9705	40	37	60-2211	10133-2209
	8	143	23	1631-9319	40	43	60-4419	10170-3729
	9	143	58	1637-9042	40	58	60-6631	10207-5929
	10	143	88	1643-8874	40	63	60-8847	10244-8809
46 ft.	11	143	118	1649-8816	40	78	61-1067	10282-2369
	1	144	3	1655-8816	40	84	61-3291	10319-6608
	2	144	61	1661-9025	40	94	61-5519	10357-1527
	3	144	94	1667-9294	40	108	61-7752	10394-7126
	4	145	0	1673-9671	40	11	61-9988	10432-3405
	5	145	38	1680-0158	40	116	62-2228	10470-0364
	6	145	62	1686-0753	41	0	62-4472	10507-8002
	7	145	96	1692-1458	41	18	62-6721	10545-6321
	8	146	1	1698-2272	41	23	62-8973	10583-5319
	9	146	41	1704-3195	41	38	63-1229	10621-4997
	10	146	72	1710-4227	41	44	63-3490	10659-5355
47 ft.	11	146	108	1716-5368	41	58	63-5754	10697-6392
	1	147	18	1722-6618	41	6	63-8023	10735-8110
	2	147	48	1728-7977	41	7	64-0295	10774-0507
	3	147	78	1734-9445	41	77	64-2572	10812-3584
	4	147	11	1741-1023	41	82	64-4853	10850-7341
	5	148	21	1747-2709	41	98	64-7137	10889-1778
	6	148	54	1753-4505	41	104	64-9426	10927-6894
	7	148	88	1759-6410	41	118	65-1719	10966-2690
	8	148	118	1765-8423	42	0	65-4016	11004-9166
	9	149	23	1772-0546	42	18	65-6317	11043-6322
	10	149	58	1778-2778	42	2	65-8621	11082-4158
48 ft.	11	149	9	1784-5119	42	26	66-0930	11121-2674
	1	150	0	1790-7569	42	38	66-3243	11160-1869
	2	150	34	1797-0128	42	43	66-5560	11199-1744
	3	150	68	1803-2706	42	58	66-7881	11238-2300
	4	150	102	1809-3284	42	63	67-0195	11277-2878
	5	150	136	1815-3862	42	68	67-2509	11316-3462
	6	150	170	1821-4440	42	73	67-4823	11355-4046
	7	150	204	1827-5018	42	78	67-7137	11394-4630
	8	150	238	1833-5596	42	83	67-9451	11433-5214
	9	150	272	1839-6174	42	88	68-1765	11472-5798
	10	150	306	1845-6752	42	93	68-4079	11511-6382

TABLE II.

Dia. in feet and inches.		Circum. in feet and inches.		Area in square feet.	Side of = square in ft. and in.	Cubic yards at one foot in depth.	Gallons at one foot in depth.	
ft.	in.	ft.	in.		ft.	in.		
48	0	150	9 $\frac{1}{2}$	1809-5574	42	6 $\frac{1}{2}$	67-0206	11277-353
	1	151	0 $\frac{1}{2}$	1815-8460	42	7 $\frac{1}{2}$	67-2536	11316-5449
	2	151	3 $\frac{1}{4}$	1822-1456	42	8 $\frac{1}{4}$	67-4869	11355-8044
	3	151	7	1828-4560	42	9 $\frac{1}{4}$	67-7206	11395-1318
	4	151	10 $\frac{1}{2}$	1834-7774	42	10	67-9547	11434-5272
	5	152	1 $\frac{1}{2}$	1841-1096	42	10 $\frac{1}{2}$	68-1892	11473-9906
	6	152	4 $\frac{1}{2}$	1847-4528	42	11 $\frac{1}{2}$	68-4242	11513-5220
	7	152	7 $\frac{1}{2}$	1853-8069	43	0 $\frac{1}{2}$	68-6595	11553-1213
	8	152	10 $\frac{3}{4}$	1860-1719	43	1 $\frac{1}{4}$	68-8953	11592-7887
	9	153	1 $\frac{1}{2}$	1866-5478	43	2 $\frac{1}{2}$	69-1314	11632-5240
	10	153	5	1872-9346	43	3 $\frac{3}{4}$	69-3679	11672-3273
49 ft.	11	153	8 $\frac{1}{2}$	1879-3324	43	4 $\frac{1}{2}$	69-6049	11712-1986
	1	153	11 $\frac{1}{2}$	1885-7410	43	5 $\frac{1}{2}$	69-8423	11752-1378
	2	154	2 $\frac{1}{2}$	1892-1605	43	6	70-0800	11792-1451
	3	154	5 $\frac{1}{2}$	1898-5910	43	6 $\frac{1}{2}$	70-3182	11832-2203
	4	154	8 $\frac{1}{2}$	1905-0323	43	7 $\frac{1}{2}$	70-5568	11872-3635
	5	154	11 $\frac{1}{2}$	1911-4846	43	8 $\frac{1}{2}$	70-7957	11912-5747
	6	155	3	1917-9478	43	9 $\frac{1}{2}$	71-0351	11952-8539
	7	155	6 $\frac{1}{2}$	1924-4218	43	10 $\frac{1}{2}$	71-2749	11993-2011
	8	155	9 $\frac{1}{2}$	1930-9068	43	11 $\frac{1}{2}$	71-5151	12033-6162
	9	156	0 $\frac{1}{2}$	1937-4027	44	0 $\frac{1}{2}$	71-7557	12074-0993
	10	156	3 $\frac{1}{4}$	1943-9095	44	1 $\frac{1}{4}$	71-9966	12114-6504
50 ft.	11	156	6 $\frac{1}{2}$	1950-4273	44	2	72-2380	12155-2695
		156	9 $\frac{1}{2}$	1956-9559	44	2 $\frac{1}{2}$	72-4798	12195-9566
		157	1	1963-4954	44	3 $\frac{1}{4}$	72-7221	12236-7116

EXAMPLES TO TABLE II.

Ex. 1. Required the circumference of a circle whose diameter is 43 feet 5 inches?

Here, in *column 1, page 32*, will be found 43 feet 5 inches, the diameter; against which, in *column 2*, will be found 136 feet 4 $\frac{1}{2}$ inches, the circumference = *Ans.*

The same result may be found by *Prob. IX, p. 50*: thus, $3.14159265 \times 43\frac{5}{12} \text{ ft.} = 136.39748 \text{ feet} = 136 \text{ ft. } 4\frac{1}{2} \text{ in.} = \text{Ans. as before.}$

Ex. 2. Required the area of a circle whose diameter is 16 feet 9 inches?

Here, in *column 1, page 26*, will be found 16 feet 9 inches, against which, in *column 3*, will be found 220.3533 square feet, the area = *Ans.*

The same result may be found as in *Ex. 1, p. 55*, taking .78539816 (*No. 7*) table of useful numbers, *page 170*, in place of .7854: thus, $16 \text{ ft. } 9 \text{ in.} = 16\frac{3}{4} \text{ ft.}$, then $.78539816 \times 16\frac{3}{4} \text{ ft.} = 13.15541918 \text{ ft.}$, then $(13.15541918 \times 16\frac{3}{4}) \text{ sq. ft.} = 220.35327 \text{ sq. ft.} = \text{Ans. as before.}$

Ex. 3. Required the side of a square whose area shall be equal to that of a circle whose diameter is 47 feet 11 inches?

Here, in *column 1, page 33*, will be found 47 feet 11 inches, against which, in *column 4*, will be found 42 feet 5 $\frac{1}{2}$ inches, the side of the equal square = *Ans.*

The same result may be found as in *Ex. 1, p. 54*, taking .886227 (*No. 56*) table of useful numbers, *page 174*, in place of .8862, then $47 \text{ ft. } 11 \text{ in.} = (48 - \frac{1}{12}) \text{ ft.}$, and $.886227 \times (48 - \frac{1}{12}) \text{ ft.} = 42.538896 \text{ ft.} = 42 \text{ ft. } 5\frac{1}{2} \text{ in.} = \text{Ans. as before.}$

Ex. 4. Required the solid content of a cylinder whose diameter is 27 feet 5 inches, and depth 1 foot?

Here, in *column 1, page 28*, will be found 27 feet 5 inches, the diameter; against which, in *column 5*, will be found 21.8653 cubic yards, the solid content = *Ans.*

The same result may be found as in *Prob. I., Rule 2, page 59*: thus, $.78539816 \times 27\frac{5}{12} \text{ ft.} = 21.53299956 \text{ ft.}$, and $(21.53299956 \times 27\frac{5}{12} \times 1) \text{ cub. ft.} = 590.36307 \text{ cub. ft.} = 21.8653 \text{ cub. yds.} = \text{Ans. as before.}$

Note 1. If the depth of the cylinder differ from 1 foot, the diameter being found in the table, multiply the tabular content by the height of the cylinder in feet, the product will be the required content of the given cylinder in cubic yards.

Ex. 5. Required the solid content of a cylinder whose diameter is 17 feet 7 inches, and depth 135 inches?

Here, the depth = 135 inches = $11\frac{1}{4} \text{ feet}$, then, in *column 1, page 26*,

will be found 17 feet 7 inches, the diameter; against which, in column 5, will be found 8.9935; whence $8.9935 \text{ cubic yards} \times 11\frac{1}{2} = 101.1769 \text{ cubic yards}$, the solid content = *Ans.*

Here, by mensuration, we have $78539816 \times 17\frac{7}{8} \text{ ft.} = 13.80991764 \text{ ft.}$, and $(13.80991764 \times 17\frac{7}{8}) \text{ sq. ft.} = 242.82438517 \text{ sq. ft.} = \text{area of base of the cylinder}$; next $(242.82438517 \times 11\frac{1}{2}) \text{ cub. ft.} = 2731.77433 \text{ cub. ft.} = 101.17683 \text{ cub. yds.} = \text{Ans. as before.}$

Ex. 6. Required the content of a cylinder in imperial gallons whose diameter is 33 feet 6 inches, and depth 1 foot?

Here, in column 1, page 30, will be found 33 feet 6 inches, against which, in column 6, will be found 5493.0598 imperial gallons = *Ans.*

The same result may be found by mensuration: thus, $78539816 \times 33\frac{1}{2} \text{ ft.} = 26.31083836 \text{ ft.}$, and $(26.31083836 \times 33\frac{1}{2} \times 1) \text{ cub. ft.} = 881.413085 \text{ cub. ft.} = \text{content of the cylinder}$; then, by No. 89, table of useful numbers, page 177, we have .003606543, the number of imperial gallons in a cubic inch, and this multiplied by 1728 (= 12^3) gives 6.23210606, the number of imperial gallons in a cubic foot; whence, $881.413085 \times 6.232106 \text{ imp. gallons} = 5493.0598 \text{ imp. gallons} = \text{Ans. as before.}$

Note 2. If the depth of the cylinder differ from 1 foot, its diameter being found in the table, multiply the tabular content by the depth of the cylinder in feet; the product will be of the same denomination as the tabular number, viz. cubic yards, or imperial gallons, according as the tabular number has been taken out from column 5, or from column 6, respectively.

Ex. 7. Required the content in imperial gallons, of a cylinder whose diameter is 41 feet 1 inch, and depth 36 feet $4\frac{3}{4}$ inches?

Here, in column 1, page 32, will be found 41 feet 1 inch; against which, in column 6, will be found 8261.4459.

Then, 4 in. = $\frac{1}{3}$ 8261.4459 imper. gall.

$$\begin{array}{r} 36 \\ 49568.6754 \\ 247843.377 \\ \hline \frac{1}{3} \text{ in.} = \frac{1}{3} \quad 2753.8153 \\ \frac{1}{4} \text{ in.} = \frac{1}{4} \quad 344.2269 \\ \hline 172.1134 \end{array}$$

300682.2080 imp. gall. = *Ans.*

Here, also, by mensuration, $78539816 \times 41\frac{1}{12} \text{ ft.} = 32.26677441 \text{ ft.}$, and $(32.26677441 \times 41\frac{1}{12} \times 1) \text{ cub. ft.} = 1325.62665 \text{ cub. feet}$, which, multiplied by 6.23210606, gives 8261.4459 imp. gall. as in the table, p. 32, for 41 ft. 1 in.; whence, as before, 300682.2080 imp. gall. = *Ans.*

TABLE III.

CONTAINING

THE SQUARE AND CUBE ROOTS OF ALL NUMBERS

FROM 1 TO 1000; AND THE

DIFFERENCE EXISTING BETWEEN EACH ROOT,

BY WHICH

**THE PROCESS FOR OBTAINING THE ROOTS OF NUMBERS, CONSISTING
OF INTEGERS AND DECIMALS, IS CONSIDERABLY FACILITATED.**

TABLE III.
CONTAINING THE SQUARE AND CUBE ROOTS OF ALL NUMBERS
FROM 1 TO 1000, &c.

No.	Sq. Rt.	Diff.	C. Rts.	Diff.	No.	Sq. Rts.	Diff.	C. Rts.	Diff.
1	1.0000		1.0000		45	6.7082		3.5569	
2	1.4142	.4142	1.2599	.2599	46	6.7823	.0741	3.5830	.0261
3	1.7321	.3179	1.4422	.1823	47	6.8557	.0734	3.6088	.0258
4	2.0000	.2679	1.5874	.1452	48	6.9282	.0725	3.6342	.0254
5	2.2361	.2361	1.7100	.1226	49	7.0000	.0718	3.6593	.0251
6	2.4495	.2134	1.8171	.1071	50	7.0711	.0711	3.6840	.0247
7	2.6458	.1963	1.9129	.0958	51	7.1414	.0703	3.7084	.0244
8	2.8284	.1826	2.0000	.0871	52	7.2111	.0697	3.7325	.0241
9	3.0000	.1716	2.0801	.0801	53	7.2801	.0690	3.7563	.0238
10	3.1623	.1623	2.1544	.0743	54	7.3485	.0684	3.7798	.0235
11	3.3166	.1543	2.2240	.0696	55	7.4162	.0677	3.8030	.0232
12	3.4641	.1475	2.2894	.0654	56	7.4833	.0671	3.8259	.0229
13	3.6056	.1415	2.3513	.0619	57	7.5498	.0665	3.8485	.0226
14	3.7417	.1361	2.4101	.0588	58	7.6158	.0660	3.8709	.0224
15	3.8730	.1313	2.4662	.0561	59	7.6811	.0653	3.8930	.0221
16	4.0000	.1270	2.5198	.0536	60	7.7460	.0649	3.9149	.0219
17	4.1231	.1231	2.5713	.0515	61	7.8102	.0642	3.9365	.0216
18	4.2426	.1195	2.6207	.0494	62	7.8740	.0638	3.9579	.0214
19	4.3589	.1163	2.6684	.0477	63	7.9373	.0633	3.9791	.0212
20	4.4721	.1132	2.7144	.0460	64	8.0000	.0627	4.0000	.0209
21	4.5826	.1105	2.7589	.0445	65	8.0623	.0623	4.0207	.0207
22	4.6904	.1078	2.8020	.0431	66	8.1240	.0617	4.0412	.0205
23	4.7958	.1054	2.8439	.0419	67	8.1854	.0614	4.0615	.0203
24	4.8990	.1032	2.8845	.0406	68	8.2462	.0608	4.0817	.0202
25	5.0000	.1010	2.9240	.0395	69	8.3066	.0604	4.1016	.0199
26	5.0990	.0990	2.9625	.0385	70	8.3666	.0600	4.1213	.0197
27	5.1962	.0972	3.0000	.0375	71	8.4261	.0595	4.1408	.0195
28	5.2915	.0953	3.0366	.0366	72	8.4853	.0592	4.1602	.0194
29	5.3852	.0937	3.0723	.0357	73	8.5440	.0587	4.1793	.0191
30	5.4772	.0920	3.1072	.0349	74	8.6023	.0583	4.1983	.0190
31	5.5678	.0906	3.1414	.0342	75	8.6603	.0580	4.2172	.0189
32	5.6569	.0891	3.1748	.0334	76	8.7178	.0575	4.2358	.0186
33	5.7446	.0877	3.2075	.0327	77	8.7750	.0572	4.2543	.0185
34	5.8310	.0864	3.2396	.0321	78	8.8318	.0568	4.2727	.0184
35	5.9161	.0851	3.2711	.0315	79	8.8882	.0564	4.2908	.0181
36	6.0000	.0839	3.3019	.0308	80	8.9443	.0561	4.3089	.0181
37	6.0823	.0828	3.3322	.0303	81	9.0000	.0557	4.3267	.0178
38	6.1644	.0816	3.3620	.0298	82	9.0554	.0554	4.3445	.0178
39	6.2450	.0806	3.3912	.0292	83	9.1104	.0550	4.3621	.0176
40	6.3246	.0796	3.4200	.0288	84	9.1652	.0548	4.3795	.0174
41	6.4031	.0785	3.4482	.0282	85	9.2195	.0543	4.3968	.0173
42	6.4807	.0776	3.4760	.0278	86	9.2736	.0541	4.4140	.0172
43	6.5574	.0767	3.5034	.0274	87	9.3274	.0538	4.4310	.0170
44	6.6332	.0758	3.5303	.0269	88	9.3808	.0534	4.4480	.0170
45	6.7082	.0750	3.5569	.0266	89	9.4340	.0532	4.4647	.0167

TABLE III.

No.	Sq. Rts.	Diff.	C. Rts.	Diff.	No.	Sq. Rts.	Diff.	C. Rts.	Diff.
89	9-4340		4-4647		137	11-7047	-0426	5-1551	-0125
90	9-4868	-0528	4-4814	-0167	138	11-7473	-0425	5-1676	-0125
91	9-5394	-0526	4-4979	-0165	139	11-7898	-0424	5-1801	-0124
92	9-5917	-0523	4-5144	-0163	140	11-8322	-0421	5-1925	-0123
93	9-6437	-0517	4-5307	-0161	141	11-8743	-0421	5-2048	-0123
94	9-6954	-0514	4-5468	-0161	142	11-9164	-0419	5-2171	-0122
95	9-7468	-0512	4-5629	-0160	143	11-9583	-0417	5-2293	-0122
96	9-7980	-0509	4-5789	-0158	144	12-0000	-0416	5-2415	-0121
97	9-8489	-0509	4-5947	-0157	145	12-0416	-0414	5-2536	-0120
98	9-8995	-0506	4-6104	-0155	146	12-0830	-0411	5-2656	-0120
99	9-9499	-0501	4-6261	-0154	147	12-1244	-0408	5-2776	-0119
100	10-0000	-0499	4-6416	-0152	148	12-1655	-0405	5-2896	-0118
101	10-0499	-0496	4-6570	-0150	149	12-2066	-0402	5-3015	-0117
102	10-0995	-0494	4-6723	-0149	150	12-2474	-0400	5-3133	-0116
103	10-1489	-0491	4-6875	-0147	151	12-2882	-0397	5-3251	-0115
104	10-1980	-0490	4-7027	-0145	152	12-3288	-0395	5-3368	-0114
105	10-2470	-0486	4-7177	-0144	153	12-3693	-0393	5-3485	-0113
106	10-2956	-0485	4-7326	-0143	154	12-4097	-0390	5-3601	-0112
107	10-3441	-0482	4-7475	-0141	155	12-4499	-0387	5-3717	-0111
108	10-3923	-0480	4-7622	-0140	156	12-4900	-0385	5-3832	-0110
109	10-4403	-0478	4-7769	-0138	157	12-5300	-0383	5-3947	-0109
110	10-4881	-0476	4-7914	-0136	158	12-5698	-0380	5-4061	-0108
111	10-5357	-0473	4-8059	-0134	159	12-6095	-0377	5-4175	-0107
112	10-5830	-0471	4-8203	-0133	160	12-6491	-0375	5-4288	-0106
113	10-6301	-0467	4-8346	-0131	161	12-6886	-0372	5-4401	-0105
114	10-6771	-0465	4-8488	-0129	162	12-7279	-0370	5-4514	-0104
115	10-7238	-0464	4-8629	-0127	163	12-7671	-0368	5-4626	-0103
116	10-7703	-0461	4-8770	-0125	164	12-8062	-0365	5-4737	-0102
117	10-8167	-0459	4-8910	-0123	165	12-8452	-0363	5-4848	-0101
118	10-8628	-0458	4-9049	-0121	166	12-8841	-0360	5-4959	-0100
119	10-9087	-0455	4-9187	-0119	167	12-9228	-0357	5-5069	-0099
120	10-9545	-0454	4-9324	-0117	168	12-9615	-0355	5-5178	-0098
121	11-0000	-0451	4-9461	-0115	169	13-0000	-0352	5-5288	-0097
122	11-0454	-0450	4-9597	-0113	170	13-0384	-0350	5-5397	-0096
123	11-0905	-0448	4-9732	-0111	171	13-0767	-0347	5-5505	-0095
124	11-1355	-0447	4-9866	-0109	172	13-1149	-0345	5-5613	-0094
125	11-1803	-0444	5-0000	-0107	173	13-1529	-0342	5-5721	-0093
126	11-2250	-0443	5-0133	-0105	174	13-1909	-0340	5-5828	-0092
127	11-2694	-0441	5-0265	-0103	175	13-2288	-0337	5-5934	-0091
128	11-3137	-0440	5-0397	-0101	176	13-2665	-0335	5-6041	-0090
129	11-3578	-0437	5-0528	-0099	177	13-3041	-0332	5-6147	-0089
130	11-4018	-0436	5-0658	-0097	178	13-3417	-0330	5-6252	-0088
131	11-4455	-0435	5-0788	-0095	179	13-3791	-0327	5-6357	-0087
132	11-4891	-0432	5-0916	-0093	180	13-4164	-0325	5-6462	-0086
133	11-5326	-0432	5-1045	-0091	181	13-4536	-0322	5-6567	-0085
134	11-5758	-0432	5-1172	-0089	182	13-4907	-0320	5-6671	-0084
135	11-6190	-0429	5-1299	-0087	183	13-5277	-0317	5-6774	-0083
136	11-6619	-0428	5-1426	-0085	184	13-5647	-0315	5-6877	-0082
137	11-7047		5-1551		185	13-6015		5-6980	

TABLE III.

No.	Sq. Rts.	Diff.	C. Rts.	Diff.	No.	Sq. Rts.	Diff.	C. Rts.	Diff.
185	13-6015		5-6980		233	15-2643		6-1534	
186	13-6382	-0367	5-7083	-0103	234	15-2971	-0328	6-1622	-0088
187	13-6748	-0366	5-7185	-0102	235	15-3297	-0326	6-1710	-0088
188	13-7113	-0365	5-7287	-0101	236	15-3623	-0326	6-1797	-0087
189	13-7477	-0364	5-7388	-0101	237	15-3948	-0325	6-1885	-0088
190	13-7840	-0363	5-7489	-0101	238	15-4272	-0324	6-1972	-0087
191	13-8203	-0361	5-7590	-0100	239	15-4596	-0323	6-2058	-0086
192	13-8564	-0361	5-7690	-0100	240	15-4919	-0323	6-2145	-0087
193	13-8924	-0360	5-7790	-0100	241	15-5242	-0321	6-2231	-0086
194	13-9284	-0358	5-7890	-0099	242	15-5563	-0322	6-2317	-0086
195	13-9642	-0358	5-7989	-0099	243	15-5885	-0320	6-2403	-0085
196	14-0000	-0357	5-8088	-0098	244	15-6205	-0320	6-2488	-0085
197	14-0357	-0357	5-8186	-0099	245	15-6525	-0319	6-2573	-0085
198	14-0712	-0355	5-8285	-0098	246	15-6844	-0318	6-2658	-0085
199	14-1067	-0355	5-8383	-0097	247	15-7162	-0318	6-2743	-0085
200	14-1421	-0353	5-8480	-0098	248	15-7480	-0317	6-2828	-0084
201	14-1774	-0353	5-8578	-0097	249	15-7797	-0317	6-2912	-0084
202	14-2127	-0351	5-8675	-0096	250	15-8114	-0316	6-2996	-0084
203	14-2478	-0351	5-8771	-0097	251	15-8430	-0315	6-3080	-0084
204	14-2829	-0349	5-8868	-0096	252	15-8745	-0315	6-3164	-0083
205	14-3178	-0349	5-8964	-0095	253	15-9060	-0314	6-3247	-0083
206	14-3527	-0348	5-9059	-0096	254	15-9374	-0313	6-3330	-0083
207	14-3875	-0347	5-9155	-0095	255	15-9687	-0313	6-3413	-0083
208	14-4222	-0346	5-9250	-0095	256	16-0000	-0312	6-3496	-0083
209	14-4568	-0346	5-9345	-0094	257	16-0312	-0312	6-3579	-0083
210	14-4914	-0344	5-9439	-0094	258	16-0624	-0311	6-3661	-0082
211	14-5258	-0344	5-9533	-0094	259	16-0935	-0310	6-3743	-0082
212	14-5602	-0343	5-9627	-0094	260	16-1245	-0310	6-3825	-0082
213	14-5945	-0342	5-9721	-0093	261	16-1555	-0309	6-3907	-0081
214	14-6287	-0342	5-9814	-0093	262	16-1864	-0309	6-3988	-0082
215	14-6629	-0342	5-9907	-0093	263	16-2173	-0308	6-4070	-0081
216	14-6969	-0340	6-0000	-0092	264	16-2481	-0307	6-4151	-0081
217	14-7309	-0339	6-0092	-0093	265	16-2788	-0307	6-4232	-0080
218	14-7648	-0338	6-0185	-0092	266	16-3095	-0306	6-4312	-0081
219	14-7986	-0338	6-0277	-0091	267	16-3401	-0306	6-4393	-0080
220	14-8324	-0337	6-0368	-0091	268	16-3707	-0305	6-4473	-0080
221	14-8661	-0336	6-0459	-0091	269	16-4012	-0305	6-4553	-0080
222	14-8997	-0335	6-0550	-0091	270	16-4317	-0304	6-4633	-0080
223	14-9332	-0334	6-0641	-0091	271	16-4621	-0303	6-4713	-0079
224	14-9666	-0334	6-0732	-0090	272	16-4924	-0303	6-4792	-0080
225	15-0000	-0333	6-0822	-0090	273	16-5227	-0302	6-4872	-0079
226	15-0333	-0332	6-0912	-0090	274	16-5529	-0302	6-4951	-0079
227	15-0665	-0332	6-1002	-0089	275	16-5831	-0301	6-5030	-0078
228	15-0997	-0330	6-1091	-0089	276	16-6132	-0301	6-5108	-0079
229	15-1327	-0331	6-1180	-0089	277	16-6433	-0300	6-5187	-0078
230	15-1658	-0329	6-1269	-0089	278	16-6733	-0300	6-5265	-0078
231	15-1987	-0328	6-1358	-0088	279	16-7033	-0299	6-5343	-0078
232	15-2315	-0328	6-1446	-0088	280	16-7332	-0299	6-5421	-0078
233	15-2643	-0328	6-1534	-0088	281	16-7631		6-5499	-0078

TABLE III.

No.	Sq. Rts.	Diff.	C. Rts.	Diff.	No.	Sq. Rts.	Diff.	C. Rts.	Diff.
281	16-7631		6-5499		329	18-1384		6-9034	
282	16-7929	·0298	6-5577	·0078	330	18-1659	·0275	6-9104	·0070
283	16-8226	·0297	6-5654	·0077	331	18-1934	·0275	6-9174	·0070
284	16-8523	·0297	6-5731	·0077	332	18-2209	·0275	6-9244	·0069
285	16-8819	·0296	6-5808	·0077	333	18-2483	·0274	6-9313	·0069
286	16-9115	·0296	6-5885	·0077	334	18-2757	·0274	6-9382	·0069
287	16-9411	·0296	6-5962	·0077	335	18-3030	·0273	6-9451	·0070
288	16-9706	·0295	6-6039	·0076	336	18-3303	·0273	6-9521	·0068
289	17-0000	·0294	6-6115	·0076	337	18-3576	·0272	6-9589	·0069
290	17-0294	·0294	6-6191	·0076	338	18-3848	·0272	6-9658	·0069
291	17-0587	·0293	6-6267	·0076	339	18-4120	·0272	6-9727	·0068
292	17-0880	·0293	6-6343	·0076	340	18-4391	·0271	6-9795	·0069
293	17-1172	·0292	6-6419	·0076	341	18-4662	·0270	6-9864	·0068
294	17-1464	·0292	6-6494	·0075	342	18-4932	·0271	6-9932	·0068
295	17-1756	·0291	6-6569	·0075	343	18-5203	·0269	7-0000	·0068
296	17-2047	·0291	6-6644	·0075	344	18-5472	·0269	7-0068	·0068
297	17-2337	·0290	6-6719	·0075	345	18-5742	·0270	7-0136	·0068
298	17-2627	·0290	6-6794	·0075	346	18-6011	·0269	7-0203	·0067
299	17-2916	·0289	6-6869	·0075	347	18-6279	·0268	7-0271	·0068
300	17-3205	·0289	6-6943	·0074	348	18-6548	·0269	7-0338	·0067
301	17-3494	·0289	6-7018	·0075	349	18-6815	·0267	7-0406	·0068
302	17-3781	·0287	6-7092	·0074	350	18-7083	·0268	7-0473	·0067
303	17-4069	·0288	6-7166	·0074	351	18-7350	·0267	7-0540	·0067
304	17-4356	·0287	6-7240	·0074	352	18-7617	·0267	7-0607	·0067
305	17-4642	·0286	6-7313	·0073	353	18-7883	·0266	7-0674	·0066
306	17-4929	·0287	6-7387	·0074	354	18-8149	·0265	7-0740	·0067
307	17-5214	·0285	6-7460	·0073	355	18-8414	·0266	7-0807	·0066
308	17-5499	·0285	6-7533	·0073	356	18-8680	·0266	7-0873	·0067
309	17-5784	·0285	6-7606	·0073	357	18-8944	·0264	7-0940	·0066
310	17-6068	·0284	6-7679	·0073	358	18-9209	·0265	7-1006	·0066
311	17-6352	·0284	6-7752	·0073	359	18-9473	·0264	7-1072	·0066
312	17-6635	·0283	6-7824	·0072	360	18-9737	·0263	7-1138	·0066
313	17-6918	·0283	6-7897	·0073	361	19-0000	·0262	7-1204	·0065
314	17-7200	·0282	6-7969	·0072	362	19-0263	·0263	7-1269	·0066
315	17-7482	·0282	6-8041	·0072	363	19-0526	·0263	7-1335	·0066
316	17-7764	·0282	6-8113	·0072	364	19-0788	·0262	7-1400	·0065
317	17-8045	·0281	6-8185	·0071	365	19-1050	·0261	7-1466	·0065
318	17-8326	·0281	6-8256	·0072	366	19-1311	·0261	7-1531	·0065
319	17-8606	·0280	6-8328	·0072	367	19-1572	·0261	7-1596	·0065
320	17-8885	·0279	6-8399	·0071	368	19-1833	·0261	7-1661	·0065
321	17-9165	·0280	6-8470	·0071	369	19-2094	·0261	7-1726	·0065
322	17-9444	·0279	6-8541	·0071	370	19-2354	·0260	7-1791	·0064
323	17-9722	·0278	6-8612	·0071	371	19-2614	·0259	7-1855	·0065
324	18-0000	·0278	6-8683	·0070	372	19-2873	·0259	7-1920	·0064
325	18-0278	·0277	6-8753	·0070	373	19-3132	·0259	7-1984	·0064
326	18-0555	·0276	6-8824	·0071	374	19-3391	·0259	7-2048	·0064
327	18-0831	·0276	6-8894	·0070	375	19-3649	·0258	7-2112	·0064
328	18-1108	·0277	6-8964	·0070	376	19-3907	·0258	7-2177	·0065
329	18-1384	·0276	6-9034	·0070	377	19-4165		7-2240	·0063

TABLE III.

No.	Sq. Rts.	Diff.	C. Rts.	Diff.	No.	Sq. Rts.	Diff.	C. Rts.	Diff.
377	19-4165		7-2240		425	20-6155		7-5185	
378	19-4422	0257	7-2304	0064	426	20-6398	0243	7-5244	0059
379	19-4679	0257	7-2368	0064	427	20-6640	0242	7-5302	0058
380	19-4936	0257	7-2432	0064	428	20-6882	0242	7-5361	0059
381	19-5192	0256	7-2495	0063	429	20-7123	0241	7-5420	0059
382	19-5448	0256	7-2558	0063	430	20-7364	0241	7-5478	0058
383	19-5704	0256	7-2622	0064	431	20-7605	0241	7-5537	0059
384	19-5959	0255	7-2685	0063	432	20-7846	0241	7-5595	0058
385	19-6214	0255	7-2748	0063	433	20-8087	0241	7-5654	0059
386	19-6469	0255	7-2811	0063	434	20-8327	0240	7-5712	0058
387	19-6723	0254	7-2874	0063	435	20-8567	0240	7-5770	0058
388	19-6977	0254	7-2936	0062	436	20-8806	0239	7-5828	0058
389	19-7231	0254	7-2999	0062	437	20-9045	0239	7-5886	0058
390	19-7484	0253	7-3061	0062	438	20-9284	0239	7-5944	0057
391	19-7737	0253	7-3124	0063	439	20-9523	0239	7-6001	0057
392	19-7990	0252	7-3186	0062	440	20-9762	0238	7-6059	0058
393	19-8242	0252	7-3248	0062	441	21-0000	0238	7-6117	0057
394	19-8494	0252	7-3310	0062	442	21-0238	0238	7-6174	0058
395	19-8746	0251	7-3372	0062	443	21-0476	0237	7-6232	0057
396	19-8997	0251	7-3434	0062	444	21-0713	0237	7-6289	0057
397	19-9249	0250	7-3496	0062	445	21-0950	0237	7-6346	0057
398	19-9499	0250	7-3558	0061	446	21-1187	0237	7-6403	0057
399	19-9750	0250	7-3619	0061	447	21-1424	0236	7-6460	0057
400	20-0000	0250	7-3681	0061	448	21-1660	0236	7-6517	0057
401	20-0250	0249	7-3742	0061	449	21-1896	0236	7-6574	0057
402	20-0499	0249	7-3803	0061	450	21-2132	0236	7-6631	0057
403	20-0749	0249	7-3864	0061	451	21-2368	0235	7-6688	0056
404	20-0998	0248	7-3925	0061	452	21-2603	0235	7-6744	0057
405	20-1246	0248	7-3986	0061	453	21-2838	0235	7-6801	0056
406	20-1494	0248	7-4047	0061	454	21-3073	0234	7-6857	0057
407	20-1742	0248	7-4108	0061	455	21-3307	0234	7-6914	0056
408	20-1990	0247	7-4169	0060	456	21-3542	0234	7-6970	0056
409	20-2237	0247	7-4229	0061	457	21-3776	0233	7-7026	0056
410	20-2485	0246	7-4290	0060	458	21-4009	0234	7-7082	0056
411	20-2731	0247	7-4350	0060	459	21-4243	0233	7-7138	0056
412	20-2978	0246	7-4410	0060	460	21-4476	0233	7-7194	0056
413	20-3224	0246	7-4470	0060	461	21-4709	0233	7-7250	0056
414	20-3470	0245	7-4530	0060	462	21-4942	0232	7-7306	0056
415	20-3715	0246	7-4590	0060	463	21-5174	0233	7-7362	0056
416	20-3961	0245	7-4650	0060	464	21-5407	0232	7-7418	0055
417	20-4206	0244	7-4710	0060	465	21-5639	0231	7-7473	0056
418	20-4450	0244	7-4770	0059	466	21-5870	0232	7-7529	0055
419	20-4695	0244	7-4829	0059	467	21-6102	0231	7-7584	0055
420	20-4939	0244	7-4889	0059	468	21-6333	0231	7-7639	0055
421	20-5183	0243	7-4948	0059	469	21-6564	0231	7-7695	0055
422	20-5426	0244	7-5007	0060	470	21-6795	0230	7-7750	0055
423	20-5670	0243	7-5067	0059	471	21-7025	0231	7-7805	0055
424	20-5913	0242	7-5126	0059	472	21-7256	0230	7-7860	0055
425	20-6155		7-5185		473	21-7486		7-7915	

TABLE III.

No.	Sq. Rts.	Diff.	C. Rts.	Diff.	No.	Sq. Rts.	Diff.	C. Rts.	Diff.
473	21-7486	-0229	7-7915	-0055	521	22-8254	-0219	8-0466	-0051
474	21-7715	-0230	7-7970	-0055	522	22-8473	-0219	8-0517	-0051
475	21-7945	-0229	7-8025	-0054	523	22-8692	-0218	8-0569	-0051
476	21-8174	-0229	7-8079	-0055	524	22-8910	-0218	8-0620	-0051
477	21-8403	-0229	7-8134	-0054	525	22-9129	-0219	8-0671	-0052
478	21-8632	-0229	7-8188	-0055	526	22-9347	-0218	8-0723	-0051
479	21-8861	-0228	7-8243	-0054	527	22-9565	-0218	8-0774	-0051
480	21-9089	-0228	7-8297	-0055	528	22-9783	-0217	8-0825	-0051
481	21-9317	-0228	7-8352	-0054	529	23-0000	-0217	8-0876	-0051
482	21-9545	-0228	7-8406	-0054	530	23-0217	-0217	8-0927	-0051
483	21-9773	-0228	7-8460	-0054	531	23-0434	-0217	8-0978	-0050
484	22-0000	-0227	7-8514	-0054	532	23-0651	-0216	8-1028	-0051
485	22-0227	-0227	7-8568	-0054	533	23-0868	-0216	8-1079	-0051
486	22-0454	-0227	7-8622	-0054	534	23-1084	-0217	8-1130	-0050
487	22-0681	-0226	7-8676	-0054	535	23-1301	-0216	8-1180	-0051
488	22-0907	-0226	7-8730	-0054	536	23-1517	-0216	8-1231	-0050
489	22-1133	-0226	7-8784	-0053	537	23-1733	-0215	8-1281	-0051
490	22-1359	-0226	7-8837	-0054	538	23-1948	-0216	8-1332	-0050
491	22-1585	-0226	7-8891	-0053	539	23-2164	-0215	8-1382	-0051
492	22-1811	-0225	7-8944	-0054	540	23-2379	-0215	8-1433	-0050
493	22-2036	-0225	7-8998	-0053	541	23-2594	-0215	8-1483	-0050
494	22-2261	-0225	7-9051	-0054	542	23-2809	-0215	8-1533	-0050
495	22-2486	-0225	7-9105	-0053	543	23-3024	-0214	8-1583	-0050
496	22-2711	-0224	7-9158	-0053	544	23-3238	-0214	8-1633	-0050
497	22-2935	-0224	7-9211	-0053	545	23-3452	-0214	8-1683	-0050
498	22-3159	-0224	7-9264	-0053	546	23-3666	-0214	8-1733	-0050
499	22-3383	-0224	7-9317	-0053	547	23-3880	-0214	8-1783	-0049
500	22-3607	-0223	7-9370	-0053	548	23-4094	-0213	8-1833	-0050
501	22-3830	-0224	7-9423	-0053	549	23-4307	-0214	8-1882	-0050
502	22-4054	-0223	7-9476	-0052	550	23-4521	-0213	8-1932	-0049
503	22-4277	-0223	7-9528	-0053	551	23-4734	-0213	8-1982	-0050
504	22-4499	-0223	7-9581	-0053	552	23-4947	-0213	8-2031	-0049
505	22-4722	-0222	7-9634	-0052	553	23-5160	-0212	8-2081	-0050
506	22-4944	-0223	7-9686	-0053	554	23-5372	-0212	8-2130	-0049
507	22-5167	-0222	7-9739	-0052	555	23-5584	-0213	8-2180	-0049
508	22-5389	-0221	7-9791	-0052	556	23-5797	-0211	8-2229	-0049
509	22-5610	-0221	7-9843	-0053	557	23-6008	-0212	8-2278	-0049
510	22-5832	-0222	7-9896	-0052	558	23-6220	-0212	8-2327	-0050
511	22-6053	-0221	7-9948	-0052	559	23-6432	-0211	8-2377	-0049
512	22-6274	-0221	8-0000	-0052	560	23-6643	-0211	8-2426	-0049
513	22-6495	-0221	8-0052	-0052	561	23-6854	-0211	8-2475	-0049
514	22-6716	-0220	8-0104	-0052	562	23-7065	-0211	8-2524	-0049
515	22-6936	-0220	8-0156	-0052	563	23-7276	-0211	8-2573	-0048
516	22-7156	-0220	8-0208	-0052	564	23-7487	-0210	8-2621	-0049
517	22-7376	-0220	8-0260	-0051	565	23-7697	-0211	8-2670	-0049
518	22-7596	-0219	8-0311	-0052	566	23-7908	-0210	8-2719	-0049
519	22-7816	-0219	8-0363	-0052	567	23-8118	-0210	8-2768	-0048
520	22-8035	-0219	8-0415	-0051	568	23-8328	-0209	8-2816	-0049
521	22-8254	-0219	8-0466	-0051	569	23-8537	-0209	8-2865	-0049

TABLE III.

No.	Sq. Rts.	Diff.	C. Rts.	Diff.	No.	Sq. Rts.	Diff.	C. Rts.	Diff.
569	23-8537		8-2865		617	24-8395		8-5132	
570	23-8747	0210	8-2913	0048	618	24-8596	0201	8-5178	0046
571	23-8956	0209	8-2962	0049	619	24-8797	0201	8-5224	0046
572	23-9165	0209	8-3010	0048	620	24-8998	0201	8-5270	0046
573	23-9374	0209	8-3059	0048	621	24-9199	0200	8-5316	0046
574	23-9583	0209	8-3107	0048	622	24-9399	0200	8-5362	0046
575	23-9792	0209	8-3155	0048	623	24-9600	0200	8-5408	0045
576	24-0000	0208	8-3203	0048	624	24-9800	0200	8-5453	0046
577	24-0208	0208	8-3251	0049	625	25-0000	0200	8-5499	0045
578	24-0416	0208	8-3300	0048	626	25-0200	0200	8-5544	0046
579	24-0624	0208	8-3348	0048	627	25-0400	0199	8-5590	0045
580	24-0832	0208	8-3396	0048	628	25-0599	0199	8-5635	0046
581	24-1039	0207	8-3443	0047	629	25-0799	0200	8-5681	0045
582	24-1247	0208	8-3491	0048	630	25-0998	0199	8-5726	0046
583	24-1454	0207	8-3539	0048	631	25-1197	0199	8-5772	0045
584	24-1661	0207	8-3587	0047	632	25-1396	0199	8-5817	0045
585	24-1868	0206	8-3634	0048	633	25-1595	0199	8-5862	0045
586	24-2074	0206	8-3682	0048	634	25-1794	0198	8-5907	0045
587	24-2281	0207	8-3730	0047	635	25-1992	0198	8-5952	0045
588	24-2487	0206	8-3777	0048	636	25-2190	0199	8-5997	0046
589	24-2693	0206	8-3825	0047	637	25-2389	0198	8-6043	0045
590	24-2899	0206	8-3872	0047	638	25-2587	0197	8-6088	0044
591	24-3105	0206	8-3919	0048	639	25-2784	0198	8-6132	0045
592	24-3311	0205	8-3967	0047	640	25-2982	0198	8-6177	0045
593	24-3516	0205	8-4014	0047	641	25-3180	0197	8-6222	0045
594	24-3721	0205	8-4061	0047	642	25-3377	0197	8-6267	0045
595	24-3926	0205	8-4108	0047	643	25-3574	0198	8-6312	0045
596	24-4131	0205	8-4155	0047	644	25-3772	0197	8-6357	0044
597	24-4336	0204	8-4202	0047	645	25-3969	0196	8-6401	0045
598	24-4540	0204	8-4249	0047	646	25-4165	0197	8-6446	0044
599	24-4745	0205	8-4296	0047	647	25-4362	0196	8-6490	0045
600	24-4949	0204	8-4343	0047	648	25-4558	0196	8-6535	0044
601	24-5153	0204	8-4390	0047	649	25-4754	0197	8-6579	0045
602	24-5357	0204	8-4437	0047	650	25-4951	0196	8-6624	0044
603	24-5561	0203	8-4484	0046	651	25-5147	0196	8-6668	0045
604	24-5764	0203	8-4530	0047	652	25-5343	0196	8-6713	0044
605	24-5967	0203	8-4577	0046	653	25-5539	0195	8-6757	0044
606	24-6171	0204	8-4623	0047	654	25-5734	0196	8-6801	0044
607	24-6374	0203	8-4670	0046	655	25-5930	0195	8-6845	0045
608	24-6577	0202	8-4716	0047	656	25-6125	0195	8-6889	0044
609	24-6779	0203	8-4763	0046	657	25-6320	0195	8-6934	0044
610	24-6982	0202	8-4809	0047	658	25-6515	0195	8-6978	0044
611	24-7184	0202	8-4856	0046	659	25-6710	0194	8-7022	0044
612	24-7386	0202	8-4902	0046	660	25-6905	0194	8-7066	0044
613	24-7588	0202	8-4948	0046	661	25-7099	0195	8-7110	0044
614	24-7790	0202	8-4994	0046	662	25-7294	0194	8-7154	0044
615	24-7992	0201	8-5040	0046	663	25-7488	0194	8-7198	0043
616	24-8193	0202	8-5086	0046	664	25-7682	0194	8-7241	0044
617	24-8395		8-5132		665	25-7876		8-7285	

TABLE III.

No.	Sq. Rts.	Diff.	C. Rts.	Diff.	No.	Sq. Rts.	Diff.	C. Rts.	Diff.
665	25-7876	-0194	8-7285	-0044	713	26-7021	-0187	8-9337	-0041
666	25-8070	-0193	8-7329	-0044	714	26-7208	-0187	8-9378	-0042
667	25-8263	-0194	8-7373	-0043	715	26-7395	-0187	8-9420	-0042
668	25-8457	-0193	8-7416	-0044	716	26-7582	-0187	8-9462	-0041
669	25-8650	-0194	8-7460	-0043	717	26-7769	-0186	8-9503	-0042
670	25-8844	-0193	8-7503	-0044	718	26-7955	-0187	8-9545	-0042
671	25-9037	-0193	8-7547	-0043	719	26-8142	-0186	8-9587	-0041
672	25-9230	-0192	8-7590	-0044	720	26-8328	-0186	8-9628	-0042
673	25-9422	-0193	8-7634	-0043	721	26-8514	-0187	8-9670	-0041
674	25-9615	-0193	8-7677	-0044	722	26-8701	-0186	8-9711	-0041
675	25-9808	-0192	8-7721	-0043	723	26-8887	-0185	8-9752	-0042
676	26-0000	-0192	8-7764	-0043	724	26-9072	-0186	8-9794	-0041
677	26-0192	-0192	8-7807	-0043	725	26-9258	-0186	8-9835	-0041
678	26-0384	-0192	8-7850	-0043	726	26-9444	-0185	8-9876	-0042
679	26-0576	-0192	8-7893	-0044	727	26-9629	-0186	8-9918	-0041
680	26-0768	-0192	8-7937	-0043	728	26-9815	-0185	8-9959	-0041
681	26-0960	-0191	8-7980	-0043	729	27-0000	-0185	9-0000	-0041
682	26-1151	-0192	8-8023	-0043	730	27-0185	-0185	9-0041	-0041
683	26-1343	-0191	8-8066	-0043	731	27-0370	-0185	9-0082	-0041
684	26-1534	-0191	8-8109	-0043	732	27-0555	-0185	9-0123	-0041
685	26-1725	-0191	8-8152	-0042	733	27-0740	-0184	9-0164	-0041
686	26-1916	-0191	8-8194	-0043	734	27-0924	-0185	9-0205	-0041
687	26-2107	-0191	8-8237	-0043	735	27-1109	-0184	9-0246	-0041
688	26-2298	-0190	8-8280	-0043	736	27-1293	-0184	9-0287	-0041
689	26-2488	-0191	8-8323	-0043	737	27-1477	-0185	9-0328	-0041
690	26-2679	-0190	8-8366	-0042	738	27-1662	-0184	9-0369	-0041
691	26-2869	-0190	8-8408	-0043	739	27-1846	-0183	9-0410	-0040
692	26-3059	-0190	8-8451	-0042	740	27-2029	-0184	9-0450	-0041
693	26-3249	-0190	8-8493	-0043	741	27-2213	-0184	9-0491	-0041
694	26-3439	-0190	8-8536	-0042	742	27-2397	-0183	9-0532	-0040
695	26-3629	-0189	8-8578	-0043	743	27-2580	-0184	9-0572	-0041
696	26-3818	-0189	8-8621	-0042	744	27-2764	-0183	9-0613	-0041
697	26-4008	-0190	8-8663	-0043	745	27-2947	-0183	9-0654	-0040
698	26-4197	-0189	8-8706	-0042	746	27-3130	-0183	9-0694	-0041
699	26-4386	-0189	8-8748	-0042	747	27-3313	-0183	9-0735	-0040
700	26-4575	-0189	8-8790	-0043	748	27-3496	-0183	9-0775	-0041
701	26-4764	-0189	8-8833	-0042	749	27-3679	-0182	9-0816	-0040
702	26-4953	-0188	8-8875	-0042	750	27-3861	-0183	9-0856	-0040
703	26-5141	-0189	8-8917	-0042	751	27-4044	-0182	9-0896	-0041
704	26-5330	-0188	8-8959	-0042	752	27-4226	-0182	9-0937	-0040
705	26-5518	-0189	8-9001	-0042	753	27-4408	-0183	9-0977	-0040
706	26-5707	-0188	8-9043	-0042	754	27-4591	-0182	9-1017	-0040
707	26-5895	-0188	8-9085	-0042	755	27-4773	-0182	9-1057	-0041
708	26-6083	-0187	8-9127	-0042	756	27-4955	-0181	9-1098	-0040
709	26-6271	-0187	8-9169	-0042	757	27-5136	-0182	9-1138	-0040
710	26-6458	-0188	8-9211	-0042	758	27-5318	-0182	9-1178	-0040
711	26-6646	-0187	8-9253	-0042	759	27-5500	-0181	9-1218	-0040
712	26-6833	-0188	8-9295	-0042	760	27-5681	-0181	9-1258	-0040
713	26-7021	-0188	8-9337	-0042	761	27-5862	-0181	9-1298	-0040

TABLE III.

No.	Sq. Rts.	Diff.	C. Rts.	Diff.	No.	Sq. Rts.	Diff.	C. Rts.	Diff.
761	27-5862		9-1298		809	28-4429	-0176	9-3179	-0038
762	27-6043	-0181	9-1338	-0040	810	28-4605	-0176	9-3217	-0038
763	27-6225	-0182	9-1378	-0040	811	28-4781	-0175	9-3255	-0039
764	27-6405	-0180	9-1418	-0040	812	28-4956	-0175	9-3294	-0038
765	27-6586	-0181	9-1458	-0040	813	28-5132	-0175	9-3332	-0038
766	27-6767	-0181	9-1498	-0040	814	28-5307	-0175	9-3370	-0038
767	27-6948	-0181	9-1537	-0039	815	28-5482	-0175	9-3408	-0039
768	27-7128	-0180	9-1577	-0040	816	28-5657	-0175	9-3447	-0038
769	27-7308	-0180	9-1617	-0040	817	28-5832	-0175	9-3485	-0038
770	27-7489	-0181	9-1657	-0040	818	28-6007	-0175	9-3523	-0038
771	27-7669	-0180	9-1696	-0039	819	28-6182	-0175	9-3561	-0038
772	27-7849	-0180	9-1736	-0040	820	28-6356	-0174	9-3599	-0038
773	27-8029	-0180	9-1775	-0039	821	28-6531	-0175	9-3637	-0038
774	27-8209	-0180	9-1815	-0040	822	28-6705	-0174	9-3675	-0038
775	27-8388	-0179	9-1855	-0040	823	28-6880	-0175	9-3713	-0038
776	27-8568	-0180	9-1894	-0039	824	28-7054	-0174	9-3751	-0038
777	27-8747	-0179	9-1933	-0039	825	28-7228	-0174	9-3789	-0038
778	27-8927	-0180	9-1973	-0040	826	28-7402	-0174	9-3827	-0038
779	27-9106	-0179	9-2012	-0039	827	28-7576	-0174	9-3865	-0037
780	27-9285	-0179	9-2052	-0040	828	28-7750	-0174	9-3902	-0038
781	27-9464	-0179	9-2091	-0039	829	28-7924	-0174	9-3940	-0038
782	27-9643	-0179	9-2130	-0039	830	28-8097	-0173	9-3978	-0038
783	27-9821	-0178	9-2170	-0040	831	28-8271	-0174	9-4016	-0038
784	28-0000	-0179	9-2209	-0039	832	28-8444	-0173	9-4053	-0037
785	28-0179	-0179	9-2248	-0039	833	28-8617	-0173	9-4091	-0038
786	28-0357	-0178	9-2287	-0039	834	28-8791	-0174	9-4129	-0038
787	28-0535	-0178	9-2326	-0039	835	28-8964	-0173	9-4166	-0037
788	28-0713	-0178	9-2365	-0039	836	28-9137	-0173	9-4204	-0038
789	28-0891	-0178	9-2404	-0039	837	28-9310	-0173	9-4241	-0037
790	28-1069	-0178	9-2443	-0039	838	28-9482	-0172	9-4279	-0038
791	28-1247	-0178	9-2482	-0039	839	28-9655	-0173	9-4316	-0037
792	28-1425	-0178	9-2521	-0039	840	28-9828	-0173	9-4354	-0038
793	28-1603	-0178	9-2560	-0039	841	29-0000	-0172	9-4391	-0037
794	28-1780	-0177	9-2599	-0039	842	29-0172	-0172	9-4429	-0038
795	28-1957	-0177	9-2638	-0039	843	29-0345	-0173	9-4466	-0037
796	28-2135	-0178	9-2677	-0039	844	29-0517	-0172	9-4503	-0037
797	28-2312	-0177	9-2716	-0039	845	29-0689	-0172	9-4541	-0037
798	28-2489	-0177	9-2754	-0038	846	29-0861	-0172	9-4578	-0037
799	28-2666	-0177	9-2793	-0039	847	29-1033	-0172	9-4615	-0037
800	28-2843	-0177	9-2832	-0039	848	29-1204	-0171	9-4652	-0037
801	28-3019	-0176	9-2870	-0038	849	29-1376	-0172	9-4690	-0038
802	28-3196	-0177	9-2909	-0039	850	29-1548	-0172	9-4727	-0037
803	28-3373	-0177	9-2948	-0039	851	29-1719	-0171	9-4764	-0037
804	28-3549	-0176	9-2986	-0038	852	29-1890	-0171	9-4801	-0037
805	28-3725	-0176	9-3025	-0039	853	29-2062	-0172	9-4838	-0037
806	28-3901	-0176	9-3063	-0038	854	29-2233	-0171	9-4875	-0037
807	28-4077	-0176	9-3102	-0039	855	29-2404	-0171	9-4912	-0037
808	28-4253	-0176	9-3140	-0038	856	29-2575	-0171	9-4949	-0037
809	28-4429	-0176	9-3179	-0039	857	29-2746	-0171	9-4986	-0037

TABLE III.

No.	Sq. Rts.	Diff.	C. Rts.	Diff.	No.	Sq. Rts.	Diff.	C. Rts.	Diff.
857	29-2746	-0170	9-4986	-0037	905	30-0832	-0166	9-6727	-0036
858	29-2916	-0171	9-5023	-0037	906	30-0998	-0166	9-6763	-0036
859	29-3087	-0171	9-5060	-0037	907	30-1164	-0166	9-6799	-0035
860	29-3258	-0171	9-5097	-0037	908	30-1330	-0166	9-6834	-0036
861	29-3428	-0170	9-5134	-0037	909	30-1496	-0166	9-6870	-0035
862	29-3598	-0170	9-5171	-0036	910	30-1662	-0166	9-6905	-0036
863	29-3769	-0171	9-5207	-0037	911	30-1828	-0165	9-6941	-0035
864	29-3939	-0170	9-5244	-0037	912	30-1993	-0166	9-6976	-0036
865	29-4109	-0170	9-5281	-0037	913	30-2159	-0165	9-7012	-0035
866	29-4279	-0170	9-5317	-0036	914	30-2324	-0166	9-7047	-0035
867	29-4449	-0169	9-5354	-0037	915	30-2490	-0165	9-7082	-0035
868	29-4618	-0170	9-5391	-0036	916	30-2655	-0165	9-7118	-0036
869	29-4788	-0170	9-5427	-0037	917	30-2820	-0165	9-7153	-0035
870	29-4958	-0169	9-5464	-0037	918	30-2985	-0165	9-7188	-0036
871	29-5127	-0169	9-5501	-0036	919	30-3150	-0165	9-7224	-0035
872	29-5296	-0170	9-5537	-0037	920	30-3315	-0165	9-7259	-0035
873	29-5466	-0169	9-5574	-0036	921	30-3480	-0165	9-7294	-0035
874	29-5635	-0169	9-5610	-0037	922	30-3645	-0164	9-7329	-0035
875	29-5804	-0169	9-5647	-0036	923	30-3809	-0165	9-7364	-0036
876	29-5973	-0169	9-5683	-0036	924	30-3974	-0164	9-7400	-0035
877	29-6142	-0169	9-5719	-0037	925	30-4138	-0164	9-7435	-0035
878	29-6311	-0168	9-5756	-0036	926	30-4302	-0165	9-7470	-0035
879	29-6479	-0169	9-5792	-0036	927	30-4467	-0165	9-7505	-0035
880	29-6648	-0168	9-5828	-0037	928	30-4631	-0164	9-7540	-0035
881	29-6816	-0169	9-5865	-0036	929	30-4795	-0164	9-7575	-0035
882	29-6985	-0168	9-5901	-0036	930	30-4959	-0164	9-7610	-0035
883	29-7153	-0168	9-5937	-0036	931	30-5123	-0164	9-7645	-0035
884	29-7321	-0168	9-5973	-0037	932	30-5287	-0164	9-7680	-0035
885	29-7489	-0169	9-6010	-0036	933	30-5450	-0163	9-7715	-0035
886	29-7658	-0167	9-6046	-0036	934	30-5614	-0164	9-7750	-0035
887	29-7825	-0168	9-6082	-0036	935	30-5778	-0163	9-7785	-0034
888	29-7993	-0168	9-6118	-0036	936	30-5941	-0164	9-7819	-0035
889	29-8161	-0168	9-6154	-0036	937	30-6105	-0163	9-7854	-0035
890	29-8329	-0167	9-6190	-0036	938	30-6268	-0163	9-7889	-0035
891	29-8496	-0168	9-6226	-0036	939	30-6431	-0163	9-7924	-0035
892	29-8664	-0167	9-6262	-0036	940	30-6594	-0163	9-7959	-0034
893	29-8831	-0167	9-6298	-0036	941	30-6757	-0163	9-7993	-0035
894	29-8998	-0168	9-6334	-0036	942	30-6920	-0163	9-8028	-0035
895	29-9166	-0167	9-6370	-0036	943	30-7083	-0163	9-8063	-0034
896	29-9333	-0167	9-6406	-0036	944	30-7246	-0163	9-8097	-0035
897	29-9500	-0166	9-6442	-0035	945	30-7409	-0162	9-8132	-0035
898	29-9666	-0167	9-6477	-0036	946	30-7571	-0163	9-8167	-0034
899	29-9833	-0167	9-6513	-0036	947	30-7734	-0162	9-8201	-0035
900	30-0000	-0167	9-6549	-0036	948	30-7896	-0162	9-8236	-0034
901	30-0167	-0166	9-6585	-0035	949	30-8058	-0163	9-8270	-0035
902	30-0333	-0167	9-6620	-0036	950	30-8221	-0162	9-8305	-0034
903	30-0500	-0166	9-6656	-0036	951	30-8383	-0162	9-8339	-0035
904	30-0666	-0166	9-6692	-0035	952	30-8545	-0162	9-8374	-0034
905	30-0832	-0166	9-6727	-0035	953	30-8707	-0162	9-8408	-0034

TABLE III.

No.	Sq. Rts.	Diff.	C. Rts.	Diff.	No.	Sq. Rts.	Diff.	C. Rts.	Diff.
953	30-8707	-0162	9-8408	-0035	977	31-2570	-0160	9-9227	-0034
954	30-8869	-0162	9-8443	-0034	978	31-2730	-0160	9-9261	-0034
955	30-9031	-0161	9-8477	-0034	979	31-2890	-0160	9-9295	-0034
956	30-9192	-0161	9-8511	-0034	980	31-3050	-0160	9-9329	-0034
957	30-9354	-0162	9-8546	-0035	981	31-3209	-0159	9-9363	-0034
958	30-9516	-0162	9-8580	-0034	982	31-3369	-0160	9-9396	-0033
959	30-9677	-0161	9-8614	-0034	983	31-3528	-0159	9-9430	-0034
960	30-9839	-0162	9-8648	-0034	984	31-3688	-0159	9-9464	-0033
961	31-0000	-0161	9-8683	-0035	985	31-3847	-0159	9-9497	-0033
962	31-0161	-0161	9-8717	-0034	986	31-4006	-0159	9-9531	-0034
963	31-0322	-0161	9-8751	-0034	987	31-4166	-0160	9-9565	-0034
964	31-0483	-0161	9-8785	-0034	988	31-4325	-0159	9-9598	-0033
965	31-0644	-0161	9-8819	-0035	989	31-4484	-0159	9-9632	-0034
966	31-0805	-0161	9-8854	-0034	990	31-4643	-0159	9-9666	-0033
967	31-0966	-0161	9-8888	-0034	991	31-4802	-0158	9-9699	-0033
968	31-1127	-0161	9-8922	-0034	992	31-4960	-0158	9-9732	-0034
969	31-1288	-0160	9-8956	-0034	993	31-5119	-0159	9-9766	-0034
970	31-1448	-0161	9-8990	-0034	994	31-5278	-0158	9-9800	-0033
971	31-1609	-0160	9-9024	-0034	995	31-5436	-0159	9-9833	-0033
972	31-1769	-0160	9-9058	-0034	996	31-5595	-0158	9-9866	-0034
973	31-1929	-0161	9-9092	-0034	997	31-5753	-0158	9-9900	-0034
974	31-2090	-0160	9-9126	-0034	998	31-5911	-0159	9-9933	-0033
975	31-2250	-0160	9-9160	-0034	999	31-6070	-0158	9-9967	-0034
976	31-2410	-0160	9-9194	-0033	1000	31-6228	-0158	10-0000	-0033
977	31-2570	-0160	9-9227	-0033					

TO FIND BY THE FOREGOING TABLE THE SQUARE OR CUBE ROOT OF ANY NUMBER NOT EXCEEDING 1000.

RULE.—Multiply the difference between the root of the integer part of the given number, and the root of the next higher integer number, by the decimal part of the given number, and add the product to the root of the integer number given, the sum is the root required.

EXAMPLE 1.—Required the square root of 53-75?

Difference by table = $\cdot 0684 \times \cdot 75 = \cdot 0513$; and the root of 53 = 7-2801 hence, $7-2801 + \cdot 0513 = 7-3314$, the root required.

EXAMPLE 2.—Required the cube root of the number 734-26?

Difference by table = $\cdot 0041 \times \cdot 26 = \cdot 001066$ and the root of 734 = 9-0205 hence, $9-0205 + \cdot 001066 = 9-021566$ or 9-0216, is the root required.

TABLE IV.

CONTAINING

THE SURFACE AND SOLIDITY OF SPHERES,

THE EDGE OR SIDE OF EQUAL CUBES,

THE LENGTHS OF EQUAL CYLINDERS,

AND THE

**WEIGHT OF EQUAL VOLUMES OF WATER IN *LBS.*
AVOIRDUPOIS.**

TABLE IV.

Dia. in inches.	Surface in sq. in.	Solidity in cub. in.	Cube in inches.	Cylinder in inches.	Water in lbs. avoir.
0 in.	sq. in.	cub. in.	in.	in.	
$\frac{1}{8}$	0.0491	0.0010	0.1007	0.0833	0.0000
$\frac{1}{4}$	0.1963	0.0082	0.2015	0.1667	0.0003
$\frac{3}{8}$	0.4418	0.0276	0.3022	0.2500	0.0010
$\frac{1}{2}$	0.7854	0.0654	0.4030	0.3333	0.0024
$\frac{5}{8}$	1.2272	0.1278	0.5037	0.4167	0.0046
$\frac{3}{4}$	1.7671	0.2209	0.6045	0.5000	0.0080
$\frac{7}{8}$	2.4053	0.3508	0.7024	0.5833	0.0127
1 in.	3.1416	0.5236	0.8060	0.6667	0.0189
$1\frac{1}{8}$	3.9761	0.7455	0.9067	0.7500	0.0269
$1\frac{1}{4}$	4.9087	1.0227	1.0075	0.8333	0.0369
$1\frac{3}{8}$	5.9396	1.3612	1.1082	0.9167	0.0491
$1\frac{1}{2}$	7.0686	1.7671	1.2090	1.0000	0.0637
$1\frac{5}{8}$	8.2958	2.2468	1.3097	1.0833	0.0810
$1\frac{3}{4}$	9.6211	2.8062	1.4105	1.1667	0.1012
$1\frac{7}{8}$	11.0447	3.4515	1.5112	1.2500	0.1245
2 in.	12.5664	4.1888	1.6120	1.3333	0.1511
$2\frac{1}{8}$	14.1863	5.0243	1.7127	1.4167	0.1812
$2\frac{1}{4}$	15.9043	5.9641	1.8135	1.5000	0.2151
$2\frac{3}{8}$	17.7205	7.0144	1.9142	1.5833	0.2530
$2\frac{1}{2}$	19.6350	8.1812	2.0150	1.6667	0.2951
$2\frac{5}{8}$	21.6475	9.4708	2.1157	1.7500	0.3416
$2\frac{3}{4}$	23.7583	10.8892	2.2165	1.8333	0.3927
$2\frac{7}{8}$	25.9672	12.4426	2.3172	1.9166	0.4487
3 in.	28.2743	14.1372	2.4180	2.0000	0.5099
$3\frac{1}{8}$	30.6796	15.9790	2.5187	2.0833	0.5778
$3\frac{1}{4}$	33.1831	17.9742	2.6195	2.1667	0.6482
$3\frac{3}{8}$	35.7847	20.1289	2.7202	2.2500	0.7256
$3\frac{1}{2}$	38.4845	22.4493	2.8210	2.3333	0.8096
$3\frac{5}{8}$	41.2825	24.9415	2.9217	2.4167	0.8995
$3\frac{3}{4}$	44.1786	27.6117	3.0225	2.5000	0.9953
$3\frac{7}{8}$	47.1730	30.4659	3.1232	2.5833	1.0988
4 in.	50.2655	33.5103	3.2240	2.6667	1.2086
$4\frac{1}{8}$	53.4562	36.7511	3.3247	2.7500	1.3254
$4\frac{1}{4}$	56.7450	40.1944	3.4255	2.8333	1.4496
$4\frac{3}{8}$	60.1320	43.8463	3.5262	2.9166	1.5813
$4\frac{1}{2}$	63.6173	47.7129	3.6270	3.0000	1.7208
$4\frac{5}{8}$	67.2006	51.8005	3.7277	3.0833	1.8682
$4\frac{3}{4}$	70.8822	56.1151	3.8285	3.1667	2.0238
$4\frac{7}{8}$	74.6619	60.6628	3.9292	3.2500	2.1878
5 in.	78.5398	65.4498	4.0300	3.3333	2.3605
$5\frac{1}{8}$	82.5159	70.4823	4.1307	3.4167	2.5420
$5\frac{1}{4}$	86.5901	75.7664	4.2315	3.5000	2.7325
$5\frac{3}{8}$	90.7626	81.3081	4.3322	3.5833	2.9324
$5\frac{1}{2}$	95.0332	87.1137	4.4330	3.6667	3.1418
$5\frac{5}{8}$	99.4020	93.1893	4.5337	3.7500	3.3609
$5\frac{3}{4}$	103.8689	99.5410	4.6345	3.8333	3.5900
$5\frac{7}{8}$	108.4340	106.1750	4.7352	3.9167	3.8280

OF = CYLINDERS, AND WEIGHT OF = VOLUMES OF WATER. [5]

TABLE IV.

Dia. in inches.	Surface in sq. in.	Solidity in cub. in.	Cube in inches.	Cylinder in inches.	Water in lbs. avoir.
6 in.	113.0973	113.0973	4.8360	4.0000	4.0789
$\frac{1}{2}$	117.8588	120.3142	4.9867	4.0833	4.3392
$\frac{1}{4}$	122.7185	127.8317	5.0375	4.1667	4.6228
$\frac{3}{4}$	127.6763	135.6561	5.1382	4.2500	4.8922
1	132.7323	143.7933	5.2390	4.3333	5.1860
$\frac{1}{2}$	137.8865	152.2496	5.3397	4.4167	5.4909
$\frac{3}{4}$	143.1388	161.0312	5.4405	4.5000	5.8077
2	148.4893	170.1440	5.5412	4.5833	6.1363
7 in.	153.9380	179.5944	5.6420	4.6667	6.4771
$\frac{1}{2}$	159.4849	189.3883	5.7427	4.7500	6.8304
$\frac{1}{4}$	165.1300	199.5320	5.8435	4.8333	7.1962
$\frac{3}{4}$	170.8732	210.0316	5.9442	4.9167	7.5749
1	176.7146	220.8932	6.0450	5.0000	7.9666
$\frac{1}{2}$	182.6542	232.1230	6.1457	5.0833	8.3716
$\frac{3}{4}$	188.6919	243.7270	6.2465	5.1667	8.7901
2	194.8278	255.7115	6.3472	5.2500	9.2223
8 in.	201.0619	268.0826	6.4480	5.3333	9.6685
$\frac{1}{2}$	207.3942	280.8463	6.5487	5.4167	10.1288
$\frac{1}{4}$	213.8246	294.0089	6.6495	5.5000	10.6036
$\frac{3}{4}$	220.3533	307.5764	6.7502	5.5833	11.0933
1	226.9801	321.5551	6.8510	5.6667	11.5970
$\frac{1}{2}$	233.7050	335.9510	6.9517	5.7500	12.1159
$\frac{3}{4}$	240.5282	350.7703	7.0525	5.8333	12.6507
2	247.4495	366.0191	7.1532	5.9167	13.2007
9 in.	254.4690	381.7035	7.2540	6.0000	13.7663
$\frac{1}{2}$	261.5867	397.8297	7.3547	6.0833	14.3479
$\frac{1}{4}$	268.8025	414.4039	7.4555	6.1667	14.9457
$\frac{3}{4}$	276.1165	431.4321	7.5562	6.2500	15.5599
1	283.5287	448.9205	7.6570	6.3333	16.1905
$\frac{1}{2}$	291.0391	466.8752	7.7577	6.4167	16.8378
$\frac{3}{4}$	298.6477	485.3024	7.8585	6.5000	17.5026
2	306.3544	504.2082	7.9592	6.5833	18.1845
10 in.	314.1593	523.5988	8.0600	6.6667	18.8838
$\frac{1}{2}$	322.0623	543.4802	8.1607	6.7500	19.6008
$\frac{1}{4}$	330.0636	563.8586	8.2615	6.8333	20.3358
$\frac{3}{4}$	338.1630	584.7402	8.3622	6.9167	21.0891
1	346.3606	606.1310	8.4630	7.0000	21.8604
$\frac{1}{2}$	354.6564	628.0373	8.5637	7.0833	22.6494
$\frac{3}{4}$	363.0503	650.4651	8.6645	7.1667	23.4593
2	371.5424	673.4206	8.7652	7.2500	24.2872
11 in.	380.1327	696.9100	8.8660	7.3333	25.1344
$\frac{1}{2}$	388.8212	720.9393	8.9667	7.4167	26.0010
$\frac{1}{4}$	397.6078	745.5147	9.0675	7.5000	26.8873
$\frac{3}{4}$	406.4926	770.6423	9.1682	7.5833	27.7936
1	415.4756	796.3283	9.2690	7.6667	28.7199
$\frac{1}{2}$	424.5568	822.5788	9.3697	7.7500	29.6662
$\frac{3}{4}$	433.7361	849.3999	9.4705	7.8333	30.6242
2	443.0137	876.7979	9.5712	7.9167	31.6221

TABLE IV.

Dia. in inches.	Surface in sq. in.	Solidity in cub. in.	Cube in inches.	Cylinder in inches.	Water in lbs. avoir.
12 in.	452-3893	904-7787	9-6720	8-0000	32-6312
$\frac{1}{8}$	461-8632	933-3486	9-7727	8-0833	33-6616
$\frac{1}{4}$	471-4352	962-5136	9-8735	8-1667	34-7135
$\frac{3}{8}$	481-1055	992-2800	9-9742	8-2500	35-7868
$\frac{1}{2}$	490-8739	1022-6539	10-0749	8-3333	36-8824
$\frac{5}{8}$	500-7404	1053-6413	10-1757	8-4167	38-0000
$\frac{3}{4}$	510-7052	1085-2485	10-2764	8-5000	39-1399
$\frac{7}{8}$	520-7681	1117-4815	10-3772	8-5833	40-3024
13 in.	530-9292	1150-3465	10-4779	8-6667	41-4877
$\frac{1}{8}$	541-1884	1183-8497	10-5787	8-7500	42-6960
$\frac{1}{4}$	551-5459	1217-9971	10-6794	8-8333	43-9276
$\frac{3}{8}$	562-0015	1252-7950	10-7802	8-9167	45-1828
$\frac{1}{2}$	572-5553	1288-2493	10-8809	9-0000	46-4613
$\frac{5}{8}$	583-2072	1324-3664	10-9817	9-0833	47-7436
$\frac{3}{4}$	593-9574	1361-1523	11-0824	9-1667	49-0905
$\frac{7}{8}$	604-8057	1398-6131	11-1832	9-2500	50-4416
14 in.	615-7522	1436-7550	11-2839	9-3333	51-8172
$\frac{1}{8}$	626-7968	1475-5842	11-3847	9-4167	53-2176
$\frac{1}{4}$	637-9397	1515-1067	11-4854	9-5000	54-6430
$\frac{3}{8}$	649-1807	1555-3287	11-5862	9-5833	56-0936
$\frac{1}{2}$	660-5199	1596-2563	11-6869	9-6667	57-5697
$\frac{5}{8}$	671-9572	1637-8957	11-7877	9-7500	59-0716
$\frac{3}{4}$	683-4928	1680-2530	11-8884	9-8333	60-5990
$\frac{7}{8}$	695-1265	1723-3343	11-9892	9-9167	62-1528
15 in.	706-8583	1767-1459	12-0899	10-0000	63-7329
$\frac{1}{8}$	718-6884	1811-6937	12-1907	10-0833	65-3395
$\frac{1}{4}$	730-6166	1856-9840	12-2914	10-1667	66-9729
$\frac{3}{8}$	742-6431	1903-0228	12-3922	10-2500	68-6333
$\frac{1}{2}$	754-7676	1949-8164	12-4929	10-3333	70-3210
$\frac{5}{8}$	766-9904	1997-3708	12-5937	10-4167	72-0361
$\frac{3}{4}$	779-3113	2045-6922	12-6944	10-5000	73-7788
$\frac{7}{8}$	791-7304	2094-7868	12-7952	10-5833	75-5494
16 in.	804-2477	2144-6606	12-8959	10-6667	77-3481

EXAMPLES TO TABLE IV.

Ex. 1. Required the surface of a sphere, whose diameter is $4\frac{1}{2}$ inches?

Here, in *column 1, page 50*, will be found $4\frac{1}{2}$ inches, the diameter; against which, in *column 2*, will be found 74·6619 square inches, the surface required.

Or by mensuration thus:—multiply the square of the diameter by π , No. 1 of the Table of Useful Numbers, *page 169*;

Here $\pi \times (4\frac{1}{2})^2 = 3\cdot1415926 \times (5 - \frac{1}{2})^2 = (15\cdot7079630 - 3926991) \times (5 - \frac{1}{2}) = 15\cdot3152639 \times (5 - \frac{1}{2}) = 76\cdot5763195 - 1\cdot9144080 = 74\cdot6619115$ square inches as before.

Ex. 2. Required the solidity of a sphere, whose diameter is $11\frac{1}{2}$ inches?

Here, in *column 1, page 51*, will be found $11\frac{1}{2}$ inches, the diameter; against which, in *column 3*, will be found 770·6423 cubic inches, the solid content required.

Or by mensuration thus:—multiply the cube of the diameter by $\frac{1}{6}\pi$, No. 8 of the Table of Useful Numbers, *page 170*;

Here $\frac{1}{6}\pi \times (11\frac{1}{2})^3 = \cdot5235988 \times (11 + \frac{1}{2} + \frac{1}{8})^3 = (5\cdot7595868 + 1\cdot308997 + \cdot0654498) \times (11 + \frac{1}{2} + \frac{1}{8})^3 = 5\cdot9559363 \times (11 + \frac{1}{2} + \frac{1}{8})^3 = (65\cdot5152993 + 1\cdot4889841 + \cdot7444920) \times (11 + \frac{1}{2} + \frac{1}{8}) = 67\cdot7487754 \times (11 + \frac{1}{2} + \frac{1}{8}) = 745\cdot2365294 + 16\cdot9371938 + 8\cdot4685969 = 770\cdot6423201$ solid inches as before.

Ex. 3. What is the length of the edge or side of a cube, equal in volume or solid content to a sphere, whose diameter is $7\frac{1}{2}$ inches?

Here, in *column 1, page 51*, will be found $7\frac{1}{2}$ inches, the diameter; against which, in *column 4*, will be found 6·1457 inches, the length required.

Or thus by mensuration:—multiply the diameter of the sphere by $\sqrt[3]{\frac{\pi}{6}}$, No 66 of the Table of Useful Numbers, *page 175*.

Here $\sqrt[3]{\frac{\pi}{6}} \times 7\frac{1}{2} = \cdot805996 \times (7 + \frac{1}{2} + \frac{1}{8}) = 5\cdot641972 + \cdot402998 + \cdot100749 = 6\cdot145719$ inches as before.

Ex. 4. What is the length of a cylinder of $4\frac{3}{4}$ inches diameter, having its volume equal to that of a sphere of the same diameter?

Here, in *column 1, page 50*, will be found $4\frac{3}{4}$ inches, the diameter; against which, in *column 5*, will be found 3·1667 inches, the length required.

Or thus:—two-thirds of the sphere's diameter = height of the cylinder, thus,

$$\begin{array}{r} \text{From } 4\frac{1}{2} = 4.75 \\ \text{Subtract } \frac{1}{3} \text{ of } 4\frac{1}{2} = 1.5833 \end{array}$$

3.1667 inches as before.

Ex. 5. Required the weight of a quantity of rain water, equal in volume to that of a sphere whose diameter is $15\frac{1}{2}$ inches?

Here, in *column 1, page 52*, will be found $15\frac{1}{2}$ inches, against which, in *column 6*, will be found 65.3395 lbs. *avoirdupois*, the weight required.

Or thus:—find the solidity of the sphere as *example 2*, which multiply by .0360654 = 10 times No. 89 of the Table of Useful Numbers, *page 177*.

$$\begin{array}{r} .5235988 = \frac{1}{6}\pi \\ \hline 5.7595868 = 11 \times \frac{1}{6}\pi \\ \hline 63.3554548 = 121 \times \frac{1}{6}\pi \\ \hline 7.9194318 = \left(\frac{121}{8}\right) \times \frac{1}{6}\pi \\ \hline 87.1137498 = 11 \times \left(\frac{121}{8}\right) \times \frac{1}{6}\pi \\ \hline 958.2512478 = 121 \times \left(\frac{121}{8}\right) \times \frac{1}{6}\pi \\ \hline 119.7814060 = \left(\frac{121}{8}\right)^2 \times \frac{1}{6}\pi \\ \hline 1317.5954660 = 11 \times \left(\frac{121}{8}\right)^2 \times \frac{1}{6}\pi \\ \hline 14493.5501260 = 121 \times \left(\frac{121}{8}\right)^2 \times \frac{1}{6}\pi \\ \hline 1811.693766 = \left(\frac{121}{8}\right)^3 \times \frac{1}{6}\pi \\ 456063 = \text{Const. No. inverted.} \\ \hline 54.350813 \\ 10.870163 \\ .108702 \\ 9058 \\ 725 \\ \hline 65.339461 \text{ lbs. av.} = \text{Ans. as before.} \end{array}$$

TABLE V.
CONTAINING THE
WEIGHT OF CYLINDRICAL COLUMNS OF WATER,
EACH ONE FOOT IN LENGTH,
AND OF VARIOUS DIAMETERS,
IN *LBS.* AVOIRDUPOIS.

TABLE V.

Dia.	Weight.	Dia.	Weight.	Dia.	Weight.
3 in.	3-0592	9 in.	27-5326	15 in.	76-4794
$\frac{1}{8}$	3-3194	$\frac{1}{8}$	28-3027	$\frac{1}{8}$	77-7594
$\frac{1}{4}$	3-5903	$\frac{1}{4}$	29-0834	$\frac{1}{4}$	79-0500
$\frac{3}{8}$	3-8718	$\frac{3}{8}$	29-8748	$\frac{3}{8}$	80-3512
$\frac{1}{2}$	4-1639	$\frac{1}{2}$	30-6768	$\frac{1}{2}$	81-6631
$\frac{5}{8}$	4-4666	$\frac{5}{8}$	31-4894	$\frac{5}{8}$	82-9855
$\frac{3}{4}$	4-7800	$\frac{3}{4}$	32-3126	$\frac{3}{4}$	84-3186
$\frac{7}{8}$	5-1039	$\frac{7}{8}$	33-1464	$\frac{7}{8}$	85-6623
4 in.	5-4385	10 in.	33-9909	16 in.	87-0166
$\frac{1}{8}$	5-7838	$\frac{1}{8}$	34-8459	$\frac{1}{8}$	88-3816
$\frac{1}{4}$	6-1396	$\frac{1}{4}$	35-7117	$\frac{1}{4}$	89-7571
$\frac{3}{8}$	6-5061	$\frac{3}{8}$	36-5880	$\frac{3}{8}$	91-1433
$\frac{1}{2}$	6-8832	$\frac{1}{2}$	37-4749	$\frac{1}{2}$	92-5401
$\frac{5}{8}$	7-2709	$\frac{5}{8}$	38-3725	$\frac{5}{8}$	93-9476
$\frac{3}{4}$	7-6692	$\frac{3}{4}$	39-2807	$\frac{3}{4}$	95-3656
$\frac{7}{8}$	8-0781	$\frac{7}{8}$	40-1995	$\frac{7}{8}$	96-7943
5 in.	8-4977	11 in.	41-1289	17 in.	98-2336
$\frac{1}{8}$	8-9279	$\frac{1}{8}$	42-0690	$\frac{1}{8}$	99-6835
$\frac{1}{4}$	9-3687	$\frac{1}{4}$	43-0197	$\frac{1}{4}$	101-1441
$\frac{3}{8}$	9-8202	$\frac{3}{8}$	43-9810	$\frac{3}{8}$	102-6152
$\frac{1}{2}$	10-2822	$\frac{1}{2}$	44-9529	$\frac{1}{2}$	104-0970
$\frac{5}{8}$	10-7549	$\frac{5}{8}$	45-9355	$\frac{5}{8}$	105-5894
$\frac{3}{4}$	11-2382	$\frac{3}{4}$	46-9286	$\frac{3}{4}$	107-0925
$\frac{7}{8}$	11-7322	$\frac{7}{8}$	47-9324	$\frac{7}{8}$	108-6061
6 in.	12-2367	12 in.	48-9468	18 in.	110-1304
$\frac{1}{8}$	12-7519	$\frac{1}{8}$	49-9719	$\frac{1}{8}$	111-6653
$\frac{1}{4}$	13-2777	$\frac{1}{4}$	51-0075	$\frac{1}{4}$	113-2108
$\frac{3}{8}$	13-8141	$\frac{3}{8}$	52-0538	$\frac{3}{8}$	114-7670
$\frac{1}{2}$	14-3611	$\frac{1}{2}$	53-1107	$\frac{1}{2}$	116-3337
$\frac{5}{8}$	14-9188	$\frac{5}{8}$	54-1783	$\frac{5}{8}$	117-9111
$\frac{3}{4}$	15-4871	$\frac{3}{4}$	55-2564	$\frac{3}{4}$	119-4991
$\frac{7}{8}$	16-0660	$\frac{7}{8}$	56-3452	$\frac{7}{8}$	121-0978
7 in.	16-6555	13 in.	57-4446	19 in.	122-7070
$\frac{1}{8}$	17-2557	$\frac{1}{8}$	58-5546	$\frac{1}{8}$	124-3369
$\frac{1}{4}$	17-8664	$\frac{1}{4}$	59-6752	$\frac{1}{4}$	125-9574
$\frac{3}{8}$	18-4878	$\frac{3}{8}$	60-8065	$\frac{3}{8}$	127-5985
$\frac{1}{2}$	19-1199	$\frac{1}{2}$	61-9484	$\frac{1}{2}$	129-2503
$\frac{5}{8}$	19-7625	$\frac{5}{8}$	63-1009	$\frac{5}{8}$	130-9126
$\frac{3}{4}$	20-4158	$\frac{3}{4}$	64-2640	$\frac{3}{4}$	132-5856
$\frac{7}{8}$	21-0796	$\frac{7}{8}$	65-4377	$\frac{7}{8}$	134-2692
8 in.	21-7542	14 in.	66-6221	20 in.	135-9635
$\frac{1}{8}$	22-4393	$\frac{1}{8}$	67-8171	$\frac{1}{8}$	137-6683
$\frac{1}{4}$	23-1350	$\frac{1}{4}$	69-0227	$\frac{1}{4}$	139-3838
$\frac{3}{8}$	23-8414	$\frac{3}{8}$	70-2389	$\frac{3}{8}$	141-1099
$\frac{1}{2}$	24-5584	$\frac{1}{2}$	71-4658	$\frac{1}{2}$	142-8466
$\frac{5}{8}$	25-2860	$\frac{5}{8}$	72-7033	$\frac{5}{8}$	144-5940
$\frac{3}{4}$	26-0243	$\frac{3}{4}$	73-9514	$\frac{3}{4}$	146-3519
$\frac{7}{8}$	26-7731	$\frac{7}{8}$	75-2101	$\frac{7}{8}$	148-1205

TABLE V.

Dia.	Weight.	Dia.	Weight.	Dia.	Weight.
21 in.	149-8997	27 in.	247-7934	33 in.	370-1605
$\frac{1}{8}$	151-6895	$\frac{1}{8}$	250-0931	$\frac{1}{8}$	372-9701
$\frac{1}{4}$	153-4900	$\frac{1}{4}$	252-4034	$\frac{1}{4}$	375-7903
$\frac{3}{8}$	155-3011	$\frac{3}{8}$	254-7244	$\frac{3}{8}$	378-6211
$\frac{1}{2}$	157-1228	$\frac{1}{2}$	257-0559	$\frac{1}{2}$	381-4625
$\frac{5}{8}$	158-9551	$\frac{5}{8}$	259-3981	$\frac{5}{8}$	384-3145
$\frac{3}{4}$	160-7980	$\frac{3}{4}$	261-7509	$\frac{3}{4}$	387-1772
$\frac{7}{8}$	162-6516	$\frac{7}{8}$	264-1143	$\frac{7}{8}$	390-0505
22 in.	164-5158	28 in.	266-4884	34 in.	392-9344
$\frac{1}{8}$	166-3906	$\frac{1}{8}$	268-8731	$\frac{1}{8}$	395-8289
$\frac{1}{4}$	168-2760	$\frac{1}{4}$	271-2684	$\frac{1}{4}$	398-7341
$\frac{3}{8}$	170-1721	$\frac{3}{8}$	273-6743	$\frac{3}{8}$	401-6499
$\frac{1}{2}$	172-0788	$\frac{1}{2}$	276-0908	$\frac{1}{2}$	404-5763
$\frac{5}{8}$	173-9961	$\frac{5}{8}$	278-5180	$\frac{5}{8}$	407-5133
$\frac{3}{4}$	175-9240	$\frac{3}{4}$	280-9557	$\frac{3}{4}$	410-4609
$\frac{7}{8}$	177-8625	$\frac{7}{8}$	283-4042	$\frac{7}{8}$	413-4192
23 in.	179-8117	29 in.	285-8632	35 in.	416-3881
$\frac{1}{8}$	181-7715	$\frac{1}{8}$	288-3328	$\frac{1}{8}$	419-3676
$\frac{1}{4}$	183-7419	$\frac{1}{4}$	290-8131	$\frac{1}{4}$	422-3577
$\frac{3}{8}$	185-7229	$\frac{3}{8}$	293-3040	$\frac{3}{8}$	425-3585
$\frac{1}{2}$	187-7146	$\frac{1}{2}$	295-8055	$\frac{1}{2}$	428-3699
$\frac{5}{8}$	189-7168	$\frac{5}{8}$	298-3176	$\frac{5}{8}$	431-3919
$\frac{3}{4}$	191-7297	$\frac{3}{4}$	300-8404	$\frac{3}{4}$	434-4245
$\frac{7}{8}$	193-7532	$\frac{7}{8}$	303-3738	$\frac{7}{8}$	437-4678
24 in.	195-7874	30 in.	305-9178	36 in.	440-5216
$\frac{1}{8}$	197-8321	$\frac{1}{8}$	308-4724	$\frac{1}{8}$	443-5861
$\frac{1}{4}$	199-8875	$\frac{1}{4}$	311-0377	$\frac{1}{4}$	446-6612
$\frac{3}{8}$	201-9535	$\frac{3}{8}$	313-6135	$\frac{3}{8}$	449-7470
$\frac{1}{2}$	204-0302	$\frac{1}{2}$	316-2000	$\frac{1}{2}$	452-8433
$\frac{5}{8}$	206-1174	$\frac{5}{8}$	318-7971	$\frac{5}{8}$	455-9503
$\frac{3}{4}$	208-2153	$\frac{3}{4}$	321-4049	$\frac{3}{4}$	459-0679
$\frac{7}{8}$	210-3238	$\frac{7}{8}$	324-0232	$\frac{7}{8}$	462-1961
25 in.	212-4429	31 in.	326-6522	37 in.	465-3350
$\frac{1}{8}$	214-5727	$\frac{1}{8}$	329-2918	$\frac{1}{8}$	468-4844
$\frac{1}{4}$	216-7130	$\frac{1}{4}$	331-9420	$\frac{1}{4}$	471-6445
$\frac{3}{8}$	218-8640	$\frac{3}{8}$	334-6029	$\frac{3}{8}$	474-8152
$\frac{1}{2}$	221-0256	$\frac{1}{2}$	337-2744	$\frac{1}{2}$	477-9965
$\frac{5}{8}$	223-1978	$\frac{5}{8}$	339-9565	$\frac{5}{8}$	481-1885
$\frac{3}{4}$	225-3807	$\frac{3}{4}$	342-6492	$\frac{3}{4}$	484-3911
$\frac{7}{8}$	227-5742	$\frac{7}{8}$	345-3525	$\frac{7}{8}$	487-6043
26 in.	229-7783	32 in.	348-0665	38 in.	490-8281
$\frac{1}{8}$	231-9930	$\frac{1}{8}$	350-7910	$\frac{1}{8}$	494-0625
$\frac{1}{4}$	234-2183	$\frac{1}{4}$	353-5262	$\frac{1}{4}$	497-3076
$\frac{3}{8}$	236-4543	$\frac{3}{8}$	356-2721	$\frac{3}{8}$	500-5633
$\frac{1}{2}$	238-7009	$\frac{1}{2}$	359-0285	$\frac{1}{2}$	503-8296
$\frac{5}{8}$	240-9581	$\frac{5}{8}$	361-7956	$\frac{5}{8}$	507-1065
$\frac{3}{4}$	243-2259	$\frac{3}{4}$	364-5733	$\frac{3}{4}$	510-3941
$\frac{7}{8}$	245-5043	$\frac{7}{8}$	367-3616	$\frac{7}{8}$	513-6923

TABLE V.

Dia.	Weight.	Dia.	Weight.	Dia.	Weight.
39 in.	517-0011	43 in.	628-4911	47 in.	750-8582
$\frac{1}{8}$	520-3205	$\frac{1}{8}$	632-1504	$\frac{1}{8}$	754-8575
$\frac{1}{4}$	523-6505	$\frac{1}{4}$	635-8204	$\frac{1}{4}$	758-8673
$\frac{3}{8}$	526-9912	$\frac{3}{8}$	639-5010	$\frac{3}{8}$	762-8878
$\frac{1}{2}$	530-3425	$\frac{1}{2}$	643-1922	$\frac{1}{2}$	766-9189
$\frac{5}{8}$	533-7044	$\frac{5}{8}$	646-8940	$\frac{5}{8}$	770-9606
$\frac{3}{4}$	537-0769	$\frac{3}{4}$	650-6064	$\frac{3}{4}$	775-0130
$\frac{7}{8}$	540-4601	$\frac{7}{8}$	654-3295	$\frac{7}{8}$	779-0760
40 in.	543-8539	44 in.	658-0632	48 in.	783-1495
$\frac{1}{8}$	547-2582	$\frac{1}{8}$	661-8075	$\frac{1}{8}$	787-2338
$\frac{1}{4}$	550-6733	$\frac{1}{4}$	665-5624	$\frac{1}{4}$	791-3286
$\frac{3}{8}$	554-0989	$\frac{3}{8}$	669-3279	$\frac{3}{8}$	795-4341
$\frac{1}{2}$	557-5352	$\frac{1}{2}$	673-1041	$\frac{1}{2}$	799-5501
$\frac{5}{8}$	560-9821	$\frac{5}{8}$	676-8909	$\frac{5}{8}$	803-6768
$\frac{3}{4}$	564-4396	$\frac{3}{4}$	680-6883	$\frac{3}{4}$	807-8142
$\frac{7}{8}$	567-9077	$\frac{7}{8}$	684-4964	$\frac{7}{8}$	811-9621
41 in.	571-3865	45 in.	688-3150	49 in.	816-1207
$\frac{1}{8}$	574-8758	$\frac{1}{8}$	692-1443	$\frac{1}{8}$	820-2899
$\frac{1}{4}$	578-3758	$\frac{1}{4}$	695-9842	$\frac{1}{4}$	824-4697
$\frac{3}{8}$	581-8864	$\frac{3}{8}$	699-8347	$\frac{3}{8}$	828-6601
$\frac{1}{2}$	585-4077	$\frac{1}{2}$	703-6959	$\frac{1}{2}$	832-8612
$\frac{5}{8}$	588-9395	$\frac{5}{8}$	707-5677	$\frac{5}{8}$	837-0729
$\frac{3}{4}$	592-4820	$\frac{3}{4}$	711-4501	$\frac{3}{4}$	841-2952
$\frac{7}{8}$	596-0351	$\frac{7}{8}$	715-3431	$\frac{7}{8}$	845-5281
42 in.	599-5989	46 in.	719-2467	50 in.	849-7716
$\frac{1}{8}$	603-1732	$\frac{1}{8}$	723-1610		
$\frac{1}{4}$	606-7582	$\frac{1}{4}$	727-0859		
$\frac{3}{8}$	610-3538	$\frac{3}{8}$	731-0214		
$\frac{1}{2}$	613-9600	$\frac{1}{2}$	734-9675		
$\frac{5}{8}$	617-5769	$\frac{5}{8}$	738-9242		
$\frac{3}{4}$	621-2043	$\frac{3}{4}$	742-8916		
$\frac{7}{8}$	624-8424	$\frac{7}{8}$	746-8696		

The Editor of the preceding tables begs to state, that in consequence of the numerous errors having been found in former impressions of the five foregoing tables, it was thought necessary to recompute the whole afresh, which has all been carefully done and it is presumed that in the present edition no errors of any consequence will be found, as the methods employed in the construction of the tables were such as nearly to preclude the possibility of error in the computations.

SAMUEL MAYNARD.

8, Earl's Court, Leicester Square,
London, September 3rd, 1852.

EXAMPLES TO TABLE V.

Ex. 1. Required the weight of a cylindrical column of water whose diameter is $47\frac{3}{8}$ inches, and length 1 foot ?

Here, in column 1, page 58, will be found $47\frac{3}{8}$ inches, against which, in the next column, will be found 762·8878 lbs. avoirdupois, the weight required.

Note 1. When the length of the cylinder differs from 1 foot, multiply the tabular number by the length in feet.

Note 2. The weight of cylindrical columns of various substances may be found by means of this Table, V., and the Table VI. of specific gravities, page 61, by multiplying the tabular number of Table V. by the length of the column in feet (when it differs from 1 foot), and by the specific gravity given in page 61, and dividing by 1000.

Ex. 2. Required the weight of a cylindrical column of cast iron, whose diameter is $7\frac{1}{8}$ inches, and length 115 inches ?

Here 115 inches = $9\frac{7}{8}$ feet, the length, and 7271 = the specific gravity of cast iron, by Table VI., page 61.

Then, by Table V., page 56, tabular No., for $7\frac{1}{8}$ in. = 17·2557 lbs. av.

		9
		<hr/>
6 in. = $\frac{1}{2}$	155·3013	
1 in. = $\frac{1}{8}$	8·6278	
	<hr/>	
	1·4380	
Weight of given volume of water	=	165·3671 lbs. av.
Table VI. page 61, Specific gravity	=	<hr/>
		7271
		<hr/>
		1653671
		11575697
		3307342
		11575697
		<hr/>
		1202384·1841 lbs. av.
		<hr/>

which, divided by 1000, gives 1202·3842 lbs. av., the weight of the cast iron column required.

Ex. 3. Required the weight of a cast iron cylindrical pipe $26\frac{1}{2}$ inches, outside diameter, and inside diameter $23\frac{3}{4}$ in., the length being $6\frac{1}{2}$ feet?

$$\begin{array}{r} \text{The tabular number for } \left\{ \begin{array}{l} 26\frac{1}{2} \\ 23\frac{3}{4} \end{array} \right. = \begin{array}{r} \text{lbs. av.} \\ 234.2183 \\ 191.7297 \\ \hline \end{array} \\ \text{Difference} = \underline{42.4886} \end{array}$$

Then, $(42.4886 \text{ lbs. av.} \times 6\frac{1}{2} \times 7271) \div 1000 = 42.4886 \text{ lbs. av.}$
 $\times 6\frac{1}{2} \times 7.271 = 2008.0750 \text{ lbs. av.} = \text{Ans.}$

TABLE VI.
COMBINING THE SPECIFIC GRAVITIES AND OTHER PROPERTIES OF BODIES. WATER THE STANDARD OF COMPARISON, OR 1000.

Names.	Specific gravity.	METALS.										STONES, EARTHS, &c.			
		Melting points in degrees of Fahrenheit.	Contraction in parts of an in. per lineal ft. from the fluid to the solid state.	Ultimate cohesive strength of an inch sq. prism in tons.	Scale of wire-drawing ductility.	Scale of laminae ductility.	Ratio of hardness.	Scale as conductors of electricity.	Ratio of power in the conduction of heat.	Names.		Specific gravity.	Weight of a cubic foot in lbs.	Cubic feet in a ton.	Tons required to crush $\frac{1}{2}$ inch cubes.
Platinum.....	21000	3280	3	1	1.8	3	3.6	Marble, average	2720	170-00	131	13.1	9.6
Pure Gold	19258	2016	1	1	1.8	3	10.0	Granite, ditto..	2651	165-68	131	13.1	6.2
Mercury	13568	1	1	1.8	3	10.0	Purebeck stone..	2651	165-68	131	13.1	9.0
Lead	11344	554	319	31	3	2	2.4	2	1.8	Portland ditto...	2570	162-56	14	14	4.5
Pure Silver.....	17774	1873	156	1.45	2	2	2.0	2	9.7	Bristol ditto...	2570	162-56	14	14	...
Bismuth	19880	476	156	1.45	2	2	2.0	2	9.7	Millstone	2484	159-62	14	14	...
Copper, cast	8788	3587	183	8.31	5	3	2.8	1	8.9	Paving stone ..	2416	159-93	14	14	5.7
Copper, " wrought	8215	15.08	5	3	2.8	1	8.9	Cragleth ditto	2392	147-62	15	15	5.0
Brass, cast	8396	3807	210	8.01	{to any (degree	...	8.6	Grindstone	2143	133-93	16	16	6.6
" sheet.....	8544	12.23	6	6	{to any (degree	...	8.6	Chalk, Brit.....	2781	123-81	12	12	0.3
Iron, cast	7271	3797	125	7.37	{to any (degree	...	3.7	Coal, Scotch	2000	125-00	18	18	0.8
bar	7788	...	137	25.00	4	8	4.7	4	3.7	" Newcastle	1300	81-15	27	27	...
" soft	7833	...	133	58.91	{to any (degree	" Standard	1270	79-37	28	28	...
Steel, hard	7816	{to any (degree	" Cannel	1240	77-50	29	29	...
" cast	7922	442	278	2.11	8	4	1.6	5	3.0		1238	77-37	29	29	...
Tin, cast	7190	700	329	5.06	7	3	1.6	7	3.6						

A concise Method of Verifying Dates, in accordance with the Julian and Gregorian Calendars; or for Finding the Day of the Week, corresponding to any proposed Date of the Month after Christ, without limitation.

By SAMUEL MAYNARD, Editor of Keith's and Bonycastle's Mathematical Works, &c. &c.

[illegible]

EXAMPLES FOR PRACTICE ON THE PRECEDING TABLE.

(1.) Find the Dominical letter for the year 1727, Old Style. Here, $1727 \div 700$ leaves a remainder of 327 years, that is, 3 centuries and 27 years; then, on the same horizontal line with 27 at the left hand of the table, among the "Remaining years of the given date less than one hundred," and opposite the 27 in the first vertical column of letters will be found the Literal Index **C**; enter this **C** in the vertical column under 3, O. S. (3 being the number of centuries of the tabular date 327), it will be found on the top line of letters under its respective tabular centuries; then, on the same horizontal line, in the first vertical column of letters will be found **A**, the Dominical letter required.

(2.) Required the day of the week; 1st of January, 1800, and 12th of February, 1852, New Style in both cases?*

Here, $1800 \div 400$ leaves a remainder of 200 years, that is, 2 centuries and 0 years, which denotes a common year, on the line of 0 in the table; the years less than one hundred will be found as in *Ex. 1*, the Literal Index **A**, enter this Index **A** under the tabular centuries 2, N. S., and it will be found on the fourth line of letters; then, on the same horizontal line, in the fourth vertical column of months will be found the given month, January (along with October), over which, in the same column and on the same horizontal line with 1, the given day of the month, will be found Wednesday, the day of the week required. Again, for 1852, we have $1852 \div 400$, remainder 252, and $\cdot 52$ in the table gives as above the Literal Index **F**, this Index **F** will be found again on the last line but one of letters in column 2, N. S., the given year being a leap year: proceed on this horizontal line to the first vertical column of months, where will be found **F**, (along with August,) viz. February, leap year, the given month; then, as in the preceding case, will be found Thursday, the day of the week required.

(3.) Required the days of the week N. S. corresponding to the dates of the following registrations:—Samuel was born December 16th, 1789; Hannah on 14th July, 1791; Rebecca on 11th October, 1800; Ann on 25th December, 1813; Alfred, on 23rd February, 1821; Augustus on 20th May, 1823; Sarah on 30th January, 1826; Newton on 6th December, 1832; and Mary Ann on 26th December, 1834.

Ans. Samuel on a Wednesday; Hannah on a Thursday; Rebecca on a Saturday; Ann on a Saturday; Alfred on a Friday; Augustus on a Tuesday; Sarah on a Monday; Newton on a Thursday; and Mary Ann on a Friday.

(4.) What day of the month did the last Friday in January, February, August, and December fall on in the year 1844, N. S.?

Ans. 26th of January, 23rd of February, 30th of August, and 27th of December.

(5.) An elderly lady, speaking of her age, says, she was born in the year 1760, N. S., but does not know on what day of the month; she only recollects hearing her father say it was the second Wednesday in February; required the day of the month she was born? *Ans.* 13th.

(6.) In what years of the 19th century, New Style, does 29th of February fall on a Friday, being leap years?

Look in the table for 29, the given day of the month, and on the same horizontal line find Friday, the given day of the week; in the

* The New Style was first introduced into Rome by Pope Gregory XIII., on October 15th, 1582, and into the British Dominions by Act of Parliament, September 14th, 1752.

same vertical column find **F**, the given month, *viz.* *February*, leap year, then, in the same horizontal line, and in the vertical column of **2, N. S.**, will be found **A**, the Literal Index of the given year; enter this Index **A** in the first vertical column of letters, then, on the same horizontal line, will be found the years of the required dates less than one hundred, selecting only the leap years, *viz.* those which are dotted, we have '0, '28, '56, '84, whence we take 1800, 1828, 1856, 1884; however, 1800 being a common year must be excluded; and were it even a leap year, it would have been excluded, since it is not in the 19th century, which does not commence before 1st January, 1801. Hence, 1828, 1856, 1884 are the years required.

(7.) In what years of the 19th century, New Style, does Midsummer Day (24th June) fall on a Sunday?

Ans. 1804, 1810, 1821, 1827, 1832, 1838, 1849, 1855, 1860, 1866, 1877, 1883, 1888, and 1894.

(8.) Required the Dominical letters, for leap years and common years, when the 1st of January and February fall on a Sunday?

Find 1, the day of the month in the table, and on the same horizontal line find Sunday, the day of the week; in the same vertical column find **J**, for *January*, leap year; then, in the same horizontal line, in the first vertical column of letters will be found **G**, the Dominical letter required; similarly for *January*, in common years, will be found **A**, the required Dominical letter; similarly for **F**, or *February*, leap year, will be found the Dominical letter **C**, and for *February*, common years, the Dominical letter will be **D**.

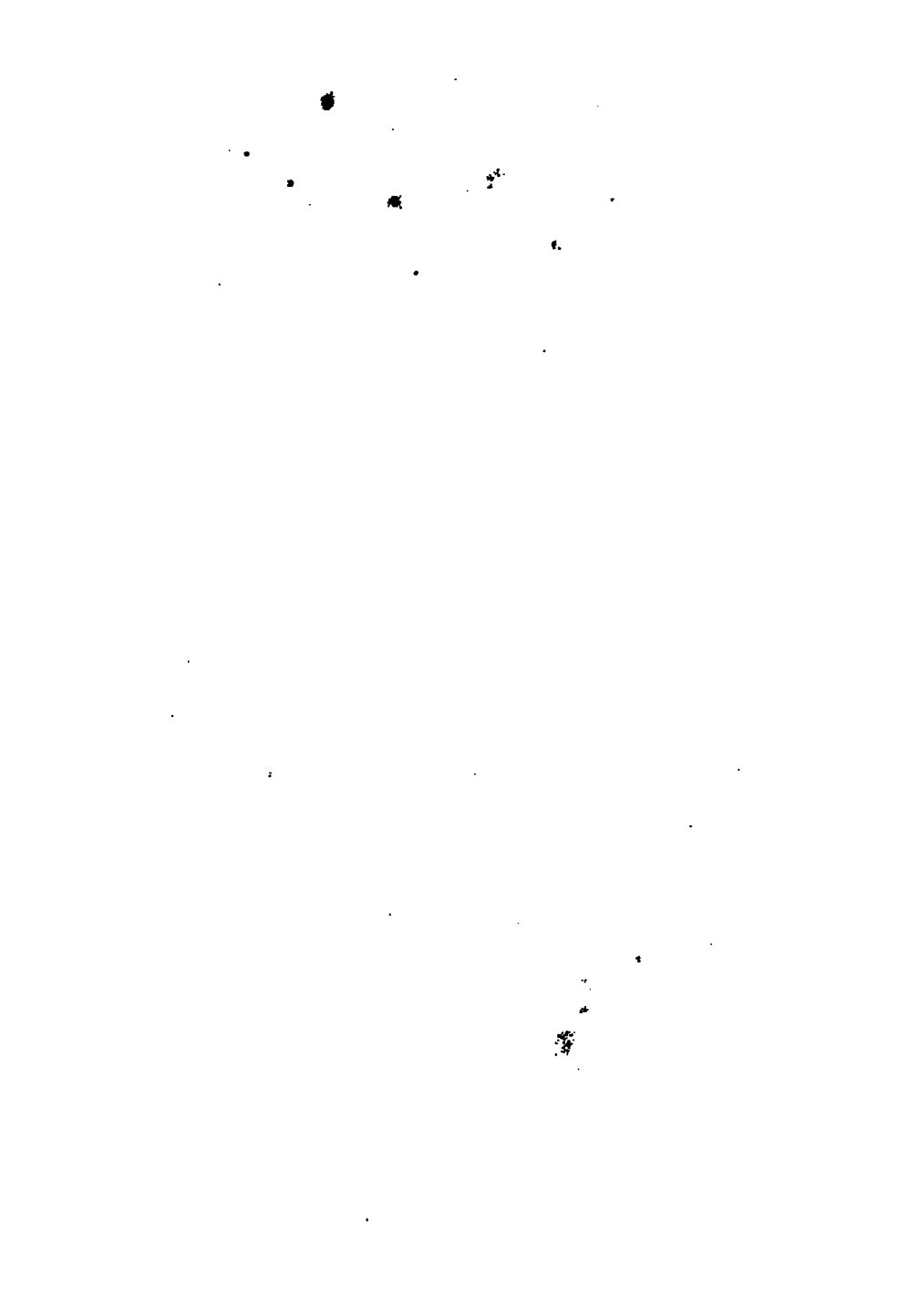
(9.) On what days of the week in the year 1724, Old Style, did 17th of *January*, leap year, and 16th December, fall? and in what other years of the 18th and 19th centuries will the same days of the month fall on the same days of the week?

Ans. In the year 1724, O. S., 17th of *January* happened on a Friday, and 16th December on a Wednesday. There is but one other leap year, O. S., *viz.* 1752, which in the 18th century answers the question, and only one leap year, N. S., in the 18th century, *viz.* 1772; similarly, in the 19th century N. S., the leap years required are 1812, 1840, 1868, and 1896; in all these instances 17th *January* falls on a Friday.

With regard to 16th December, all years before 1752 are to be taken by Old Style; these years are 1702, 1713, 1719, 1724, 1730, 1741, and 1747, O. S.; the years after 1751 are to be taken by New Style, which, in the 18th century, are 1761, 1767, 1772, 1778, 1789, and 1795. In the 19th century the years are 1801, 1807, 1812, 1818, 1829, 1835, 1840, 1846, 1857, 1863, 1868, 1874, 1885, 1891, and 1896, in all which cases 16th December falls on a Wednesday.

N.B. Those who may be desirous of acquainting themselves with further particulars on the perpetuity of the Civil and Ecclesiastical Calendars, may with advantage consult Bonnycastle's Arithmetic and Key, by Samuel Maynard, Edition 1851, and Key, 1852, and all subsequent editions.

THE END.





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